Bernie

Cum #326

This cume is based on the paper "Which Globular Clusters Contain Intermediate Mass Black Holes" by Baumgardt, Makino and Hut 2005 ApJ 620,238

Astronomical constants
$G = 6.67 \times 10^{-8}$ cm^3/(g-sec^2), 1 pc = 3.1×10^{18} cm, 1 solar mass = 2×10^{33} g

Some parts of the paper have been omitted to save you reading time and to avoid influencing your opinion on various cume questions. Particularly relevant sections to the below questions are noted in the paper's margins.

1. (10 pts) In the Introduction to this paper, the authors give four lines of evidence supporting the existence of intermediate mass black holes.
   a. 4pts) Offer a counter argument to their suggestion about the Magorrian relation.
   a. 6 pts) Why would high luminosity and strong variability suggest that the ULXs are IMBHs?

2.(15pts) The mass of the central region of a GC largely consists of neutron stars and heavy white dwarfs. This is somewhat surprising since the cluster stellar population is dominated by lower mass stars. This phenomenon is called mass segregation. Discuss how mass segregation is achieved in a cluster.

3. (15pts) The authors define the characteristics of the GCs they will model in the section of the paper marked by the number 3 (see attachment). Do globular clusters have 4.5×10^4 solar masses. Isn't 250 solar masses too low for an IMBH? Explain this choice of parmeters.

4. (15 pts) What point are the authors making with Figure 1b?

5. This question is worth 20 points
   a. 5 pts) Use the virial theorem to predict the slope of the dotted line in Figure 3.
   b. 10 pts) Show that the random velocity of a star used in the virial theorem, V, and a star's escape velocity from the cluster, v, are related as v = 2V.
   c. 5 pts) Assuming a reasonable cluster mass and a radius R = 5 pc, estimate V.

6. (25 pts) Table 2 provides a list of GC candidates that could contain an IMBH. Suppose you wished to pick the best candidate from this list to conduct a search for an IMBH. Your study will use radial velocities obtained from a ground-based telescope. Which object would you select and why? Based on 5c, how many stars would have to be measured to achieve a S/N = 50 in V?

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WHICH GLOBULAR CLUSTERS CONTAIN INTERMEDIATE-MASS BLACK HOLES?

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ABSTRACT

1. INTRODUCTION

Over the last few years, four lines of evidence have accumulated pointing to the possible presence of a $\sim 10^3\ M_{\odot}$ black hole in some globular clusters. The first hint follows from an extrapolation of the M_{BH}–M_{bulge} relation found for supermassive black holes in galactic nuclei (Magorrian et al. 1998), which leads to a prediction of a typical central black hole mass of $\sim 10^3\ M_{\odot}$ for globular clusters (Kormendy & Richstone 1995; van der Marel 2001). The empirical M_{BH}–M_C relation also comes naturally from rapid mass segregation and the Spitzer instability applied to a standard initial mass function (IMF) in young, dense star clusters (Gürkan et al. 2004).

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The second hint is related to the discovery of a new class of ultraluminous, compact X-ray sources (ULXs). Their high luminosities and strong variability suggest that they are intermediate-mass black holes (IMBHs), rather than binaries containing a normal stellar-mass black hole, and they may occur preferentially in young star clusters (Zezas et al. 2002).

The third hint stems from an analysis of the central velocity dispersions of specific globular clusters. Gerssen et al. (2002, 2003) and Gebhardt et al. (2002) have published evidence for black holes in M15 and G1 with masses of the order of 10^3 and $10^4\ M_{\odot}$, respectively (since M31's G1 is 1 order of magnitude more massive than typical globular clusters in our Galaxy, both values fall on the M_{BH}–M_{bulge} relation).

The fourth hint is based on detailed N-body simulations by Portegies Zwart et al. (2004) of the evolution of a young (~ 10 Myr) star cluster in M82, the position of which coincides with an ULX with luminosity $L > 10^{40}$ ergs s^{-1}. They found that runaway merging of massive stars could have led to the formation of an IMBH of $\sim 10^3\ M_{\odot}$. Since globular clusters in their youth may have resembled this type of star cluster, it is altogether likely that at least some globular clusters harbor IMBHs.

