The Astrophysical Journal, 795:79 (6pp), 2014 November 1 doi:10.1088/0004-637X/795/1/79

ON SUNSPOT AND STARSPOT LIFETIMES

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Received 2014 September 3; accepted 2014 September 15; published 2014 October 15

ABSTRACT

We consider the lifetimes of spots on the Sun and other stars from the standpoint of magnetic diffusion. While normal magnetic diffusivity predicts lifetimes of sunspots that are too large by at least two orders of magnitude, turbulent magnetic diffusivity accounts for both the functional form of the solar empirical spot–lifetime relation and for the observed sunspot lifetimes, provided that the relevant diffusion length is the supergranule size. Applying this relation to other stars, the value of turbulent diffusivity depends almost entirely on supergranule size, with very weak dependence on other variables such as magnetic field strength and density. Overall, the best observational data for other stars is consistent with the extension of the solar relation, provided that stellar supergranule sizes for some stars are significantly larger than they are on the Sun.

Key words: magnetic fields – starspots – stars: variables: T Tauri, Herbig Ae/Be – Sun: photosphere – sunspots

1. INTRODUCTION

Starspots are ubiquitous features of stellar surfaces and markers of stellar activity, yet we know relatively little about the physical processes that govern their lifetimes. Spot lifetimes are important to quantify because (1) if lifetimes are long enough, stellar light curves from different epochs may be phased over multiyear timescales, which facilitates rotation and angular momentum studies of young stars; (2) measurements of spot lifetimes allow stellar activity cycles to be tracked and studied; and (3) lifetime measurements yield insights into the physics of stellar convection zones. Sunspot lifetimes may also provide clues to the time dependence of the heating mechanism in solar active regions. For example, Ugarte-Urra & Warren (2012) found that young active regions are consistent with more dynamic, burst-like low-frequency heating, whereas the energy deposition in older active regions may have evolved to a relatively steady-state configuration corresponding to high-frequency heating.

An obvious question to consider is whether commonalities exist between the natures of sunspots and starspots, and there is some evidence that they do. We possess a certain amount of phenomenological information based on long-term photometric observations of stellar brightness variations; for example, the lifetimes of smaller spots are proportional to their sizes (Berdyugina 2005), which is also true of sunspots (Gnevyshev 1938; Waldmeier 1955; Petrovay & Van Driel-Gesztelyi 1997). In addition, long-term monitoring of stellar light curves has revealed what appear to be the analog of solar cycles that operate on timescales of decades (Oláh et al. 2009). Like the Sun, spotted stars are also well known to have enhanced X-ray activity associated with magnetic coronal heating (e.g., Feigelson et al. 2005).

Significant differences between sunspots and starspots also exist. For example, sunspots typically last for days to a month, and cover at most a few percent of the Sun's surface area, while spots on young, pre-main-sequence, solar mass (T Tauri) stars may persist for several years and cover up to 20% of the surface (Hall & Henry 1994; Strassmeier et al. 1999a). There is a greater brightness contrast between spot regions and the surrounding photosphere on hotter stars, which suggests that the temperature difference between them increases with stellar temperature. The temperature difference is about 2000 K in stars of spectral type G0 and only 200 K in stars of type M4. This property persists between active dwarfs and giant stars, where cooler dwarfs tend to have stronger magnetic fields covering larger areas (Berdyugina 2005). Another difference between the solar and stellar cases is that some stars exhibit long-lived polar spots (Strassmeier et al. 1999a).

One stellar property that might act to limit spot sizes and hence their lifetimes is differential rotation, which causes shearing that breaks up large spots that cover a broad latitudinal range into fragments that subsequently form groups of smaller spots with correspondingly shorter lifetimes. Hall & Busby (1990) studied the lifetimes of a sample of 40 spots on 17 different stars and found that small spots ($r_s < 20°$) disappear before they are subjected to significant shearing, whereas larger spots appear to be more disrupted by differential rotation. The rotational shear is defined as the difference in the rate of rotation between the equator and the pole, given by $\Delta\Omega$. In solar-type differential rotation, the equatorial region rotates faster than the poles with $\Delta\Omega = 0.055$ rad day^{-1}. The strength of the differential rotation is quantified by $\tau_L = 2\pi/\Delta\Omega$ days, where τ_L ($\sim$115 days for the Sun) is the time taken for the equator to lap the poles. Clearly, if $\tau_L < \tau_S$, where τ_S is the spot lifetime, then differential rotation is likely to be important in determining the longevity of the spot. In the case of the Sun τ_L is significantly more than the longest observed spot lifetime of approximately one month ($\approx$30 days), so the differential rotation is probably not a factor.

