R.T.J McAteer **CUME #464** **28 Oct 2022**

This CUME is motivated by the research presented in the accompanying pdf, "*Reconnection-Driven Energy Cascade in Magnetohydrodynamic Turbulence*".

- At the end of your exam time, upload your answers to the canvas page.
- There are 50 total points available. A total of 35 points is *expected* to be a passing grade.
- Take a new page for each question. This makes it much easier to award partial credit for incorrect answers.
- Be careful with your time management. Do not allow yourself to get stuck on any one question. If you find yourself spending a long time at one question without success, move on and come back to it later. In particular I suggest you try to get through all questions inside the first ~90 minutes and then use the remaining ~30 minutes to go back and complete any parts you have skipped.
- Calculators are only to be used for calculations. You may not store equations.
- You may not use your cell phone at any time. You may not use the internet. You may not consult any notes from classes.
- Show all work for full points. Attempt all parts of all questions. Unless informed otherwise, use cgs units in your calculations.
- Show the derivation of any constants not provided.
- Some equations you may need
  - Faraday's Law, $\nabla \times \mathbf{E} = -(1/c)(\partial \mathbf{B}/\partial t)$
  - Ohm's Law, $\eta \mathbf{J} = \mathbf{E} + (1/c)(\mathbf{v} \times \mathbf{B})$
  - Ampere's Law, $\mathbf{J} = (c/4\pi)(\nabla \times \mathbf{B})$ (neglects displacement current)
  - Vector identity for any vector $\mathbf{A}$, $(\nabla \times (\nabla \times \mathbf{A})) = \nabla(\nabla . \mathbf{A}) - \nabla^2 \mathbf{A}$
  - The divergence theorem, $\iiint_V \nabla . v \, dV = \oiint_S v.n \, da$
- Adopt the following values for constants

  Mass of a proton, $m_p = 1.7 \times 10^{-24}$ g

  Boltzmann constant, $k_B = 1.4 \times 10^{-16}$ erg K^{-1}

  For the Sun, surface gravity $g = 2.7 \times 10^4$ cm s^{-2}

  For the Sun, photospheric temperature $T \sim 5800$K

  1 solar radii = 7×10^{10} cm

  The Sun subtends an angle of 0.5^o at 1AU

1. This abstract is intended for any astrophysics audience, so define the following terms *from an astrophysics undergraduate student perspective* . Use 2-3 sentences for each one and include a brief sketch each time to help you in your definition.

- (a) “magnetohydrodynamic”
- (b) “turbulence”
- (c) “magnetic reconnection”
- (d) “magnetic flux rope”
- (e) “energy cascade”
- (f) “anisotropy”

3 points each

2. The magnetic Reynolds number is a key component of these simulations. It governs the evolution of a plasma structure and can be derived as the ratio of the advective term to the diffusive term the induction equation.

a) Describe Faraday's law, Ohm's law and Ampere's law in 2-3 sentences each. Include a sketch each time to assist you in your description.

6 points

b) Using Faraday's law, Ohm's law and Ampere's law, derive the induction equation, $(\partial \mathbf{B}/\partial t) = \nabla \times (\mathbf{v} \times \mathbf{B}) + D_\eta (\nabla^2 \mathbf{B})$, where D_η is magnetic diffusivity. Ensure you state all assumptions.

5 points

c) Derive the equation for the magnetic Reynolds number, $R_m \approx vL / D_\eta$, where v is velocity and L is length scale. Ensure you state your simplifications.

2 points

d) In Section 'Results', they quote a $R_m \approx 10^6$. The typical value of the magnetic diffusion coefficient in the photosphere is $D_\eta \approx 10^7$ cm^2 s^{-1}. Propose a combined set of v and L (hint, see figure 8), that would create such a high R_m. Justify those choices of velocity and length with references to features on the Sun.

4 points

3. Focus in on the model description section (see Materials and Methods).

The equation of mass conservation (equation 3) states that the change of the mass in any volume element is given by the flow of mass into, or out of, the element, i.e.

$$\frac{d}{dt}\int_V \rho dV = -\int_{loop} \rho \mathbf{u} \,.\, d\mathbf{S}$$

a) Draw a sketch of the volume element and provide a few sentences to explain this equation. In your sketch include a label to each of dV, $\mathbf{u}$, and d**S**

5 points

b) Use the divergence theorem to derive equation 3 in the paper.

3 points

c) Equation 4 is the generalized equation of motion. Lets focus in on two terms, the first term on the left, $\partial_t(\rho\mathbf{u})$ and the first term on the right, $-\nabla(p + \frac{B^2}{2})$

Show that each of $\partial_t(\rho\mathbf{u})$ and $-\nabla(p)$ are actually Force per Volume

4 points

d) Cleary $\frac{B^2}{2}$ is a magnetic pressure. Describe a general comparison of magnetic pressure to gas pressure that explains the topology of the types of features that can be found in Fig 1 and discussed in that caption.

3 points