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2. MODELING METHOD

The reason why clusters with unresolved cores have been the primary candidate for harboring an IMBH is that there

TABLE 1
Results of the N-Body Runs

M_{BH} ($M_\odot$)	N	W_0	$R_{h,\text{init}}$ (pc)	$M_{c,\text{fin}}$ ($M_\odot$)	$R_{h,\text{fin}}$ (pc)	$R_{h,\text{pro}}$ (pc)	R_C (pc)	$R_C/R_{h,\text{pro}}$	$\log T_{r,h}$ (yr)
125	65536	5	2.03	21451.3	10.12	4.14	0.55	0.13	9.54
125	65536	9	2.03	21749.6	12.34	6.20	0.65	0.11	9.67
125	131072	7	4.91	45534.8	12.31	5.98	0.81	0.14	9.82
250	131072	7	4.91	45311.2	12.60	6.46	0.71	0.11	9.84
500	131072	7	4.91	44771.1	13.70	7.76	0.64	0.08	9.89
1000	131072	7	4.91	45300.4	14.07	7.96	0.58	0.08	9.91

should be a density cusp with $\rho \propto r^{-7/4}$ around the black hole. The formation of such a cusp was first predicted by Bahcall & Wolf (1976) and later confirmed by numerical simulations (Cohn & Kulsrud 1978; Marchant & Shapiro 1980; Baumgardt et al. 2004a; Preto et al. 2004). The projected density profile therefore should have a cusp with slope $-3/4$.

2 A cusp in density, however, does not necessarily imply the existence of a cusp in luminosity, since there is no guarantee that M/L is constant. Quite the contrary, numerical simulations of core collapse have demonstrated that in post–core-collapse clusters, M/L shows a sharp rise toward the center: neutron stars and heavy white dwarfs dominate the central regions as a result of mass segregation (Baumgardt et al. 2003a). A similar rise in M/L must exist in the density cusp around an IMBH.

In previous studies, Baumgardt et al. (2004a, 2004b) have followed the evolution of star clusters with central black holes with masses of 1%–10% of the cluster mass. They found a distinct density cusp around the central black hole but no clear luminosity cusp, since the central cusp is dominated by remnant stars. The projected luminosity profile was effectively flat at the center, and the evolved clusters looked just like normal King clusters. Unfortunately, the black hole mass used in Baumgardt et al. (2004b) was too large to allow a direct comparison with observations of globular clusters. In this paper, we report the results of new simulations, starting with a more realistic central black hole with a mass of 0.1%–1% of the total cluster mass.

The setup of our runs is similar to that of the runs made by Baumgardt et al. (2004a, 2004b), and we refer the reader to these papers for a detailed description. We simulated the evolution of star clusters using the collisional N-body program NBODY4 (Aarseth 1999) on the GRAPE-6 computers at Tokyo University. Our simulations include two-body relaxation, stellar evolution, and the tidal disruption of stars by the central black hole. Initially, no binaries were present in our models. Each cluster started with a spectrum of stellar masses between 0.1 and 30 $M_\odot$, distributed according to a Kroupa (2001) mass function, and massive central black holes initially at rest at the cluster center. The initial density profile for most models was given by a King $W_0 = 7$ model with half-mass radius $R_h = 4.8$ pc. Stellar evolution was modeled according to the fitting formulae of Hurley et al. (2000), assuming a retention fraction of neutron stars of 15%. Simulations were run for $T = 12$ Gyr.

If the M_{BH}–M_{bulge} relation found by Magorrian et al. (1998) for galactic nuclei holds for globular clusters as well, the mass expected for the central black hole in an average globular cluster of mass $M = 1.5\times10^5\ M_\odot$ would be around 1000 $M_\odot$. Black hole masses of 10^3–$3\times10^3\ M_\odot$ were also found as the end result of runaway merging of massive stars in the dense star cluster MGG-11 by Portegies Zwart et al. (2004), although their values are likely to be upper limits, since stellar mass loss of the runaway star was not included.

