CUME 382

Useful values

Mass of the Sun = 2×10^{33} g
Luminosity of the Sun = 3.8×10^{33} ergs/sec
Mass of Hydrogen = 1.7×10^{-24} g
k = 1.4×10^{-16} ergs/K

Short answer questions (5 pts each)

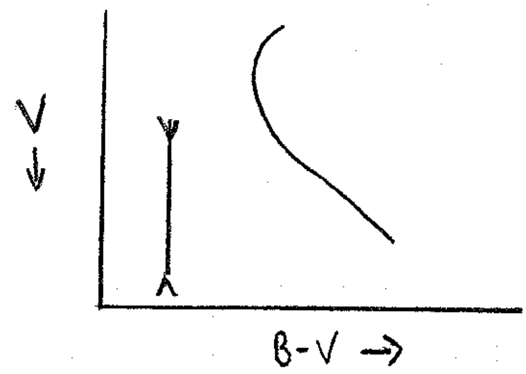
1. Describe the physical characterists of a white dwarf.
2. In your own words, define the term M_{crit} and then explain why it is of interest.
3. What are the effective wavelengths of the UBV filters?
4. Define "apparent distance modulus".
5. If M35 has an apparent distance modulus of 10.5 magnitudes and E(B-V)= 0.26, what is the distance to this cluster?

Paper specific questions

6. (20 pts) Explain, in your own words, what is meant by the expression "initial-final mass relation".
   a. How are the initial and final masses determined?
   b. Why are cluster's useful targets for the calibration of this relation?
7. (20 pts) Assume that a white dwarf has $M_{WD} = 1\ M_{\odot}$, $L = 0.01\ L_{\odot}$, a uniform temperature of 17×10^6 K, and is entirely composed of helium atoms. Derive an approximate expression for the cooling time and then compute its value.
8. (15 pts) Figure 1 shows the positions of the white dwarfs in M35. They are shown as fill-squares in the bottom-left of the figure. At a given B-V, it is not obvious which of these points designates the more massive white dwarf. This question allows us to predict where the more massive white dwarfs will be located relative to others at the same B-V.

   a. Write the equation of hydrostatic equilibrium.
   b. In a crude manner, how does the pressure P vary with M and R?
   c. The equation of state for a white dwarf is $P \propto \rho^{5/3}$. How does the radius of the white dwarf depend on its mass?

d. Suppose I have two cluster white dwarfs with the same B-V, indicate their relative positions along the vertical line in the below diagram and then explain how you determined these placements.
e. Explain why it would be difficult to observationally study massive white dwarfs in an old cluster.



9. (20 pts) This study studied a handful of white dwarfs in M35. This question asks you to estimate how many other white dwarfs might this cluster contain. Assume a Salpeter mass function of the form

$$dN/dM = \Phi(M) \propto M^{-2.4}$$

a. Derive an expression for the number of expected white dwarfs as a function of M_{crit}, $M_{turnoff}$, the minimum mass star in M35, M_{min}, and the number of stars, N_*, between $M_{turnoff}$ and M_{min}.
b. If $M_{crit} = 6.0$, $M_{turnoff} = 4.0$, $M_{min} = 0.4$ and $N_* = 320$, what is the predicted number of white dwarfs?

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AN EMPIRICAL INITIAL-FINAL MASS RELATION FROM HOT, MASSIVE WHITE DWARFS IN NGC 2168 (M35)

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ABSTRACT

The relation between the zero-age main-sequence mass of a star and its white dwarf remnant (the initial-final mass relation) is a powerful tool for the exploration of mass-loss processes during stellar evolution. We present an empirical derivation of the initial-final mass relation based on spectroscopic analysis of seven massive white dwarfs in NGC 2168 (M35). Using an internally consistent data set, we show that the resultant white dwarf mass increases monotonically with progenitor mass for masses greater than 4 $M_\odot$, one of the first open clusters to show this trend. We also find two massive white dwarfs foreground to the cluster that are otherwise consistent with cluster membership. These white dwarfs can be explained as former cluster members moving steadily away from the cluster at speeds of $\lesssim$0.5 km s^{-1} since their formation and may provide the first direct evidence of the loss of white dwarfs from open clusters. Based on these data alone, we constrain the upper mass limit of white dwarf progenitors to be $\geqq$5.8 $M_\odot$ at the 90% confidence level for a cluster age of 150 Myr.

