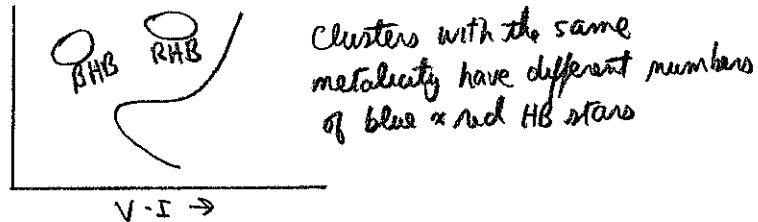


1. 10 pts. What is the second parameter problem? Illustrate this problem using a sketch of a color-magnitude diagram.

Globular clusters with similar compositions have different HB morphologies. The difference highlighted here is the number of red and blue HB stars.



2. 5 pts. What was the rationale for selecting NGC 288 and NGC 362 for this study?
The compositions of these two clusters are believed to be similar thus eliminating a possible second parameter. Globular clusters can vary in $[Fe/H]$ by more than a factor of 10.
3. 5 pts. Why are the authors troubled about NGC 288 and what impact might this problem have on their conclusions?

Figure 4 shows that the model fit to the HB of NGC 288 is not very good when compared to the fit for NGC 362. If the model is incorrect, its predictions about the morphology of the HB are also suspect. The comparison of model and observational data in Figure 5 would then be questionable.

4. 10 pts. Are the authors suggesting that the mass of the HB stars in these two clusters should be the same, that all HB stars have the same mass? Offer an explanation about why the masses of these stars might differ.

No, they are not suggesting all HB stars have the same mass although the range of these masses is likely to be small. NGC 288 is about 2 Gyr older than NGC 362. Therefore, the turnoff mass for NGC 288 is lower than for NGC 362. The HB masses in these clusters differ by about 0.1 solar masses. This difference could be explained by the higher mass of the progenitor stars in NGC 362.

A. Subject matter questions

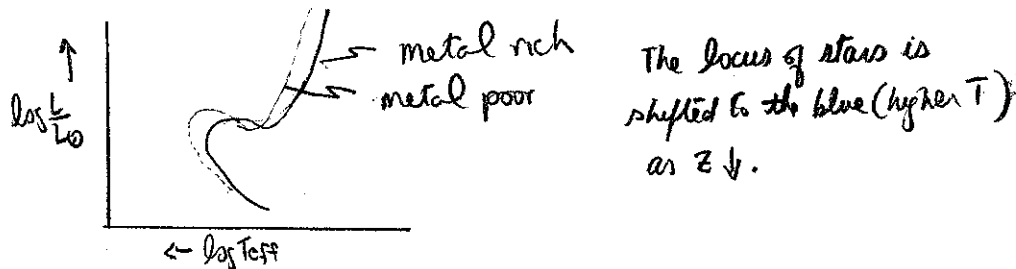
1. 40 pts. Cluster distances and ages.
 - a. 5 pts. Using the values given in the upper left panel of Figure 1 compute the distance to NGC 362. Assume $E(B-V) = E(V-I)$. Using Figure 1 as a guide, what other method could be used to determine the distance to this cluster?

Apply $(m-M)_v = 5 \log(d/10) + A_v$ $A_v = 3.1E(B-V)$. Figure 1 gives $(m-M) = 14.79$ and $E(V-I) = 0.04$. When completed, this computation should yield $d = 6.9$ kpc. Figure 1 also shows a population of RR Lyrae stars. An alternate estimate of the cluster distance could be obtained by measuring the light curves of its RR Lyrae stars and then using their period-luminosity relation.

- b. 10 pts. How can one measure the reddening to an open cluster using color indexes from for example the UBV system? Why wouldn't this work for a globular cluster? How might the reddening to a globular cluster be measured?

One could use Johnson U-B vs B-V color plot and the relation $E(U-B)/E(B-V)=0.72$ to determine the cluster's reddening. It would be difficult to use this method for a globular cluster because their high mass stars have left the main sequence. One would be forced to use lower mass main sequence stars where the two color plot becomes doubly or triply degenerate in yielding a reddening. Other techniques for measuring the reddening include using the blue and red boundaries of the RR Lyrae instability strip or using IR dust maps or radio measurements of neutral hydrogen. These alternatives are subject to rather large errors.

- c. 5 pts. Sketch the color-magnitude diagram of a metal poor and rich globular cluster.



- d. 5 pts. HB branch evolution occurs after the "Helium flash". What is this, why does it occur and do all stars undergo this event?

The helium flash occurs when helium burns into carbon in the degenerate core of the red giant. This event marks the end of the red giant phase. A flash occurs because the degenerate matter does not respond to increases in temperature as an ideal gas. This leads to an explosive core event. Upper and low mass main sequence stars do not undergo a helium flash. The former stars do not burn helium in a degenerate core and the lower main sequence stars do not reach a high enough core temperature.

