This cume is based on the paper **OSSOS Finds an Exponential Cutoff in the Size Distribution of the Cold Classical Kuiper Belt**, by Kavelaars *et al.* (2021, ApJL, 920, L28). The paper is a short letter and should be a quick read (you may "skim-read" Sect 2.1 and 2.2).

The exam contains 12 questions, worth 55 total points. 41 points (75%) guarantees a pass.

Please make sure your writing is legible. I am not fluent in hieroglyphs and I cannot grade what I cannot read! Also, please show all work and do attempt each problem, showing your thought process even if you cannot solve it completely. If during the exam you have any question please email me (wlyra@nmsu.edu). I will also be accessible at the virtual zoom office `https://nmsu.zoom.us/j/7631131283`.

At the end of the exam, put your responses in question order, scan or photograph them, and upload them to the canvas assignment.

1. The classical Kuiper belt is loosely defined as comprised of objects free from significant perturbation from Neptune. Based on your reading of the paper, explain

   (a) (3 points) what you understand by *cold* classical Kuiper belt, (*Comprehension*)

   The cold classical Kuiper belt is a sub-population of the classical belt that is dynamically cold, i.e., low inclination and low eccentricity orbits, like the eight major planets.

   1 for primordial; 1 for some mention to dynamics: low eccentricity, low inclination, or no past interaction with Neptune; 1 for low frequency of collisions. Full 3 points for explictly identifying that cold means *dynamically* cold.

   (b) (3 points) why this population is supposedly primordial. (*Comprehension*)

   The classical Kuiper belt does not experience intense interaction with Neptune today, but the hot population shows signs of previous interaction with the giant planets, that lead them into orbits of high inclination and moderately high eccentricity. The hot population has also colors similar to the Jupiter trojans, which evidences they come from a similar population. This population was scattered by the giant planets, some ending up at 5AU around Jupiter and others ending up beyond Neptune. Conversely, the orbits of the cold population indicate that they never experienced gravitational interaction with the giant planets. Binaries among the cold KBOs are also so weakly bounded that they would not have survived scattering by the giant planets. This evidences that the cold classicals formed in situ. The weakly bound binaries, as well as low crater count, also evidence that this population does not have a history of collisions.

   1 for each line of evidence.

2. (6 points) The graphs below (Fig. 1) show the results of a planetesimal formation model by streaming instability (Schäfer et al. 2017, A&A, 597, 69, "Initial mass function of planetesimals formed by the streaming instability"), referenced in the cume paper.

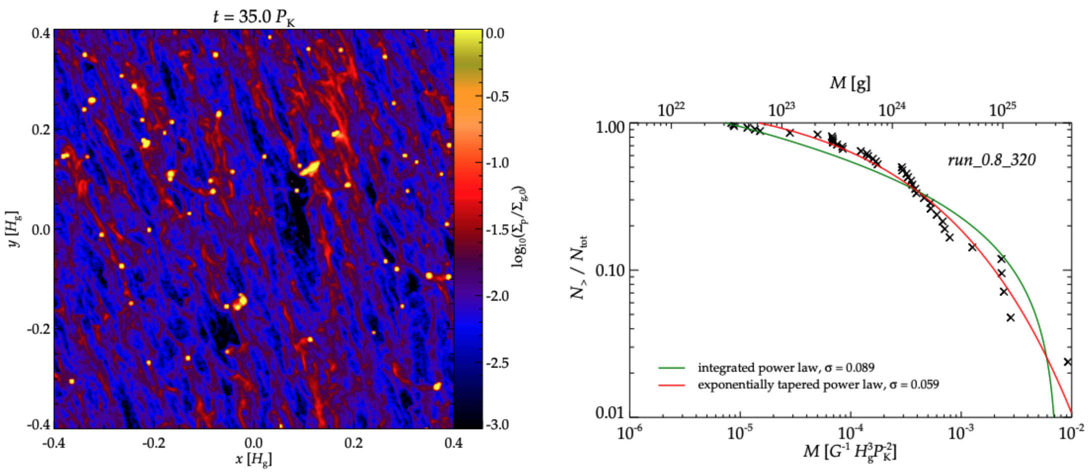

Figure 1: *Left:* Pebble column density Σ_p, integrated over the vertical dimension of the simulation box, as a function of radial location x and azimuthal location y at $t = 35$ orbits (P_K). The box is centered at 2.5 AU, and the length unit H_g is roughly 0.1 AU. Bright dots indicate locations of planetesimal formation. *Right:* the mass distribution of planetesimals formed in the simulation, well-fit by an exponentially-tapered power law. Reproduced from Schäfer et al. (2017).