Subject headings: open clusters and associations: individual (NGC 2168) — white dwarfs

1. INTRODUCTION

White dwarfs (WDs) are the final state of stellar evolution for the vast majority of intermediate- and low-mass stars. The upper mass limit of WD progenitor stars, M_{crit}, is also the lower mass limit of core-collapse supernova progenitors. The current best observational estimate for M_{crit} comes from spectroscopic analysis of WDs in the open cluster NGC 2516 (Koester & Reimers 1996) and is $M_{crit} \approx 8 \pm 2\ M_\odot$. Owing to the steepness of the initial mass function, however, this range results in a factor of $\approx$2 uncertainty in the number of supernovae and duration of supernova-driven winds resulting from bursts of star formation. This uncertainty in turn has a large impact on understanding the star formation rate in galaxies (e.g., Somerville & Primack 1999), the evolution of starbursts (e.g., Leitherer et al. 1999), and the fate of low-mass dwarf galaxies at early times (e.g., Dekel & Silk 1986).

The best observational constraints on M_{crit} are obtained from studies of WD populations in open clusters with ages $\lesssim$150 Myr (Williams 2002). NGC 2168 (M35) is one of the richest, compact, and nearby open clusters in this age range, with age determinations ranging from ~100 Myr (von Hippel et al. 2000) to ~200 Myr (Sung & Bessell 1999). The WD cooling sequence of NGC 2168 has been discussed often in the literature, with recent photometric analyses by von Hippel et al. (2002) and Kalirai et al. (2003).

With spectroscopy of cluster WD candidates, it is possible to determine unambiguously if the objects are WDs and, for the bona fide WDs, to determine T_{eff} and $\log g$ and to derive the cooling age (τ_{cool}), mass, and luminosity. For those WDs with cooling ages smaller than the cluster age and distance modulus consistent with cluster membership, subtraction of the WD cooling age from the age of the open cluster results in the lifetime of the progenitor star, and stellar evolutionary models can then be used to determine the progenitor star mass. Reimers & Koester (1988) applied this algorithm to two NGC 2168 WD candidates identified on photographic plates by Romanishin & Angel (1980); however, their signal-to-noise ratio was too low to determine the WD properties precisely.

As part of our ongoing program to identify and spectroscopically analyze the WD populations of open clusters, the Lick-Arizona White Dwarf Survey (LAWDS; K. A. Williams, M. Bolte, D. Koester, & M. A. Wood 2004, in preparation), we have obtained high signal-to-noise ratio spectra of eight candidate massive WDs in NGC 2168. Six or seven of the observed WDs are hot, high-mass WDs likely to be cluster members. This sample doubles the number of known open cluster WDs with high-mass progenitors. A detailed photometric and spectroscopic analysis of the entire WD population of this cluster will be presented in a later paper. In this Letter, we present the constraints on M_{crit} and the upper end of the initial-final mass relation for WDs based on the high-mass WDs already analyzed.

2. OBSERVATIONS AND ANALYSIS

UBV imaging of NGC 2168 was obtained in 2002 September with the Lick Observatory Shane 3 m reflector and the PFCam prime-focus imager; additional *UBV* imaging of a larger field center on the cluster was obtained in 2004 January with the KPNO 4 m MOSAIC camera. Point-spread function fitting photometry was obtained using the DAOPHOT II program (Stetson 1987). Candidate WDs were selected by their blue excess in *UBV* color space. Figure 1 shows the color-magnitude diagram of all objects detected in *U*, *B*, and *V* across the entire MOSAIC field. Several very blue, faint objects are observed in the diagram; these are our candidate WDs. All four WD candidates of Romanishin & Angel (1980) are recovered, as is the WD candidate in von Hippel et al. (2002). Astrometry and