Since it is not yet possible to perform a full N-body simulation of a massive globular cluster over a Hubble time, we have to scale down our simulations. Scaling down can be achieved by simulating either a smaller N cluster while keeping the mass of the central black hole unchanged, as done by Baumgardt et al. (2004b), or by scaling down the black hole mass and the cluster mass simultaneously while keeping the ratio of both constant. The first method has the advantage that the ratio of black hole to stellar mass is the same as in a real cluster, allowing a study of black hole wandering and relaxation processes in the cusp around the central black hole, while the second method is most suitable to compare the final velocity and density profile of a star cluster with observations. In the present paper we decided to employ both strategies and made several runs of star clusters containing $N = 65{,}536$ (64K) and $N = 131{,}072$ (128K) cluster stars and central black holes with masses of $M_{BH} = 125$, 250, 500, and 1000 $M_\odot$, respectively. For a $N = 128$K star cluster the final cluster mass is around $M_C = 45{,}000\ M_\odot$, so a cluster 3
with a 250 $M_\odot$ IMBH would follow the Magorrian et al. (1998) relation.

The results of our runs are listed in Table 1, which gives, respectively, the mass of the black hole, the initial number of cluster stars, the initial depth of the central potential, the initial half-mass radius, the final cluster mass, the final half-mass radius, the projected final half-light radius, the final core radius, the ratio of the last two quantities, and the logarithm of the final half-mass relaxation time (Spitzer 1987):

$$T_{r,h} = 0.138\frac{\sqrt{M_C}R_h^{3/2}}{\langle m\rangle\sqrt{G}\ln\Lambda}, \tag{1}$$

where $\langle m\rangle$ is the mean mass of all stars in the cluster, and $\ln\Lambda$ is the Coulomb logarithm and is of order 10. The core radius was determined as the radius where the surface density has dropped to half its central value.

3. RESULTS

3.1. *Density Profile*

We first discuss the density profile of a cluster with an IMBH after it has evolved for a Hubble time. Portegies Zwart & McMillan (2002) and Rasio et al. (2004) have shown that a globular cluster has to start with a short enough central relaxation time to form an IMBH by runaway merging of main-sequence stars. Specific examples have been provided by Portegies Zwart et al. (2004), who found that the star cluster MGG-11 in the starburst galaxy M82, which has a mass of $3.5\times10^5\ M_\odot$ and an initial half-mass radius of $R_h = 1.2$ pc, could have formed an