Briefly ($\sim$ 5 sentences) explain the main result of the cume paper, in terms of the observational result and the implications for models of planetesimal formation.

The paper constructs a luminosity function for the cold classical Kuiper Belt objects, a population with strong evidence of being primordial. As such, these objects should be examples of "planetesimals", the leftover bricks of planet formation. Thus, this population provides the ideal comparison sample to validate or falsify computational models of planetesimal formation. The luminosity function (which for constant albedo reflects the size distribution) seems to be complete at the bright, high-mass, end, showing a sharp decline at this end, with no object bigger than 400 km. Importantly, while previous works suggested that the luminosity distribution was a broken power law, the better statistics provided by the OSSOS survey allows the authors to infer that the high-mass end is not described by a power-law, but by an exponential cutoff. This is in excellent agreement with hydrodynamical computational models of planetesimal formation via streaming instability, which result in mass functions following exponentially-tapered power laws. Although the shape of the function is well-matched, there is a problem with the mass, with computational models necessitating much high masses to trigger the streaming instability than what the CCKBOs show. The paper discusses possible solutions to the discrepancy.

1 for luminosity function, 2 for exponentially tapered power law in the high-mass

end, 2 for comparison to computational models of streaming instability, 1 for noting mass discrepancy.

3. (3 points) The subscript r in the absolute magnitude H_r indicates that the observation is done in the r' SDSS photometric band (5640–6850 Å). Given that Kuiper belt objects are at temperature $\approx$40 K, their thermal emission peaks in the far infrared (70μm). Why is the observation done in optical and not in the far infrared? (*Analysis*)

   KBOs searches are done in reflected light, in the optical, usually with a red filter. Like Pluto and Triton, KBOs are red. Although the thermal emission KBOs peaks in far infrared (and the thermal luminosity is higher than the refleted, given the low albedo), the infrared glow from Earth's atmosphere is much brighter, which translates into less sensitivity. Eliminating the atmosphere (going to space) is prohibitively expensive, and still requires cooling the detector. Searching in reflected light is a good balance between brightness, atmospheric interference, and expense.

   1 for reflected light, 1 for brighter in infrared, 1 for why we would prefer the fainter but cheaper.

4. Absolute magnitudes of minor bodies are quoted not with respect to a distance of 10 pc but of 1 AU (=1.496×10^8 km), more appropriate for bodies in the solar system. Thus, the absolute magnitude H is defined as the magnitude that a body would have if it was 1 AU from the Earth and 1 AU from the Sun, while having a phase angle of 0°.

   (a) (3 points) Explain why this physical configuration is impossible. (*Application*)
   The phase angle is zero when a body is either in conjunction or in opposition (for superior bodies), or inferior conjunction (for inferior bodies). To be at both 1 AU from the Sun and the Earth, the body would necessarily have to be at Earth's L4/L5 points.
   2 for identifying what phase angle is, 1 for geometry.

   (b) (8 points) Considering that the apparent magnitude of the Sun is -26.76, show that

   $$H \approx 5\log_{10}\left(\frac{1330\,\mathrm{km}}{D\sqrt{p}}\right) \quad (1)$$

   where p is the geometric albedo and D is the diameter of the object. (*Application*)
   Solar system magnitudes are defined in the same way as stellar, except that now the absolute magnitude is the flux at 1 AU

   $$H = -2.5\log_{10} F_{1\mathrm{AU}} + C \quad (2)$$

   For the Sun, the apparent magnitude and absolute magnitude are the same, since the Sun is 1 AU away, so $H_\odot = -26.76$. This can be equated to

$$H_\odot = -2.5 \log_{10} F_{\odot,1\mathrm{AU}} + C \tag{3}$$

The flux of an object at 1AU is given by the solar flux $F_{\odot,1\mathrm{AU}}$ reflected (with albedo p) off the object and scaled by its distance from Earth (also 1 AU)

$$F_{1\mathrm{AU}} = pF_{\odot,1\mathrm{AU}} \frac{R^2}{(1\,\mathrm{AU})^2} \tag{4}$$

where R is the radius of the object. Substituting radius for diameter and putting all non-flux terms under the same square

$$F_{1\mathrm{AU}} = F_{\odot,1\mathrm{AU}} \left(\frac{\sqrt{p}D}{2\,\mathrm{AU}} \right)^2 \tag{5}$$