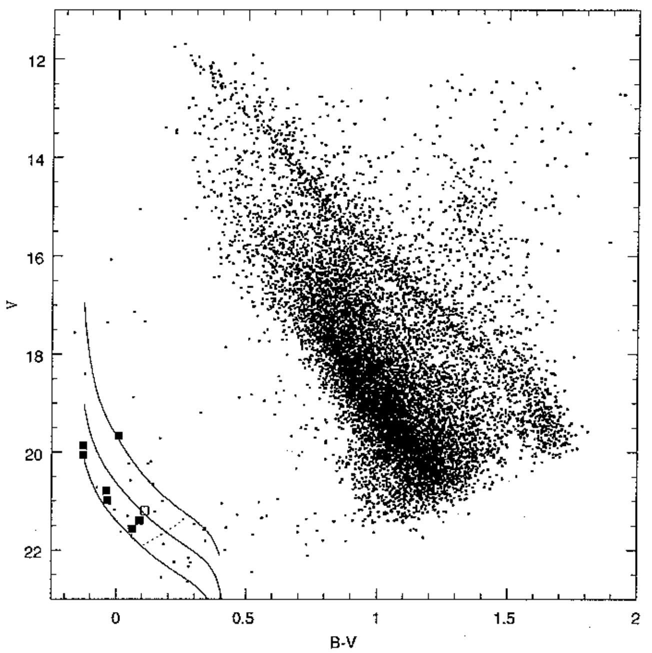

FIG. 1.—$B-V$, V color-magnitude diagram for NGC 2168. Squares indicate WDs presented in this study, both cluster members (*filled squares*) and the nonmember (*open square*). Solid curves indicate cooling curves for WDs with masses of 0.4 $M_\odot$ (*top curve*), 0.7 $M_\odot$ (*middle curve*), and 1.0 $M_\odot$ (*bottom curve*) at the distance and reddening of NGC 2168. The dotted line indicates the location of WDs with $\log \tau_{\rm cool} = 8.15$, the assumed age of NGC 2168.

photometry for the WD candidates in this Letter are presented in Table 1.

Spectroscopic observations of selected WD candidates were obtained with the blue camera of the Low-Resolution Imaging Spectrometer on the Keck I 10 m telescope (Oke et al. 1995). A 1″-wide long slit at parallactic angle was used with the 400 line mm^{-1}, 3400 Å blaze grism for a resulting spectroscopic resolution of ~6 Å. The spectra were extracted and a relative spectrophotometric calibration was applied using standard *IRAF* routines.

The $T_{\rm eff}$ and $\log g$ were determined for each WD using simultaneous Balmer line fitting (Bergeron et al. 1992). The model spectra are updated versions of those in Finley et al. (1997). The WD evolutionary models of Wood (1995) were used to calculate the mass ($M_{\rm WD}$) and cooling age ($\tau_{\rm WD}$) of each WD. A distance modulus to each WD was measured by comparing the observed V magnitude to the absolute magnitude M_V calculated from the best-fitting model atmosphere and the appropriate WD cooling model. Errors in the fits were determined empirically by adding the noise measured for each spectrum to the best-fitting model spectrum convolved with the instrumental response. These simulated spectra were fitted by the same method; nine iterations were used to calculate the scatter in $T_{\rm eff}$, $\log g$, $M_{\rm WD}$, and $\tau_{\rm WD}$. The fitting procedure is discussed in depth in an upcoming paper on the open clusters NGC 6633 and NGC 7063 (K. A. Williams, M. Bolte, D. Koester, & M. A. Wood 2004, in preparation). The atmospheric fits and derived WD masses and ages are given in Table 2. The Balmer line fits are shown in Figure 2.

A systematic error in the fits of the hottest ($T_{\rm eff} \gtrsim 50{,}000$ K) WDs became apparent, as it was not possible to simultaneously fit all the Balmer lines. In these cases, the fits were limited to Hβ, Hγ, and Hδ. For LAWDS 22, no satisfactory convergence was achieved with these limited fits; the best-fit models are used in the analysis. WDs with $T_{\rm eff} \gtrsim 50{,}000$ K are known to exhibit metals in the atmosphere and non-LTE effects (Napiwotzki 1992; Holberg et al. 1998), neither of which are included in the models.