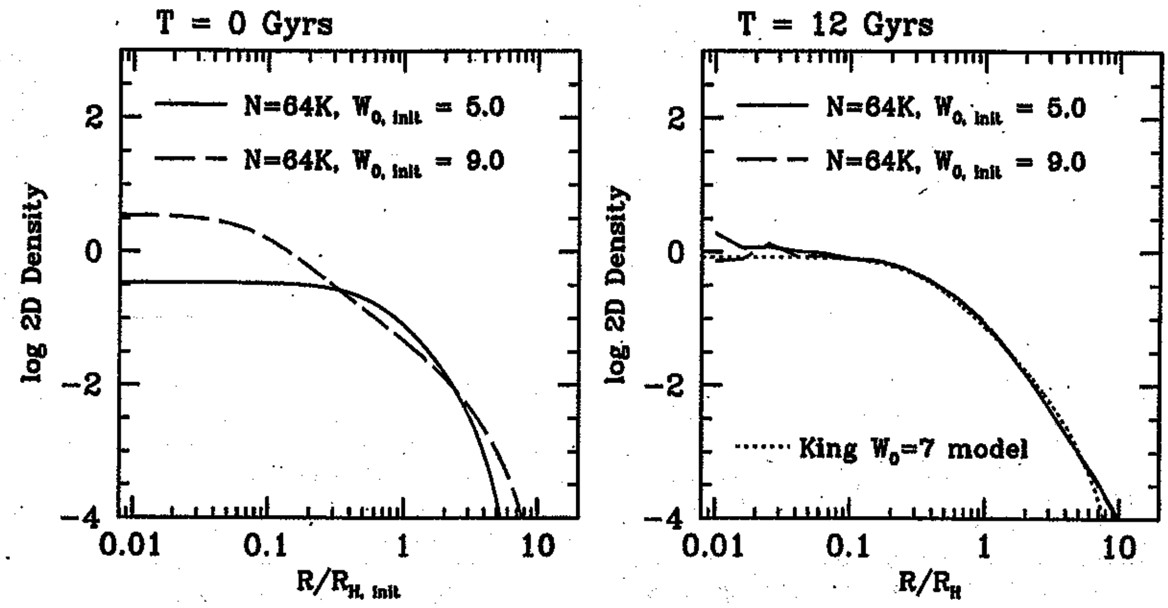


FIG. 1.—Two-dimensional density profile of bright stars for two $N = 64\mathrm{K}$ clusters starting with half-mass radii of $R_h = 2.0$ pc but different values for the initial central potential W_0. The initial relaxation time was short enough that both clusters expanded by a factor $R_h \sim 5R_{h,\mathrm{init}}$ and evolved toward the same density profile after a Hubble time.

IMBH while in the slightly larger cluster MGG-9 with a half-mass radius of $R_h = 2.6$ pc; the time for spiral-in of heavy mass stars was already too long and no runaway merging occurred. In order to study the dynamical evolution of clusters concentrated enough to form IMBHs, we first simulated two $N = 64\mathrm{K}$ clusters starting with a three-dimensional half-mass radius of $R_h = 2.0$ pc. These clusters have half-mass relaxation times equal to those of MGG-11. Both clusters start with black holes of 125 $M_\odot$, in agreement with the Magorrian relation. Since the initial density profile could in principle influence the final density profile and the dynamical evolution of the cluster, we simulated two clusters starting from a King $W_0 = 5$ and a higher concentration $W_0 = 9$ model, respectively.

4) Figure 1 depicts the projected density profile of bright stars in both clusters at the start of the simulations and after a Hubble time. We defined bright stars to be all giants and all main-sequence stars with masses larger than 90% of the mass of turn-off stars. In order to improve statistics, we overlaid 10 snapshots spaced by 50 Myr and centered at 12 Gyr. Although initially quite

In order to improve statistics for the inner parts, it was necessary to add more particles to the simulation. We have simulated a set of $N = 128\mathrm{K}$ clusters, containing a range of IMBH masses between $125 \leq M_{\mathrm{BH}} \leq 1000\ M_\odot$. The starting density profile was chosen to be a King $W_0 = 7$ model, close to the equilibrium profile found above. These calculations are quite challenging: the total amount of computing time used for the runs reported in this letter is more than half of a teraflops year, or well over 10^{19} floating point operations. It was only through the use of the GRAPE-6 system in the Astronomy Department of Tokyo University that we were able to perform these simulations.

Figure 2 depicts the projected density distribution of bright stars after the cluster evolution was simulated for 12 Gyr. Between $0.1 < R/R_h < 3$, the final profiles can be fitted by King $W_0 = 7$ profiles and by almost flat power-law profiles $\rho \sim r^\alpha$ inside $R/R_h = 0.1$. The measured slopes α lie between -0.1