Differential rotation can be identified by long-term photometric observations of the stellar brightness modulation, where changes in the seasonal rotation period are indicative of differential rotation and the gradual migration of spot latitudes (Henry et al. 1995). A comparison between the range of seasonal rotation rates and the mean rotation rate of the star suggests that slower rotators exhibit stronger differential rotation. Significantly, a majority of stars are found to have weaker rotational shear than is observed on the Sun. This was confirmed by Reiners & Schmitt (2003a, 2003b), who used a Fourier transform method to study the differential rotation of a sample of rapidly rotating F0–G0 dwarf stars and found that it is strongest in slower rotators. Moreover, the differential rotation of the more rapidly rotating stars in their sample fell below their method's

threshold of sensitivity, rendering it undetectable. The differential rotation of a small sample of active G2–M2 dwarfs was analyzed by cross-correlating successive Doppler images (Petit et al. 2004; Barnes et al. 2005), a more sensitive technique than the previous Fourier transform method, which revealed that $\Delta\Omega$ is effectively negligible in M dwarfs and strong in G dwarfs. A particularly large brightness variation (0.65 mag) was observed for V410 Tau, where the lifetime of the associated spot has been estimated to be at least 20 yr (Strassmeier et al. 1997; Hatzes 1995), indicating that differential rotation may not play a significant role, especially if, like V410 Tau, the star is a relatively fast rotator.

In light of the possibility that the underlying nature of starspots and the physical processes determining their lifetimes may be similar from star to star, in this paper we investigate whether the proportional relationship between the lifetime of a spot and its size is consistent with anomalous magnetic diffusivity operating at the supergranule scale. In Section 2, we discuss the encapsulation of this relationship by the Gnevyshev–Waldmeier rule and how it may be applied to the Sun and T Tauri stars. In Section 3, we discuss the reasons why classical magnetic diffusivity cannot account for the observed spot lifetimes, and in Section 4, we introduce the need for anomalous magnetic diffusivity to explain the relative lifetimes of spots on the Sun and other stars. We consider what is known about spot lifetimes for the best-observed cases in Section 5, and summarize the paper in Section 6.

2. THE GNEVYSHEV–WALDMEIER RULE

The Gnevyshev–Waldmeier rule (Gnevyshev 1938; Waldmeier 1955, hereafter G-W) is a simple linear empirical relationship between the area of a sunspot and its lifetime:

$$A = WT, \tag{1}$$

where A is the maximum size of the spot, T is the spot lifetime and $W = 10$ MSH day^{-1} for the Sun. The unit of area Micro Solar Hemisphere (MSH) is $(5\farcs8)^2$, where $1'' \approx 725$ km on the Sun. Hence, the rate of area shrinkage of the spot, $W = 3.04 \times 10^{13}$ m^2 day^{-1} is the quantity encapsulating the physics that determines the lifetime of the spot and is of direct interest to us in the present work. Many sunspots persist for less than one day, and lifetimes of more than a week or two are relatively uncertain since the spots disappear over the limb and observations are interrupted. The data used to calculate W are probably biased toward smaller values of T than are obtained for the longest-lived sunspots.

Applying the G-W rule to a spot that might be typical of a T Tauri star can yield an estimate of the spot's longevity, which we expect to be significantly extended in comparison with the average lifetime of a sunspot. A typical T Tauri star has a radius of 3 $R_\odot$ and light curve amplitudes that range from a few percent to 10%. The total light curve amplitude will depend on the relative amounts of starspots on opposite sides of the star, but for the purpose of calculation, if we take an individual spot to cause a light variation of 0.01 mag and assume the spot to be completely dark, then the spot area is 1.4×10^{17} m^2, which corresponds to a lifetime of $\approx$12 yr using Equation (1). These lifetime estimates are roughly in line with what we expect based on observational studies of T Tauri stars, that is, the relationship between the sizes and lifetimes of spots provides values that fall within the right orders of magnitude. We will now consider the physics that underlies the value of W, and assess if there are theoretical reasons why the solar relation should hold for active stars in general.