The progenitor mass for each WD was calculated by subtracting the WD cooling age from the cluster age. The age difference is the total lifetime of the progenitor star. The lifetimes of stars as a function of mass and metallicity are calculated from the stellar isochrones of Girardi et al. (2002). The progenitor mass for each WD likely to be a cluster member is given in Table 3 for an assumed cluster age of 150 Myr and for three different stellar evolutionary models: $Z = 0.008$ and $Z = 0.019$, both with modest convective overshoot, and $Z = 0.019$, without convective overshoot. Upper and lower errors are for 1 σ differences in $\tau_{\rm WD}$.

3. DISCUSSION

All eight of the observed WDs are much more massive than the typical WD mass of ≈0.56 $M_\odot$ (Bergeron et al. 1992), and five of the WDs have apparent distance moduli $(m-M)_V \approx 10.5$. It is therefore reasonable to assume that at least five of these objects are members of NGC 2168. LAWDS 11 is almost certainly *not* a cluster member. Its age is likely older than that of the cluster as a whole, and while the uncertainties in the spectral fits leave open the possibility that it is younger (which would require it to be hotter), the observed colors are more consistent with the cooler (and older) interpretation. The distance modulus of this WD is foreground to the cluster by little more than 1 σ, but the cooler temperature favored by the $B-V$ color again favors the foreground interpretation.

LAWDS 15 has a mass and age consistent with cluster membership, but the calculated distance modulus is inconsistent with that of the other WDs by ~$4\sigma_{M_V}$. Assuming that the distance modulus is correct and that there is no difference in reddening

TABLE 1
WDs IN THE FIELD OF NGC 2168: PHOTOMETRY

LAWDS Identification	McCook & Sion Designation[a]	R.A. (J2000.0)	Decl. (J2000.0)	V	σ_V	$B-V$	σ_{B-V}	Previous References
NGC 2168: LAWDS 1	WD J0608+242	6 08 38.79	24 15 06.9	20.989	0.019	−0.035	0.028	1
NGC 2168: LAWDS 2	WD J0608+241	6 08 42.30	24 10 17.7	21.569	0.032	0.061	0.044	
NGC 2168: LAWDS 5	WD J0609+244.1	6 09 11.54	24 27 20.9	20.065	0.017	−0.128	0.024	1, 2
NGC 2168: LAWDS 6	WD J0609+244.2	6 09 23.48	24 27 22.0	19.863	0.016	−0.128	0.023	1, 2
NGC 2168: LAWDS 11	WD J0609+241	6 09 42.79	24 11 05.4	21.198	0.025	0.110	0.037	
NGC 2168: LAWDS 15	WD J0609+240	6 09 11.63	24 02 38.5	20.785	0.022	−0.039	0.032	
NGC 2168: LAWDS 22	WD J0608+245	6 08 24.65	24 33 47.6	19.657	0.016	0.008	0.023	
NGC 2168: LAWDS 27	WD J0609+243	6 09 06.26	24 19 25.3	21.398	0.026	0.090	0.039	3

NOTE.—Units of right ascension are hours, minutes, and seconds, and units of declination are degrees, arcminutes, and arcseconds.
[a] Format as in McCook & Sion (1999).
REFERENCES.—(1) Romanishin & Angel 1980; (2) Reimers & Koester 1988; (3) von Hippel et al. 2002.