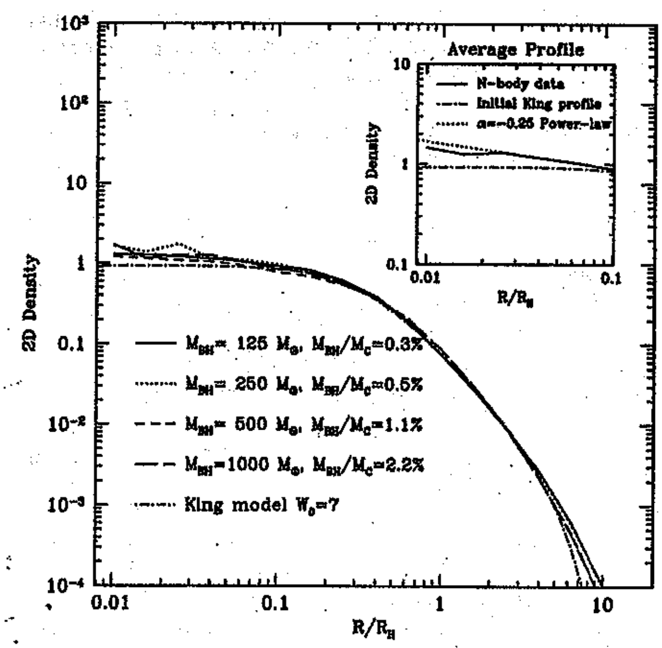


FIG. 2.—Projected density distribution of bright stars after $T = 12$ Gyr for four clusters containing black holes between $125 \leq M_{\mathrm{BH}} \leq 1000\ M_\odot$ and $N = 128\mathrm{K}$ stars. All clusters can be fitted by profiles in which the density is equal to a King $W_0 = 7$ model outside $R = 0.1R_h$, followed by a density increase in the inner parts. The profiles in the inner parts are nearly the same for all models. The inset shows the average profile of all N-body runs. Between $0.01 < R/R_h < 0.1$, it can be fitted by a power law with slope $\alpha = -0.25$. This is significantly flatter than the value found for galactic core-collapsed clusters, $\alpha = -0.8$. [*See the electronic edition of the Journal for a color version of this figure.*]