Table 1
Lifetimes of Sunspots and Starspots Assuming Classical Magnetic Diffusivity

Type	Dimension		Lifetime	
	($R_\odot$)	(km)	(days)	(yr)
Sunspot	0.04	2.8×10^4	1.5×10^4	41
Starspot	0.33	2.3×10^5	1.0×10^6	2700

3. MAGNETIC DIFFUSIVITY

Our aim is to identify the physical nature of the sunspot diffusion so that we can know how to scale it and apply it to other stars. Spots on active stars are regions of relatively strong magnetic field, so we begin our investigation by determining whether their observed lifetimes are consistent with classical magnetic diffusivity. The rate of change of field strength in the diffusive limit is given by the induction equation,

$$\frac{\partial B}{\partial t} = \eta \nabla^2 B, \tag{2}$$

where η is the magnetic diffusivity. The G-W rule implies a single value for the magnetic diffusivity—this would be the actual diffusivity that operates in the Sun and does not depend on the spot size. To order of magnitude, the lifetime of a spot due solely to the rate of diffusion of its magnetic field is given by

$$\tau = \frac{R_{\rm spot}^2}{\eta}, \tag{3}$$

where $R_{\rm spot}$ is the radius of the spot. The classical magnetic diffusivity is the inverse of the product of the magnetic permeability μ_0 and the electrical conductivity σ due only to Coulomb collisions:

$$\eta = \frac{1}{\mu_0 \sigma}, \tag{4}$$

where $\mu_0 = 4\pi \times 10^7$ N A^2. It is straightforward to calculate the electrical conductivity in the case of a fully ionized plasma, but estimating σ in the partially ionized photosphere is more difficult and its value depends to some extent on the model atmosphere on which the calculation is based. Kovitya & Cram (1983) calculated the conductivity using a second-order Chapman–Enskog method (which differs from the fourth-order approximation by less than 0.2%; Devoto 1967) and the sunspot model of Avrett (1982). The ionization-equilibrium calculation used by Kovitya & Cram (1983) predicted a somewhat weaker charge state (lower electron density) than in the original sunspot model, and a slightly lower value for σ. To be conservative, we choose the smallest value of σ in the coolest part of the sunspot, which in Table II of Kovitya & Cram (1983) is $\sigma = 1.3$ S m^{-1}. Inserting this quantity into Equation (4) we find that $\eta = 5.29 \times 10^{10}$ m^2 day^{-1}, much smaller than the G-W value ($\eta_{\rm GW} \sim W/\pi$) of $\eta_{GW} = 9.68 \times 10^{12}$ m^2 day^{-1} ($\eta_{\rm GW}/\eta \approx 180$). Larger values of σ make the comparison even worse.

Table 1 shows the sunspot and starspot lifetimes expected for a given spatial scale based on the classical magnetic diffusivity and the electrical conductivity in the photosphere. It is clear that the lifetimes are gross overestimates (by $\gtrsim$two orders of

Table 2
Spot Lifetimes for the Sun and a T Tauri Star Assuming Turbulent Magnetic Diffusion

Size Scales[a] (km)	η (km^2 s^{-1})	η (m^2 day^{-1})	u' (m s^{-1})	$\tau_\odot$[b] (yr)	τ_{star}[c] (yr)
116	0.87	7.5×10^{10}	23	28	1900
440	4.40	3.8×10^{11}	30	5.5	380
1400	18.0	1.6×10^{12}	39	1.4	93
11300	112	9.68×10^{12}	30	0.22	15

Notes.
[a] Maximum turbulent length scale.
[b] For a sunspot with $R_{spot} = 0.04\ R_\odot$ and the turbulent scale size in Column 1, $\tau = R_{spot}^2/\eta$.
[c] For a starspot with $R_{spot} = 0.33\ R_\odot$ and the turbulent scale size in Column 1.

magnitude). The electrical conductivity depends on the composition of the photosphere and the dominant term in calculating its value is the elastic scattering of electrons from neutral hydrogen (Kovitya & Cram 1983). In consequence, provided that the photosphere is composed primarily of hydrogen and spot interior temperatures are not hugely variable, then there should not be large variations in σ from star to star.