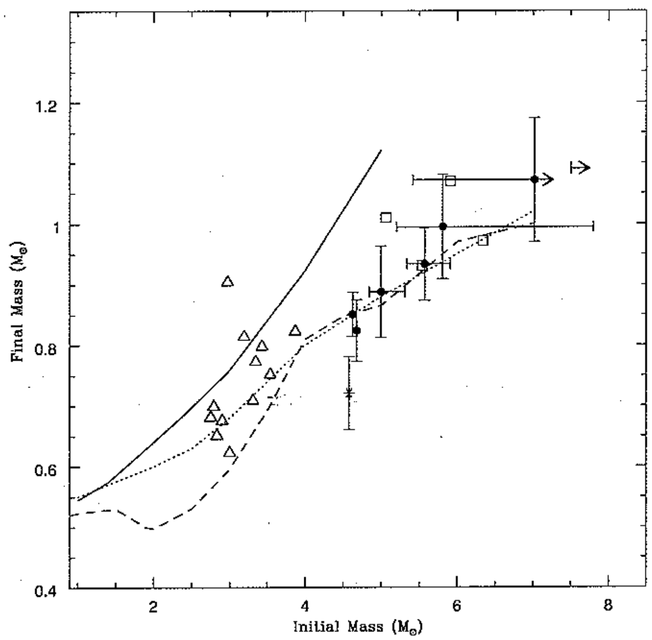

FIG. 3.—Initial-final mass relation. Filled circles with error bars are WDs from this study; other symbols are from the Hyades and Praesepe (Claver et al. 2001; *open triangles*), NGC 2516 (Koester & Reimers 1996; *open squares*), and the Pleiades (Claver et al. 2001; upper limit). The asterisk with error bars is LAWDS 22, discussed in the text. Curves represent the theoretical initial-final mass relation from Girardi et al. (2000; solid curve), the core mass at the first thermal pulse from the same models (*dashed curve*), and the quasi-empirical relation from Weidemann (2000; *dotted line*). Slight horizontal offsets have been applied to the three WDs with $M_{\rm init} \approx 4.6\ M_\odot$ for the sake of clarity. Statistical error bars in the initial masses for these three WDs are smaller than the points.

Based on the NGC 2168 data alone, it is possible to place lower limits on the value of $M_{\rm crit}$. Making the assumption that errors in the WD ages in Table 2 are Gaussian, we calculate that the oldest cluster WD ages are $\log \tau_{\rm cool} \gtrsim 7.76$ with 90% confidence. For $\log \tau_{\rm cl} = 8.15$ and $Z = 0.008$, this corresponds to $M_{\rm crit} \geq 5.81\ M_\odot$ at a 90% confidence level. This value is in agreement with that obtained by Koester & Reimers (1996).

Improved constraints on the initial-final mass relation, including its intrinsic scatter and metallicity dependence, require improvements on existing observations. First and foremost, the ages of open clusters such as NGC 2168 must be determined to higher precision. Alternatives to main-sequence fitting such as lithium depletion studies or activity/rotation studies may provide the necessary constraints. Second, large samples of WDs from individual open clusters are needed to reduce the effect of systematics (such as errors in assumed ages) that plague the comparison of intercluster samples. The WD sample presented here is a start to that end, and the initial-final mass relation derived from these stars alone provides dramatic confirmation of the existence of an initial-final mass relation, an idea that was strongly hinted at from previous open cluster studies and from other theoretical and observational work. Assuming that the majority of the remaining massive NGC 2168 WD candidates are cluster WDs, planned spectroscopic observations of these objects will soon result in a sample of nearly a dozen WDs originating from a single stellar population, permitting, for the first time, studies of the intrinsic dispersion of the initial-final mass relation at high masses.

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TABLE 2
WDs in the Field of NGC 2168: Spectral Fits