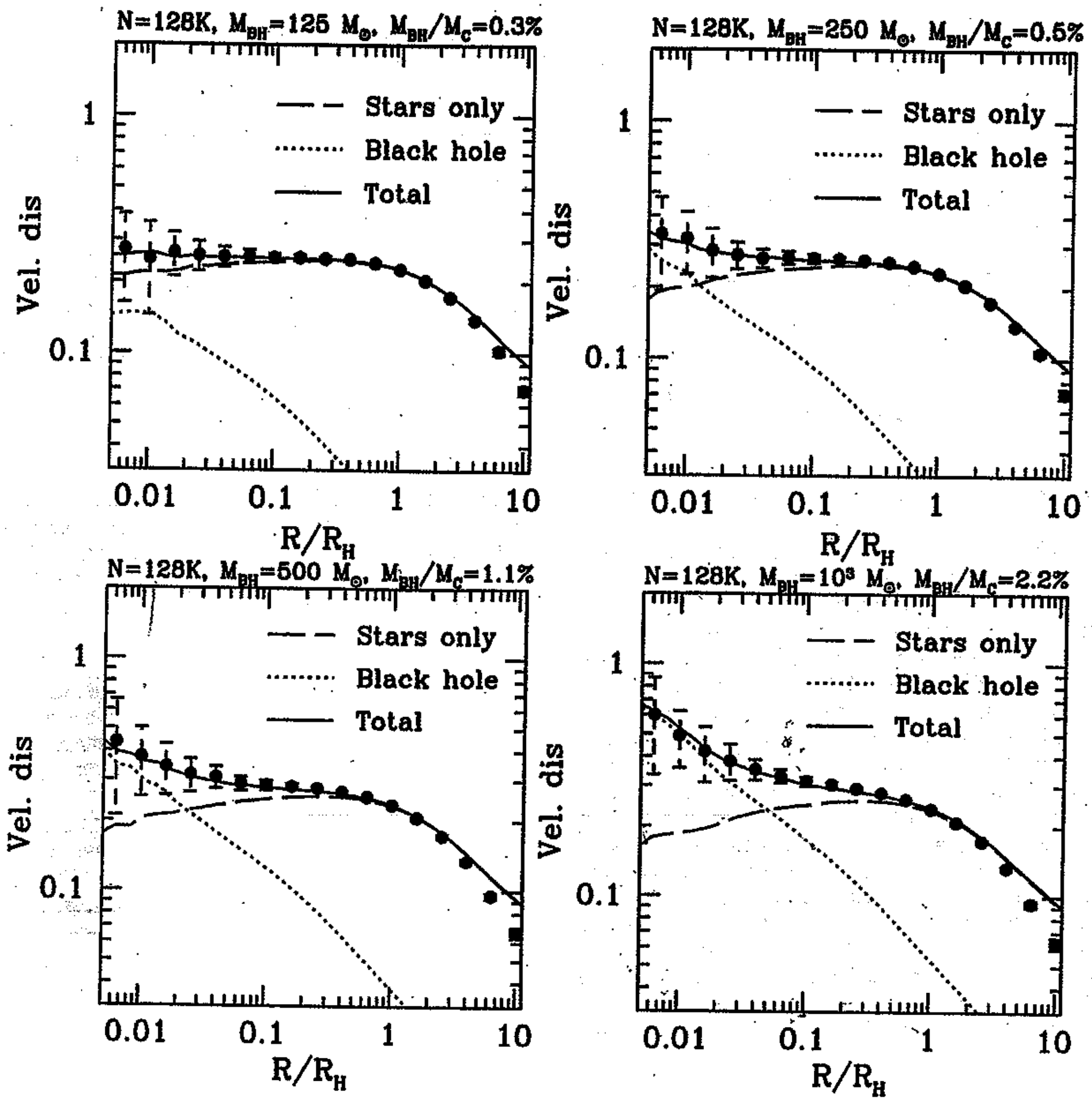


FIG. 3.—Velocity dispersion profiles for four cluster simulations that started with $N = 128$K stars and $M = 125$, 250, 500, and 1000 $M_\odot$ black holes. Filled circles with error bars are the velocity dispersion of visible stars in the N-body runs. The influence of the central black hole grows with increasing mass. Also shown are estimates for observational error bars for a cluster with 5×10^5 stars in which the brightest 5% of all stars can be observed all the way into the center.

and −0.3 for the different models, with no clear trend with the mass of the central black hole. The mean profile of all models has a slope of $\alpha = -0.25$ (see inset). A look at the projected profiles of other stars shows that the heavy-mass stars, i.e., the heaviest white dwarfs and neutron stars, follow significantly steeper slopes near −0.5, reflecting their strong mass segregation. The overall density profile in the center, however, is quite close to the density profile of the bright stars, since the mass of the bright stars is close to the average mass of the stars in the core.

According to Noyola & Gebhardt (2004), slopes of the central surface brightness profiles of galactic globular clusters span a range of values between those for constant density core models and those for models with steep cusps up to $\alpha = -0.8$. The latter value would correspond to the luminosity profile expected for a cluster in core collapse (Baumgardt et al. 2003a). The above results show that core-collapse density profiles are too steep for clusters that contain IMBHs, but that several clusters in the list from Noyola & Gebhardt (2004) have slopes compatible with the assumption that they contain IMBHs. We will come back to this point in § 4.

3.2. *Velocity Dispersion Profile*

We next discuss the chances of detecting an IMBH through observations of the radial velocity or proper motion profiles of a star cluster. The filled circles in Figure 3 show the projected velocity dispersion profile for four $N = 128$K star clusters. The lines show the predicted profiles calculated from the Jeans equation under the assumption that the velocity dispersion is isotropic, using as input the potential from the black hole and the cluster stars. These predictions form a very good fit to the N-body data, including the region where the influence of the black hole begins to dominate that of the stars, an effect that becomes stronger with increasing black hole mass.

Equations (2) and (3) of Baumgardt et al. (2004a) predict a linear relation between the radius R where the stellar

TABLE 2

GC CANDIDATES THAT COULD CONTAIN IMBHs

Name	Central Slope	$\log M_C$ ($M_\odot$)	$R_{h,\rm pro}$ (pc)	R_C (pc)	$R_C/R_{h,\rm pro}$	c	$\log T_{r,h}$ (yr)
NGC 5286	-0.20 ± 0.02	5.67	2.44	0.18	0.08	1.46	9.72
NGC 5694	-0.21 ± 0.10	5.35	3.28	0.34	0.10	1.84	9.76
NGC 5824[a]	-0.38 ± 0.08	5.15	3.35	0.20	0.06	2.45	9.67
NGC 6093	-0.13 ± 0.04	5.51	1.89	0.24	0.13	1.95	9.48
NGC 6266	-0.15 ± 0.04	5.90	1.92	0.20	0.08	1.70	9.68
NGC 6388	-0.14 ± 0.03	5.99	1.53	0.20	0.10	1.70	9.58
NGC 6397[a]	-0.29 ± 0.03	4.87	1.94	0.03	0.02	2.50	9.17
NGC 6541[a]	-0.36 ± 0.07	5.56	2.42	0.13	0.05	2.00	9.67
NGC 6715[a]	-0.16 ± 0.07	6.23	3.58	1.30	0.34	1.84	10.26

NOTE.—From the List of Noyola & Gebhardt (2004).