Plugging into Eq 2 and subtracting $H_\odot$

$$H - H_\odot = -2.5 \log_{10} \left(\frac{\sqrt{p}D}{2\,\mathrm{AU}} \right)^2 \tag{6}$$

$$H = 5 \log_{10} \left[\frac{10^{H_\odot/5}\ (2\,\mathrm{AU})}{\sqrt{p}D} \right]^2 \tag{7}$$

$$10^{H_\odot/5}(2\,\mathrm{AU}) \approx 1330\,\mathrm{km} \tag{8}$$

2 for writing down magnitude correctly, 2 for comparing with the magnitude of the Sun, 2 for scaling the reflected flux correctly (1 for albedo, 1 for area/distance scaling), 2 for correct result

(c) (3 points) Given that the abstract of the paper states that the range $H = 5 - 12$ in figures 1 and 2 implies diameters 400-20 km, what is the typical geometric albedo of minor Kuiper belt objects? (*Application*)

Solve for p in the magnitude equation

$$p \approx \left[10^{-H/5} \frac{1330\,\mathrm{km}}{D} \right]^2 \tag{9}$$

leads to 0.07 and 0.11; average 0.09.

2 for derivation, 1 for correct value.

5. Pluto and Charon have absolute magnitudes -0.7 and 1.

   (a) (6 points) What is the combined absolute magnitude when the system is unresolved? (*Application*)

$$
\begin{aligned}
H_1 &= -2.5 \log_{10} f_1 + C \quad \therefore \quad f_1 = 10^{-0.4(H_1 - C)} & (10) \\
H_2 &= -2.5 \log_{10} f_2 + C \quad \therefore \quad f_2 = 10^{-0.4(H_2 - C)} & (11) \\
H &= -2.5 \log_{10} (f_1 + f_2) + C \\
&= -2.5 \log_{10} \left[10^{-0.4(H_1 - C)} + 10^{-0.4(H_2 - C)} \right] + C \\
&= -2.5 \log_{10} \left[10^{0.4C} \left(10^{-0.4H_1} + 10^{-0.4H_2} \right) \right] + C \\
&= \cancel{-2.5 \log_{10} \left(10^{0.4C} \right)} - 2.5 \log_{10} \left(10^{-0.4H_1} + 10^{-0.4H_2} \right) + \cancel{C} \\
H &= -2.5 \log_{10} \left(10^{-0.4H_1} + 10^{-0.4H_2} \right) & (12)
\end{aligned}
$$

Combined $H = -0.9$.

4 for solution (1 for setting up the magnitude equation, 2 for summing the fluxes, 1 for canceling the constant rigorously), 2 for correct value. Hopefully no one will sum the magnitudes...

(b) (4 points) What is the apparent magnitude of the system when in perihelic opposition, seen from Earth 28 AU away? (*Application*)

$$
m = H + 5 \log_{10} \left(\frac{29\,\text{AU} \times 28\,\text{AU}}{1\text{AU}^2} \right) \approx 13.6 \qquad (13)
$$

1 for setting distance modulus, 2 for scaling the incoming flux with distance, 1 for scaling the reflected flux

6. (6 points) The article notes that a discrepancy exists between the model predictions and the observations, with respect to the mass of the objects. Explain the authors' preferred solution for the discrepancy in light of of Fig. 2, which shows the dust continuum emission from the disk around MWC 758, a young (3.5 ± 2 Myr) A3 star. (*Analysis*)

The discrepancy is that the mass needed for streaming instability is 100× larger than allowed by the column density of the cold classical Kuiper belt. If the CCKB is primordial and has not evolved since formation, that invalidates the explanation of the size distribution as having formed from streaming instability. However, the discrepancy relies on the assumption that the Solar nebula was homogeneous, or at least azimutally symmetric (the argument of spreading the mass into a ring of constant column density). However, observations show that some disks around young stars have extreme deviations from axisymmetry, as dramatically shown in the observation of MWC 758, where the dust is concentrated in two blobs. Inside these blobs the concentration of dust can locally achieve 100× higher than the average density, and trigger planetesimal formation by streaming instability. Once the planetesimals are formed, they leave the dust trap and disperse azimuthally.