Identification	$T_{\rm eff}$ (K)	$dT_{\rm eff}$ (K)	$\log g$	$d \log g$	$M_{\rm WD}$ ($M_\odot$)	$dM_{\rm WD}$ ($M_\odot$)	$\log \tau_{\rm WD}$	$d \log \tau_{\rm WD}$	M_V	dM_V	$(m - M)_V$
LAWDS 1	32400	512	8.40	0.125	0.888	0.075	7.386	0.208	10.454	0.215	10.535
LAWDS 2	31700	1800	8.74	0.191	1.072	0.102	7.929	0.291	11.132	0.328	10.437
LAWDS 5	52600	1160	8.24	0.095	0.824	0.051	6.158	0.025	9.542	0.184	10.523
LAWDS 6	55200	897	8.28	0.065	0.851	0.036	6.094	0.027	9.564	0.108	10.299
LAWDS 11	19900	2792	8.48	0.367	0.921	0.213	8.288	0.463	11.534	0.769	9.664
LAWDS 15	29900	318	8.48	0.060	0.934	0.037	7.693	0.089	10.754	0.111	10.031
LAWDS 22	54400	1203	8.04	0.121	0.721	0.060	6.169	0.054	9.155	0.219	10.502
LAWDS 27	30900	500	8.58	0.164	0.995	0.086	7.760	0.220	10.866	0.310	10.532

between LAWDS 15 and NGC 2168, LAWDS 15 is ~185 pc closer than the cluster. Based on the spectral fits, LAWDS 15 has a cooling age of ~50 Myr. If the WD has been moving away from the cluster since its formation at a steady rate of only 0.4 km s^{-1}, it will have covered this distance. Therefore, it is possible that LAWDS 15 was once a cluster member and has escaped the cluster.

The likelihood that a massive, hot WD would be found foreground along the line of sight to the cluster can be estimated from the luminosity function in Figure 16 of Liebert et al. (2004). The luminosity function gives the space density of WDs with $M > 0.8\ M_\odot$ and $\tau_{\rm cool} \le 100$ Myr as $\sim 10^{-5.3}$ pc^{-3} 0.5 mag^{-1}. This results in an estimated 0.1 hot, massive WDs in the ~30′ × 30′ MOSAIC field to a distance of 1 kpc. Therefore, it is unlikely, but not impossible, that LAWDS 15 is a field WD. Based on these arguments, we will consider LAWDS 15 to be a cluster WD for this discussion. For similar arguments, we retain LAWDS 6 ($\sim 2\sigma_{M_V}$ closer than the other cluster WDs) as a likely cluster member.

Figure 3 shows the initial-final mass relation of these seven cluster members, along with that of WDs from the Hyades, Praesepe, and the Pleiades (Claver et al. 2001) and from NGC 2516 (Koester & Reimers 1996). Also shown are theoretical and semiempirical data from plots in Claver et al. (2001) and sources therein. For the sake of consistency, the initial and final masses of each WD from the literature have been redetermined using our WD models and the published $T_{\rm eff}$ and $\log g$.

From the figure, it can be seen that the NGC 2168 WDs form a monotonic sequence of more massive WDs originating from more massive progenitors. This conclusion is robust, as changes in the assumed age of NGC 2168 do not affect the relative positions of the points, only the absolute initial masses. Figure 3 also shows that the NGC 2168 initial-final mass relation agrees with the empirical relation derived from previous clusters. This need not be expected a priori, as NGC 2168 has a significantly lower metallicity than the other clusters in the diagram. Model core masses decrease with increasing metallicity, and the efficiency of mass-loss processes could change with metallicity.

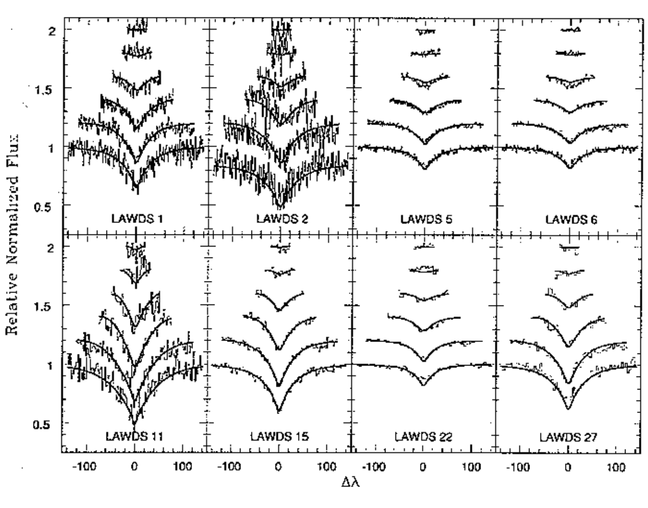


Fig. 2.—Balmer line fits for WD candidates in NGC 2168. The panels show the Balmer lines from Hβ (*bottom row*) to H9 (*top row*), with the curves showing the best-fit models.