[a] Unlikely to contain IMBHs, see text.

velocity dispersion is affected more by the IMBH than by the stars alone: $R/R_h = \gamma M_{\rm BH}/M_C$, where M_C is the cluster mass. The results in Figure 3 are compatible with this relation and give $\gamma \approx 2.5$. The error bars in Figure 3 show the statistical error for a star cluster containing 5×10^5 stars and in which the brightest 5% of all stars can be observed in the center. For the cluster with the lowest mass black hole, the black hole dominates only at radii $R < 0.005R_h$, corresponding to radii of $R < 0\farcs5$ for a typical globular cluster. There are too few stars inside this radius, so the velocity error is too large to discern between the black-hole and the no-black-hole case. IMBHs with masses $M_{\rm BH}/M_C < 0.3\%$ can therefore not be detected by radial velocity measurements in star clusters. For the cluster with a $M_{\rm BH} = 250\ M_\odot$ black hole, the detection might be possible at the 2 σ level, and higher mass black holes might be detected even under less favorable conditions. Nevertheless, even the largest simulated IMBH with 1000 $M_\odot$, which has a mass significantly above the Magorrian et al. relation, creates a central rise that is hardly significant if only the brightest cluster stars can be observed. Observational detection of an IMBH in a star cluster will therefore be a challenging task.

4. GALACTIC GLOBULAR CLUSTER CANDIDATES

In this section, we compare the projected density profile of bright stars in our simulations with the observed central surface brightness profiles of galactic globular clusters. Noyola & Gebhardt (2004) have determined surface brightness profiles for 37 globular clusters from previously published *Hubble Space Telescope* (*HST*) WFPC2 images. They found that the slopes of central surface brightness profiles follow a range of values, from 0 (i.e., flat cores) to −0.8. As was shown in § 3.1, the most promising candidate clusters for IMBHs have central surface brightness slopes of −0.25 and outer profiles that can be fitted by King models with $W_0 = 7$, corresponding to a concentration parameter $c = 1.5$. Slightly different values for W_0 and c might be possible if the tidal field plays an important role and removes stars in the halo.

Table 2 lists all clusters whose profiles are compatible with a central slope between −0.2 and −0.3 and incompatible with a flat core from the list of Noyola & Gebhardt (2004). We have also listed the central concentration c of the clusters, the projected half-light radii as given by Trager et al. (1995), and the core radii as determined by Noyola & Gebhardt (2004). Core and half-light radii were transformed into physical units with the cluster distances from Harris (1996).[4] The final column gives the half-mass relaxation times, calculated from the cluster masses and the half-light radii, assuming that the (three-dimensional) half-mass radius is twice as large as the (two-dimensional) half-light radius. This is approximately the case in our runs.

We thank Karl Gebhardt for sending us his draft prior to publication. We also thank the referee Fred Rasio for comments that improved the presentation of the paper.

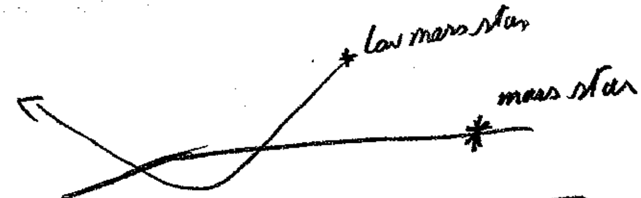
[4] Catalog available at http://physun.physics.mcmaster.ca/Globular.html.

Answers to Como 326

1. a. The Magorrian relation extrapolates th M_{BH} - M_{Bulge} relation found for supermassive black holes in galactic nuclei to IMBHs. There is no assurance that such an extrapolation is valid. The enviromental conditions are likely to be quite different in GC

b The high luminosity suggests material is falling into a deep potential well This could be caused by a IMBH. The rapid variability suggests the source of this variability is small. Deep potential well + v. small size → IMBH?