4. ANOMALOUS MAGNETIC DIFFUSIVITY

Because classical magnetic diffusivity based on Coulomb collisions is not of sufficient magnitude to explain the lifetimes of observed spots, we must appeal to a form of anomalous diffusivity if magnetic diffusion governs spot lifetimes. Chae et al. (2008) have suggested that the magnetic diffusivity is of turbulent origin and this should be accounted for in the rate of flux cancelation at the photosphere. They measured values of the diffusivity by applying the induction equation to pairs of magnetograms taken at different times by *Hinode* Solar Optical Telescope (SOT) and *Solar and Heliospheric Observatory* Michelson Doppler Imager (MDI) at pixel sizes of 116 km (SOT), 440 km (high-resolution MDI magnetograms), and 1400 km (full disk MDI magnetograms). By solving the induction equation for η at these different length scales, they were able to estimate values for the magnetic diffusivity and found them to be consistent with a turbulent cascade from supergranular and granular scales, ending at a resistive dissipation scale of about 30 km (the scale at which the magnetic diffusivity equals the classical collisional value).

Table 2 provides some estimates of sunspot and starspot lifetimes based on the Chae et al. (2008) values of the anomalous magnetic diffusivity. The lifetimes predicted for sunspots and starspots in Table 2 are still too long at the scales listed in the first column. We may then ask: what is the length scale of η consistent with the G-W value? Chae et al. (2008) show that the observed cascade follows the Iroshnikov–Kraichnan model for magnetohydrodynamic turbulence:

$$\eta = \left(\frac{l}{260\ \mathrm{km}}\right)^{5/4}\ \mathrm{km^2\ s^{-1}}, \quad (5)$$

where l is the turbulent scale length. We match the G-W value ($\eta_{GW} = 112$ km^2 s^{-1}) for $l = 11300$ km, which is the size of a typical supergranule (Leighton et al. 1962; Chae et al. 2008).

Thus, turbulent magnetic diffusivity at supergranule size scales that cascades down to a scale at which collisional diffusivity operates and energy is dissipated appears to provide a viable explanation for the lifetimes of sunspots and T Tauri starspots. Other authors (e.g., Petrovay & Moreno-Insertis 1997) have found that magnetic diffusion operating on the granular scale ($l \approx 1500$ km) is adequate to explain observed sunspot lifetimes. However, this conclusion results from using a much larger value for η. Petrovay & Moreno-Insertis (1997) quote a granular value of $\eta = 1000$ km^2 s^{-1} (which is a factor of $\sim$9 larger than η_{GW}) in comparison to the later measurements of Chae et al. (2008), who found that $\eta = 18$ km^2 s^{-1} at $l = 1400$ km (see also Table 2).

In the Iroshnikov–Kraichnan model, the turbulent magnetic diffusion is given by

$$\eta = (\epsilon V_A)^{1/4} l^{5/4}, \quad (6)$$

so that the scaling of the diffusion coefficient only depends weakly on the rate of energy injection ϵ and the Alfvén speed V_A. The weak scaling with the Alfvén speed implies a weak scaling with the magnetic field strength B and the particle number density n ($V_A \propto B/\sqrt{n}$), quantities that may vary significantly from star to star. This result suggests that as long as the size of supergranules remains the same, the magnitude of the turbulent diffusivity in other stellar photospheres could be similar to that of the Sun. Otherwise, one should be able to use the solar G-W law suitably scaled to the supergranule size for other stars.

Leighton (1964) was the first to associate the dispersal and migration of photospheric magnetic regions with a random-walk, diffusive process driven by supergranulation-scale convection currents. He estimated very similar dispersal rates due to granular and supergranular convection, but argued that only the more deeply rooted supergranular motions are coupled strongly enough with the magnetic field lines to move them about. Evidence for this is the chromospheric network, which is described by the distribution of magnetic flux that corresponds to the boundaries of supergranule cells. Simon & Leighton (1964) suggested that one possible mechanism for the breakup of sunspots at this scale is the progressive fragmentation of the magnetic field by the action of the supergranulation convection currents.