LAWDS 15 and LAWDS 6, the potential escaped cluster WDs discussed above, fit the observed initial-final mass relation, providing additional evidence that they are cluster members. If these WDs are indeed escaped cluster members, they are crucial pieces of evidence that WDs can receive velocity kicks during mass loss and perhaps explain the observed deficit of WDs in other open clusters (see Williams 2004 and references therein). This and other potential explanations for these objects (e.g., binarity) will be discussed more fully in the later paper on WDs in NGC 2168.

The WD in Figure 3 with the apparently low final mass is LAWDS 22. As mentioned above and visible in Figure 2, the spectral fitting did not converge satisfactorily, despite the high signal-to-noise ratio of the spectrum. This star is a close visual double with a redder companion ($V = 19.12$, $B - V = 1.27$, $U - B = 0.93$) 2″ to the north. While resolved, this double is close enough that the spectrum of LAWDS 22 is likely contaminated by light from the neighbor star, resulting in the unsatisfactory fit. Light from the neighboring star may also be contaminating the photometric colors of LAWDS 22, which would also explain why the star lies redward of the 1 $M_\odot$ cooling track in Figure 1. Given its measured $T_{\rm eff}$, LAWDS 22 likely suffers from an extension of the systematic issue in the high-$T_{\rm eff}$ WD spectral fits described above. Other explanations for the location of this point could include magnetic fields (although no splitting is observed) or a low-mass, unresolved companion, but contamination by the neighboring star seems to be the most likely cause.

TABLE 3
Progenitor Masses for NGC 2168 WDs

Identification	$Z = 0.008$ ($M_\odot$)	$Z = 0.019$ ($M_\odot$)	$Z = 0.019$[a] ($M_\odot$)
LAWDS 1	$5.00^{+0.31}_{-0.16}$	$5.08^{+0.29}_{-0.15}$	$4.86^{+0.28}_{-0.14}$
LAWDS 2	$7.02^{+\infty}_{-1.60}$	$6.96^{+\infty}_{-1.49}$	$6.63^{+\infty}_{-1.40}$
LAWDS 5	$4.628^{+0.001}_{-0.001}$	$4.734^{+0.002}_{-0.001}$	$4.540^{+0.001}_{-0.001}$
LAWDS 6	$4.625^{+0.001}_{-0.001}$	$4.732^{+0.001}_{-0.001}$	$4.537^{+0.001}_{-0.010}$
LAWDS 15	$5.57^{+0.34}_{-0.24}$	$5.62^{+0.31}_{-0.22}$	$5.37^{+0.30}_{-0.21}$
LAWDS 22	$4.629^{+0.003}_{-0.003}$	$4.735^{+0.003}_{-0.002}$	$4.540^{+0.003}_{-0.002}$
LAWDS 27	$5.81^{+1.99}_{-0.61}$	$5.84^{+1.81}_{-0.56}$	$5.59^{+1.46}_{-0.54}$

[a] No convective overshoot.

Answers

1 A white dwarf is a star that consists of electron-degenerate matter. They are a final endpoint of stellar evolution and have masses $\sim 0.6 M_\odot$, $L \sim 10^{-2} L_\odot$ and $T_{eff} \sim 7000-50{,}000\,K$

2 M_{crit} is the highest mass star that can become a white dwarf. Above this value the star is a supernova candidate

3 $U \sim 3600\,Å$
$B \sim 4400\,Å$
$V \sim 5400\,Å$

4 Apparent distance modulus = reddened value of true distance modulus. It has not accounted for interstellar extinction.

5 If M35 has an apparent distance modulus of $(m-M)_V = 10.5$, its true modulus is $10.5 - 3.2E(B-V) =$

$$10.5 - 3.2(0.26) = 9.67$$

This corresponds to a distance of $9.67 = 5\log\frac{d}{10} =$

$$d = 10^{\frac{9.67}{5}+1} = 859\,pc.$$

6 The initial-final mass relation relates the white dwarf mass [final mass] to the mass of its main sequence progenitor. It does not address how mass is lost of when it was lost.

a final mass → determined from model fits to position in CMD and measurements of g

initial mass → use difference in cooling time and cluster age to identify progenitor MS star with this lifetime

b For a cluster the distance is known and L can be determined. Reddening somewhat easier to measure, but not a major pt.