2 Mass segregation results from the interaction of stars of different mass. During a gravitational encounter energy is transfered between the stars



The trajectory of the high mass star is not altered much, whereas th low mass star's trajectory can be substantially changed. In the core region, the low mass stars would be ejected, leaving the higher mass stars behind

3. GC typically have masses of several hundred thousand solar masses. An IMBH is 'expected' to have a mass of $1-2\times10^{3} M_\odot$. The values used in the computations are simply scaled versions of these values. $M_c \approx \frac{1}{5}-\frac{1}{10}$ mass of a typical GC, 250 IMBH $\sim \frac{1}{5}-\frac{1}{10}$ mass of a real IMBH.

4. The point of Fig 1b is that even if a GC starts out with very different properties, they evolve toward the same density profile (Fig 1b). Therefore, (Fig 1a) how a GC looks today (compact, low surface brightness) is not a good guide to its initial conditions. It is, therefore, not correct to think that an IMBH is preferentially located in compact GC as opposed to some other sort of GC. The formation history is not revealed by the current appearance of the GC.

5. Let $\left.\begin{array}{l} V = \text{typical random velocity in the cluster} \\ R = \text{Average separation between cluster stars} \end{array}\right\}$. Assume for simplicity that all cluster stars have the same m. The gravitational potential energy of a pair is

$$-Gm^2/R$$

There are $N(N-1)/2$ possible pairings of N stars. Therefore the total gravitational potential energy is $\sim -\frac{N(N-1)}{2}\frac{Gm^2}{R}$. The virial theorem then gives

$$(1) \quad \frac{NmV^2}{2} = \frac{N(N-1)}{2}\frac{Gm^2}{2R}$$

$$V \sim \frac{1}{R^{1/2}} \quad \log V \sim \frac{1}{2}\log R \quad \text{slope} \sim \frac{1}{2}.$$

The typical escape velocity of a star from the cluster is

$$② \quad \frac{m v_e^2}{2} \sim \frac{G[(N-1)m]\, m}{R}$$

Comparing ① and ②

$$V = \frac{1}{2} v_e$$

$$\boxed{v_e \sim 2V}$$

5c. Taking $R = 5\,pc$, $m \sim 2\times10^5 M_\odot$, $N \sim 5\times10^5$, $m \sim 0.5 M_\odot$

$$V \sim \frac{1}{2} v_e \sim \frac{1}{2}\left(\frac{2G(N-1)m}{R}\right)^{1/2}$$

$$V \sim \frac{1}{2}\left(\frac{2\cdot 6.67\times10^{-8}\cdot 5\times10^5\cdot 2\times10^{33}}{5\times10^{18}\cdot 2}\right)^{1/2}$$

$$V \sim \frac{1}{2}\left(13.3\times10^{12}\right)^{1/2} \sim 1.8\times10^6 \frac{cm}{sec}$$

$$\boxed{V \sim 1.8\times10^6 \frac{cm}{sec} \times \frac{km}{10^5 cm} = 18\ km/sec}$$

6a This question could be answered in a variety of ways

NGC	Central Slope too Steep	"high" c	low $M_\odot$	
NGC 5286	→	→	x	
5694	→	x		
5829	x			
6093	→	→	x	
6266	→	→	→	equally good candidates given data in Table 2
6388	→	→	→	
6397	x			
6541	x			
6715	x			

6b $V \sim 18$ km/sec Computed from 5c

$\sigma \sim \sqrt{V} \sim 4$

$\frac{S}{N} \sim \frac{18}{4} \sim 4.5$ per object 4

Desired goal is $\frac{S}{N} \sim 50 \rightarrow$ factor of 11 increase in $\frac{S}{N}$

$\therefore$ # of targets is $\sim (11)^2 \sim \underline{121}$ since $\frac{S}{N} \sim \sqrt{\text{Number of objects}}$

$$\boxed{N \sim 121}$$