Returning again to a classical formalism, the magnetic diffusivity depends on the length scale for dissipation l and the gas velocity u' by (e.g., Leighton 1964)

$$\eta = \frac{1}{3} u' l. \quad (7)$$

If turbulent flow dominates, then u' should be the magnitude of velocity fluctuations. The first three lines of Table 2 show the observed diffusivities for the size scales reported by Chae et al. (2008), and the corresponding velocities inferred from Equation (7). These velocities compare well with observations of actual velocity fluctuations in the solar photosphere. Lites et al. (1998) measured oscillations of Doppler velocity at photospheric heights in a sunspot based on observational data taken by the High Altitude Observatory/National Solar Observatory Advanced Stokes Polarimeter. This instrument provides the complete Stokes polarization vector profiles of several spectral lines (e.g., Fe I 630.15 nm, 630.25 nm) and allows vector magnetic fields and Doppler velocities in sunspots and active regions to be measured to a high degree of accuracy. Table I of Lites et al. (1998) reports velocity fluctuations of $\sim$40–50 m s^{-1} in the umbral and penumbral regions in the five-minute band (the three-minute band is the chromospheric/transition region). Lifetimes of spots for the scale sizes in the first three lines of Table 2 are still much longer than the observed ones, so for turbulent

Table 3
Observed Starspot Sizes and Lifetimes

Star	Spectral Type	P_{rot} (days)	R_{star} ($R_\odot$)	Spot Radius (10^5 km)	Lifetime (days)	References
Kepler 17	G2V	11.89	1.05	1.2	50	a
CoRoT 6	F9V	8.88	1.03	1.2	35	b
CoRoT 2	G7V	4.54	0.90	1.1	31	c
V410 Tau	K4IV	1.87	3.46	6.3	>7300	d
HD 199178	G5III-IV	3.32	6.5	16	>4400	e
Sun	G2V	25.4	1.00	0.06	3.7	

References. (a): Bonomo & Lanza 2012; (b): Lanza et al. 2011; (c): Silva-Valio et al. 2010; Silva-Valio & Lanza 2011; (d): Hatzes 1995; Carroll et al. 2012; and (e): Strassmeier et al. 1999a.

diffusion to explain the observations the length scales must be larger than those observed by Chae et al. (2008). Using the value of η required by the G-W law, and taking l to be a supergranular scale of $l = 11{,}300$ km, the velocity given by Equation (7) is $u' = 30$ m s^{-1}, reasonably consistent with the observed fluctuations.

However, once the dissipation scale size exceeds the size of the spot, diffusion becomes a non-local process and Equation (7) may not apply. The transport of magnetic flux in a weakly electrically conducting plasma could be analogous to the transport of heat in a plasma where the mean free path approaches the physical scale of the system. The temperature and ionization several thousand kilometers below the photosphere indicate a high electrical conductivity and a slow rate of diffusion, which may serve to limit the displacement of "free-streaming" field lines at the surface, causing a localized build-up of flux and then strong dissipation due to the enhanced field gradient. Alternatively, the scales relevant to determining the physics that sets the lifetime of the sunspot may not be visible at the solar surface; the photosphere is only the layer at which the optical depth of light at ~550 nm approaches 2/3, but does not have to be the layer at which key physical scales relevant to magnetic activity are revealed. Finally, it may simply be that the velocity fluctuations discussed above (usually averaged out in the data), that are essentially scale-independent up to at least a supergranular scale, lead to an effective diffusion of magnetic flux concentrations (Litvinenko 2011).

5. OBSERVATIONAL DATA AND DISCUSSION

While a large amount of evidence for spots on other stars exists, usually in the form of small-amplitude periodic variations in the stellar light curves, it can be very difficult to infer the sizes and lifetimes of spots accurately from photometry alone owing to degeneracies in reconstructed maps of the stellar surfaces (Walkowicz & Basri 2013; Lanza 2013). However, when combined with the techniques of Doppler imaging and eclipse mapping by transiting exoplanets, photometric data yields a much more robust assessment of the size and longevity of starspots, albeit only for a limited selection of objects. To augment the solar observations, in Table 3 we compile starspot data from recent papers of three active main-sequence stars that the *Kepler* and *CoRoT* satellites have found to have transiting planets. These values derive from maps of the starspot distributions that typically show the spots last for a month or two and cover something like 20° in longitude. In order to sample the largest range of parameters possible, we also include two other well-known stars that have been reported to host large spots. This sample is not intended to be complete, but rather