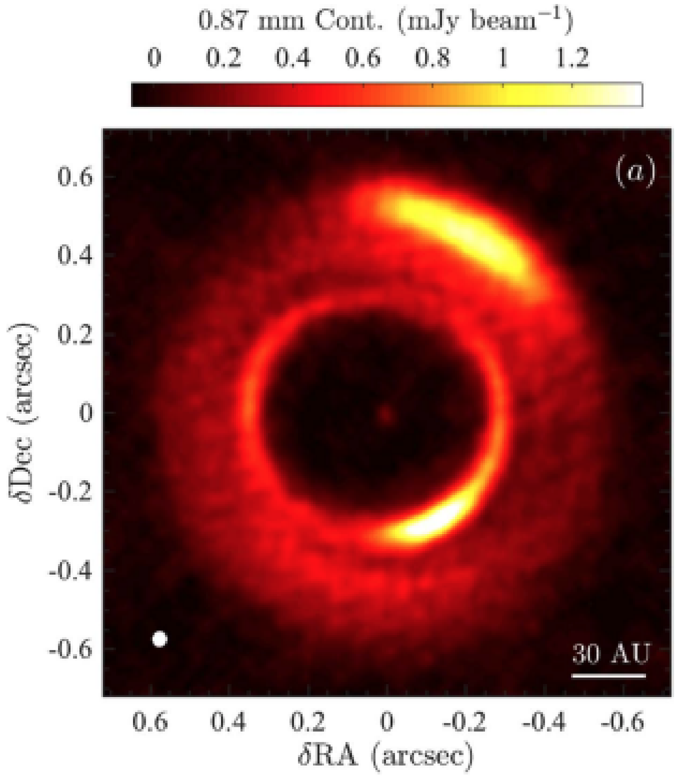

Figure 2: ALMA 0.87 mm dust continuum emission from MWC 758 with a beam size 43 × 39 mas (6.9 × 6.2 au; labeled at the lower left corner). North is up and east is to the left. Reproduced from Dong *et al.* (2018, ApJ, 860, 124).

3 for noting the discrepancy relies on axisymmetry, 3 for understanding that the observation shows non-axisymmetric distribution of dust, where density can be much higher.

7. A recent model suggests that Jupiter formed beyond 30 AU, and perhaps as far out as 45 AU (Öberg & Wordsworth 2019, AJ, 158, 194, "Jupiter's Composition Suggests its Core Assembled Exterior to the N_2 Snowline"). This is based on the idea that the nitrogen and argon abundance of Jupiter are explained if these elements were accreted as solids, which would happen if Jupiter formed beyond the N_2 and Ar snowlines.

   (a) (4 points) How does the existence of the cold classical KBO population contradict this idea? (*Analysis*)

   If formed at 45 AU and migrated to 5 AU, Jupiter would have obliterated the cold classical population as it migrates *through* it. Formation at 30 AU is less disruptive, since Jupiter's gravitational radius of influence (Hill radius) is about 10% of the semimajor axis (so, up to 33 AU). In principle, Jupiter forming at 30 AU can leave the CCKB undisturbed. Yet, the formation of Jupiter so close to the CCKB could lead to other disturbances (resonances, gap formation, dynamical excitation, evection, etc), that need to be quantified.

Ideally would constrast formation at 45 AU (disruptive, 2 points) with formation at 30 AU (less disruptive, still dangerous and needs to be quantified, 2 points). But will accept 4 points for disruptive if formed at 45 AU.

(b) (6 points) Suggest a theoretical study to test if the "far Jupiter" hypothesis can be reconciled with the cold classical Kuiper Belt. (*Synthesis*)

1. A hydrodynamical simulation to check if Jupiter's gap if formed at 30 AU reaches the boundary of the CCKB, preventing objects from forming by depleting the gas.
2. A N-body model of Jupiter at 30 AU and the CCKBOs, to check if their inclinations get excited beyond $i = 4^\circ$.

3 for a plausible idea, 3 for a well-thought experiment.