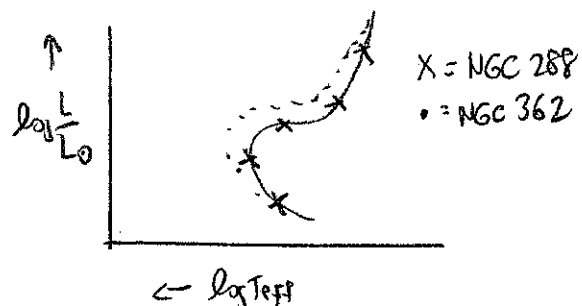
- e. 5 pts. What is the typical age and metal abundance of a globular star cluster?

Typical values are 10^{10} years and $Z = 0.001$ to 0.002 .

- f. 10 pts. Suppose that NGC 362 were younger than NGC 288 by 2 Gyr, but they both had identical compositions. Using crosses for NGC 288 and dots for NGC 362, sketch the color magnitude diagram of these two clusters (in a single figure) from the main sequence to the top of the red giant branch. Which region is most sensitivity to age? Discuss one complication in using this feature to determine the age of a globular cluster.

The region of highest sensitivity is at the cluster turnoff. Complications using this region include metallicity differences compared to that used to construct the isochrones. Alternately binaries broaden the turnoff location and make fits to it more difficult

The patterns join on the main sequence and giant branch. Elsewhere the younger cluster's pattern is above that of the older cluster.



2. 10 pts. Mass segregation

- a. What is mass segregation and why does it occur?

The higher the mass of the stellar group, the more concentrated these stars are toward the cluster center. Mass segregation forms as a result of stellar interactions. The velocity of the lower mass stars increase through interactions with higher mass stars. These interactions send lower mass stars into a larger orbit. They can even be ejected from the cluster. The higher mass stars to move toward the cluster center.

- b. Since only the star's magnitudes and colors are usually measured, how is mass segregation measured?

To quantify this phenomenon in a star cluster one must measure a magnitude, color, and position. The magnitude and color are used in conjunction with the HR diagram to provide a mass. Stars within a mass range are then identified in the cluster and counted as a function of r . One then plots a normalized histogram of the number of stars in a series of radial bins versus r . Alternately, a plot of the cumulative distribution (normalized sum of the number of stars within a mass range versus r) is often used.

3. 10 pts. Stellar structure

Iben and Renzini (1984) find that the mass of the helium core of a star at the onset of the helium flash is

$$M_{\text{core}} = 0.476 - 0.221(Y-0.3) - 0.009(3+\log(Z)) - 0.023(M-0.8)$$

where M is in progenitor main sequence mass in solar units, Y and Z are the mass fractions. For reasonable values of Y , Z , and M , what is the predicted value of the core mass of a HB star? Using $Z=0.001$, the authors find that NGC 362 and NGC 288 have $\langle M_{\text{HB}} \rangle = 0.667$ and 0.594 solar masses respectively. Suppose a HB star had a constant density. What is the core's radius in terms of the star's radius? What does this suggest the next evolutionary state of a low mass HB star might be?

Apply the above relation with values provided in Figures 1 or 4 for Y and Z . Assume M is near 0.8 since the HB mass is close to this value and it is multiplied by a small number, 0.023 .

$$\text{Predicted core mass} = 0.476 - 0.221(.23-.3) - 0.009(0) - 0.023(0) = 0.478$$

The HB mass the authors derive was about 0.6 solar masses. If the HB star had a constant density, then $M(r) = \int 4\pi r^2 \rho dr$. For r = core radius, and R = radius of the star, one has $M_c/M_R = (r_c/R)^3$ or $r_c = (.48/.60)^{1/3} R = 0.93 R$.

This star is essentially all core. It suggests two type of objects, a white dwarf or an AGB star.

4. 10 pts. Mass loss

- a. Suppose a solar-like star spends about 10% of its lifetime on the giant branch. If all of the mass loss after leaving the main sequence occurred on the giant branch, what does this paper suggest the mass loss rate is on the giant branch?

The age of a globular cluster is about 10^{10} years, so 10% of this lifetime of 10^9 years. The HB mass they derived, 0.6 solar masses, suggests that about 0.6 solar masses has been lost since the star left the main sequence. About 0.4 solar masses would have been

lost for a solar like star. The mass loss rate would be $0.4/10^9 = 4 \times 10^{-10}$ solar masses/yr.

- b. Assume the giant star mass loss rate is $dM/dt = 1.5 \times 10^{-14}$ LR/M solar masses per year. where L, R and M are in solar units. What absolute magnitude will the Sun have when it becomes a giant with $R = 80$ solar radii? What is its expected effective temperature?

From the last question, $dM/dt = 4 \times 10^{-10}$ solar masses/yr $= 1.5 \times 10^{-14}$ LR/M. Take $M = 0.8$ as the average mass along the RGB, $R = 80$, and solve for L. This gives $L = 266$ solar luminosities.

The absolute mag will be $2.512^{\Delta m} = 266$, $\Delta m = 6.0$ so $M = 4.7 - 6.0 = -1.3$. The expected effective temperature is

$T/T(\text{sun}) = [L/L(\text{sun}) \times (R(\text{sun})/R)^2]^{1/4}$ or $T = 0.45 T(\text{sun}) = 2,600\text{K}$ (a low value indicating level of uncertainty in the proportionality constant)