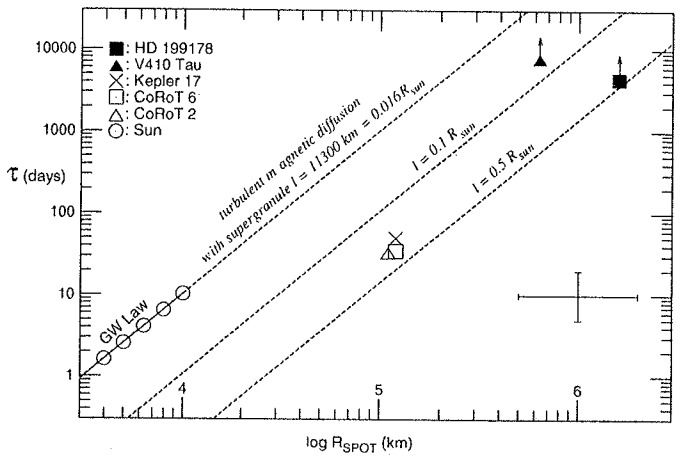

Figure 1. Spot lifetime plotted against size. The G-W law (solid line) and its extension (dashed line) are shown for the Sun, and for two cases with larger supergranule sizes l. The error bar represents a systematic error of a factor of two in both axes.

to provide an indication as to how the parameters vary among different stars.

The lifetimes and sizes of starspots on the main-sequence transiting planet objects Kepler 17, CoRoT 6, and CoRoT 2 (Bonomo & Lanza 2012; Lanza et al. 2011; Silva-Valio et al. 2010; Silva-Valio & Lanza 2011, respectively) in Table 3 resemble scaled-up versions of the Sun. For the two stars with larger radii, Doppler imaging of the rapidly rotating young star V410 Tau shows a large spot near the pole that has persisted for at least 20 yr (Hatzes 1995; Carroll et al. 2012), and the FK Comae giant HD 119178 has an even larger polar spot that has lasted for more than 12 yr (Strassmeier et al. 1999a). While the origin of these polar spots may differ from those at lower latitudes, it is clear that, like sunspots, starspots live longer when they are larger, as expected if a diffusion process governs their longevity.

Plotting the lifetime of the starspots against their size, we obtain Figure 1. The extension of the solar G-W law using a constant supergranule size lies above the observed points for the solar analogs with transiting planets, but is consistent with the lower limits of the ages for the large polar spots on the subgiant and giant stars. Taken at face value, matching the lifetimes of the solar analog stars requires a supergranule diffusion length much larger than the Sun, covering ~23° in longitude. However, one should keep in mind that estimates of spot sizes on these objects are limited by spatial resolution, so the spot sizes could be considerably smaller than estimated in Table 3 if they collect loosely into larger active areas, and this behavior would reduce

the required supergranule size. The systematic error bar of a factor of two in the figure represents the approximate uncertainty in assigning a lifetime and a spot size from the reconstructed images of the stellar surfaces.

Explaining the origin of supergranulation remains one of the outstanding challenges of modern solar physics (Rieutord & Rincon 2010). It is not clear what sets the lifetime and, most importantly for the work presented here, the spatial dimension of the supergranule cells. We therefore do not have a good idea, or even much intuition, of how this quantity might vary from star to star. The typical size scale for supergranulation on the Sun is 10,000–30,000 km (Hirzberger et al. 2008), with a depth of not more than 5000 km (Sekii et al. 2007) as determined by local helioseismology, which suggests that supergranules are in fact rather shallow structures. Their depth may be linked to the ionization depth of He or He^+ (2000–7000 km; Simon & Leighton 1964) in a similar manner to the depth of granulation cells being connected with the location of hydrogen ionization below the photosphere. The average size of supergranules is sensitive to the method used to define them (e.g., as coherent, diverging flow cells at the solar surface) and their vertical profile is at present unconstrained. One potential clue to the mechanism that sets the size scale of supergranulation may lie in observations of its dynamical interaction with magnetic field in the quiet Sun. Supergranules are strongly associated with the magnetic network and correlations between their sizes and the strength of network and internetwork fields has been observed (Meunier et al. 2007).