7. Cooling time calculation (Estimate)

Heat source of white dwarf is thermal motion

$$E = \frac{3}{2} kT \quad \text{per atom}$$

$$\#\text{ of atoms in the star is } \frac{M_{WD}}{\text{average mass per atom}} = \frac{M_{WD}}{4 \cdot M_H}$$

Cooling time is approximately given by

$$t_{cool} \sim \frac{\frac{3}{2} kT \frac{M_{WD}}{4M_H}}{L_{WD}} \quad \text{[This is off by about a factor of 3]}$$

$$t_{cool} \quad \frac{3}{2}\left[\frac{\frac{1.4\times10^{-16} \times 17\times10^{6} \times 2\times10^{33}}{4 \cdot 1.67\times10^{-24}}}{3.8\times10^{+33} \times 10^{-2}}\right] = \frac{2.8\times10^{16}\,\text{sec}}{3\times10^{7}\,\text{sec/yr}}$$

$$\sim 0.9\times10^{9}\,\text{years} = 9\times10^{8}\,\text{years}$$

8 a) $\frac{dP}{dr} = -\frac{G M_r \rho}{r^2}$

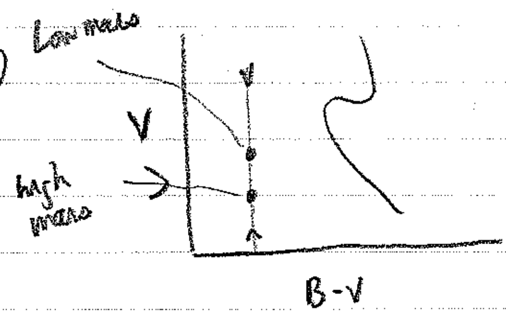
b) $P \sim \frac{M}{R^2}\frac{M}{R^3} R \sim \frac{M^2}{R^4}$

c) $P \sim \rho^{\frac{5}{3}} \sim \left(\frac{M}{R^3}\right)^{\frac{5}{3}} = \frac{M^{5/3}}{R^5}$

$$\frac{M^2}{R^4} \sim \frac{M^{\frac{5}{3}}}{R^5} \Rightarrow R \sim M^{-\frac{1}{3}}$$

$R \sim M^{-\frac{1}{3}}$ higher mass white dwarfs have smaller radii

d)



$L \sim R^2 T_e^4$ same B-V $\Rightarrow$ same T_{eff}
$\therefore$ L depends on R. Since $R \sim M^{-\frac{1}{3}}$
a larger M means a smaller R, L and V.

e. Older white dwarfs will be farther down (fainter V) on their cooling curve. High mass white dwarfs will be faint

9 Computing M35 white dwarf population

(a) $$N_* = \int_{M_{min}}^{M_{to}} \Phi(M)\,dM = \int_{M_{m}}^{M_{to}} M^{-2.4}\,dM = \left[M_{min}^{-1.4} - M_{to}^{-1.4}\right] C$$

$$N_{WD} = \int_{M_{to}}^{M_c} \Phi(M)\,dM = \int_{M_{to}}^{M_c} M^{-2.4}\,dM = \left[M_{to}^{-1.4} - M_{crit}^{-1.4}\right] C$$

Combining

$$N_{WD} = N_* \frac{M_{to}^{-1.4} - M_{crit}^{-1.4}}{M_{min}^{-1.4} - M_{to}^{-1.4}}$$

Using numbers provided in question yields

$N_{WD} \sim 5$ i.e. close to the 4 already detected.