Results from numerical simulations have been similarly inconclusive. Global simulations have yielded a supergranulation-like pattern, but the scale of the pattern was extremely close to the chosen grid scale (DeRosa et al. 2002). Local simulations of hydrodynamic convection do not exhibit excess power at supergranulation scales (Stein et al. 2009), but local simulations of magnetohydrodynamic convection lead to the formation of a magnetic network at supergranulation scales in which magnetic flux concentrations may play an important role in the scale selection process (Ustyugov 2009) as suggested by observations of the quiet Sun.

Lacking a robust physical understanding of what sets the spatial scale of the supergranulation makes estimating this scale on other stars extremely difficult. We can calculate the value of η required to explain the lifetime of the observed spots, but without an estimate of the supergranulation scale or the velocity fluctuations we face a problem of degeneracy in understanding exactly what determines η. One approach to estimating l might be to calculate the interior temperature structure of the star based on observations of its surface properties and the standard stellar structure equations, in order to find the depth of He and He^+ ionization as a proxy for the depth of a supergranule. If $R = D/H$, where D is the cell depth and H its width, Simon & Leighton (1964) suggested that the depth of a solar supergranule could be estimated from its horizontal scale through comparisons with other examples of non-stationary convection found in nature, where $R \sim 5$–10 over a very large range of densities and sizes. Conversely, one can estimate the horizontal scale of a stellar supergranule given its depth.

6. CONCLUSIONS

Starspot lifetimes are of great interest to the astronomical community both from the standpoint of providing a means to measure rotation periods and activity cycles and from a more fundamental plasma physics point of view. Motivated by the recent improvements in stellar tomography made possible by Doppler imaging and exoplanet transits, we have attempted to explain the emerging correlation of stellar spot ages and sizes by considering well-known laws that govern the phenomenon in the Sun.

The solar case provides insight into the physics of spot lifetimes in several ways. First, it is clear that classical diffusion processes lead to spot lifetimes that are too large by at least two orders of magnitude. Recent observations of anomalous, turbulent-driven magnetic diffusion, however, are far more promising, and can account for the solar data if supergranules represent the maximum scale size for diffusion. Theoretically, the magnetic diffusion law should be relatively insensitive to changes in photospheric properties such as the density and magnetic field strength, and should be governed primarily by the supergranule scale length. However, while extrapolation of the solar law is consistent with the longevity of large polar spots on giant and subgiant stars, applying the solar law to active main-sequence stars overestimates the ages of these spots. Increasing the supergranule/diffusion scales on these stars to cover $\sim 20°$ in longitude brings the observations in accord with theory. Hagenaar et al. (1999) hypothesized the presence of a much larger diffusive scale associated with giant cells of 10 times the supergranular length scale. It is also possible, however, that the spot sizes are overestimated on these stars, and the active regions consist of many smaller spots grouped together, which would then reduce the supergranule size needed to explain the observations. This would be the case if, for example, the spots were fragmented due to buffeting by supergranule-scale convection currents.

A much larger sample of stars will be needed to further work along these lines, a difficult task because of inherent ambiguities associated with inverting light curves to generate stellar surface maps. Exoplanet transit mapping combined with Doppler imaging seems to produce the most robust maps, and with the large efforts underway to study such systems, one of the byproducts of these efforts should be a greater understanding of the physical processes at work as starspots grow and dissipate.

The authors thank the referee for comments and suggestions for improvements to the original manuscript.

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As Set

R.T.J McAteer **CUME #429** **Nov 2018**

This CUME is based on the accompanying paper
Bradshaw and Hartigan, 2014, ApJ, 795, 79
"On Sunspot and Starspot Lifetimes "
This is a 5-page short paper with 3 tables and only 1 figure
Attempt all questions.

There are 50 total points available.
A total of 38 points is *expected* to be a passing grade.
Do not allow yourself to get stuck on any one question. Q1 tests your explanatory ability of terms you should know. Q2 and Q3 are mostly calculations. Q4 is a derivation using Maxwell's equations, and Q5 asks for you to create your own non-solar model.

If you find yourself spending a long time at one question, move on and come back to it later. In particular note that
Q1 has 5 points
Q2 has 5 points
Q3 has 15 points
Q4 has 15 points
Q5 has 10 points

I suggest you try to get through all questions inside the first ~90 minutes (so do about 2 minutes per point) and then use the remaining ~30 minutes to go back and complete any parts you have skipped.

Calculators are only to be used for calculations. You may not store equations. You may not use your cell phone at any time. Show all work for full points. Attempt all parts of all questions.

Take a new page for each question. This makes it much easier to award partial credit for incorrect answers.

1: In this question, I would like you to show me you understand the research presented. In 2-3 sentences, describe each of these key terms or phrases from throughout the paper at a level that could be understood by someone with a physics undergraduate degree. Provide an additional sketch for any of the terms if will assist.
(1 point each)

(a) 'diffusion lengthscale is the supergranule size'
(b) 'differential rotation causes shearing over a broad latitude range'
(c) 'pre-main sequence T-Tauri star'
(d) 'Functional form of an empirical relationship'
(e) 'optical depth of light at 550nm approaches 2/3'

5 points

2. Have a look at the Introduction, 4th paragraph. It states that "differential rotation is probably not a factor" in the determination of sunspot lifetime.

Calculate the value of rotation period at the equator that would be necessary for differential rotation to become a factor in determining sunspot lifetime.
(Assume the polar rotation stays at about 33 days)

5 points

3. Lets now move onto Section 2: the Gnevyshev-Waldmeier Rule

(a) The common unit used for measuring sunspot area is MSH. The clue for this measurement unit is in the name - micro solar hemisphere. Show that 1MSH is $(5".8)^2$. (5.8 arcsec squared)

3 points

(b) Equation 1 is derived from observations of the Sun. Sketch a plot that could be used to determine this rule. Include axis titles, units, range of values you would expect to obtain on each axis, data points, and a description of how you would derive this equation and the value for W from your plot.

7 points

(c) Show by calculation that a "if we take an individual spot to cause a light variation of 0.01mag and assume the spot to be completely dark, then the spot area is 1.4×10^{17} m^2"

5 points

4. Lets derive some equations in Section 3. You will need the following

Faraday's Law, $\nabla \times \mathbf{E} = -(\partial \mathbf{B}/\partial t)$

Ohm's Law, $(1/\sigma)\mathbf{J} = \mathbf{E} + (\mathbf{v} \times \mathbf{B})$

Ampere's Law, $\mu_0 \mathbf{J} + (1/c^2)\, \partial \mathbf{E}/\partial t = \nabla \times \mathbf{B}$

(a) In 1 sentence, describe what each of the laws means using terms and phrases that would be understandable to any astronomy PhD student.

6 points

(b) Using Faraday's law, Ohm's law and Ampere's law, derive the induction equation as

$$(\partial \mathbf{B}/\partial t) = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta\, (\nabla^2 \mathbf{B})$$

(Vector Identity $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla . \mathbf{A}) - \nabla^2 \mathbf{A}$)
Ensure you state all assumptions
(HINT: some slowly varying currents should be neglected)

7 points

(c) The equation (2) in the paper differs from the induction equation you derived in 4(a) above. What physical feature of the plasma must be true in order to use the simplified equation (2) in the paper

2 points

5. Study Table 3 and Figure 1.
Star V410 Tau, is a T-Tauri star, and HD199178 may have evolved from a short period binary. Yet both stars have developed large polar spots - a feature that is not observed on the Sun. In this question I want you to show me that you can explain a basic solar model, you can then change that basic solar model to provide some explanation of polar spots, and you can then suggest what observations agree with, or test your new model for polar spots. For any part, provide a sketch if that helps with your description.

(a) Describe how polar fields do develop on the Sun.(i.e., discuss typical sizes, timesscales and velocities of various flows and how they interact with observed field strengths) **3 points**

(b) Suggest what change (or changes) could be made (i.e, what parameter or parameters could be altered) to this solar model to create large polar spots with strong magnetic fields **4 points**

(c) Which feature of V410 Tau or HD199178 agree with your suggestion in 5(b) above, (or if none, then what measurement would need to be made)

3 points