Solution and Rubric

Overview

This cume is based on the paper **$^{13}C^{17}O$ suggests gravitational instability in the HL Tau disc**, by Booth & Ilee 2020. The paper is a short letter and should be a quick read.

The exam contains 9 questions or subquestions, worth 52 total points. 39 points (75%) guarantees a pass.

If during the exam you have any question I will be accessible at the virtual zoom office `https://nmsu.zoom.us/j/7631131283`.

Context

1. (4 pts) We cannot directly measure the masses of protoplanetary disks. Explain why based on your reading of the article. (Comprehension).

   The bulk (80%) of the mass is H_2, which is invisible. Mass is inferred by proxies, such as dust or CO, and scaled by their expected abundance relative to H_2.

   3 pt if mentioned cannot measure main component (or must use a tracer). 1 pt if H_2 is identified.

Background

2. Let us now consider how the observations tie to physical conditions.

   (a) (6 pts) Assuming that vertically the gas is in hydrostatic equilibrium, show that the density is

$$\rho(z) = \rho_0 \exp\left(-\frac{z^2}{2H^2}\right) \tag{1}$$

   where H is the pressure scale height, ρ_0 is the midplane density, and z is the height above the midplane. (Knowledge)

   According to the diagram of forces for a gas parcel, in Fig 1, the force equilibrium in the z direction is (appropriate substitutions shown in grey font)

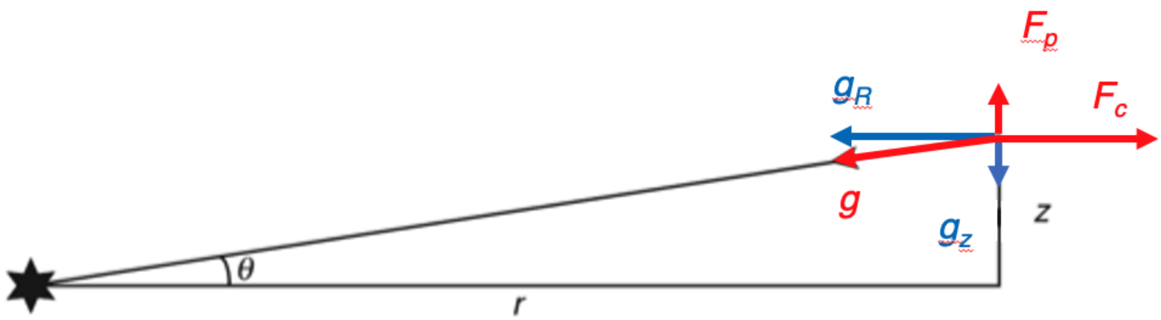

Figure 1: Auxiliary force diagram. F_p is the pressure force, g is the stellar gravity, and F_c the centrifugal force.

$$
\begin{aligned}
\frac{1}{\rho}\frac{dp}{dz} &= g_z \\
\frac{1}{\rho}\frac{dp}{dz} &= -\frac{GM}{r^2}\cos\theta \\
\frac{1}{\rho}\frac{dp}{dz} &= -\frac{GM}{r^3}z \qquad \left(\cos\theta = \frac{z}{r}\right) \\
\frac{d\ln\rho}{dz} &= -\frac{\Omega^2}{c_s^2}z \qquad \left(p = \rho c_s^2 \quad \text{and} \quad \Omega \equiv \sqrt{\frac{GM}{r^3}}\right) \\
\rho &= \rho_0 \exp\left\{-\frac{\Omega^2 z^2}{2c_s^2}\right\} \\
\rho &= \rho_0 \exp\left\{-\frac{z^2}{2H^2}\right\} \qquad \left(H \equiv \frac{\Omega}{c_s}\right)
\end{aligned}
\tag{2}
$$

2 pt if correctly identified forces; 2 pt if 3rd equation but not identify H; 2 pt full

(b) (6 pts) The paper states that "HL Tau is a class I-II protostar surrounded by both a circumstellar disk and an envelope", and that it is accreting at a rate of $\dot{M} = 10^{-7} M_\odot\ \mathrm{yr}^{-1}$. What can you infer from this about the evolutionary stage of HL Tau compared to other young stellar objects? (Application)

Having an envelope, the source is probably very young, still accreting from the molecular cloud. That the accretion rate is high ($10^{-7} M_\odot$/yr, when the T-Tauri average is $10^{-8} M \odot$/yr) corroborates the youth. Indeed, HL Tau is estimated to be 0.5 Myr old.

3 pt if noting the presence of the envelope (or class I for that matter) means it is accreting from the cloud, so it is still very young; 3 pt if realizing the accretion

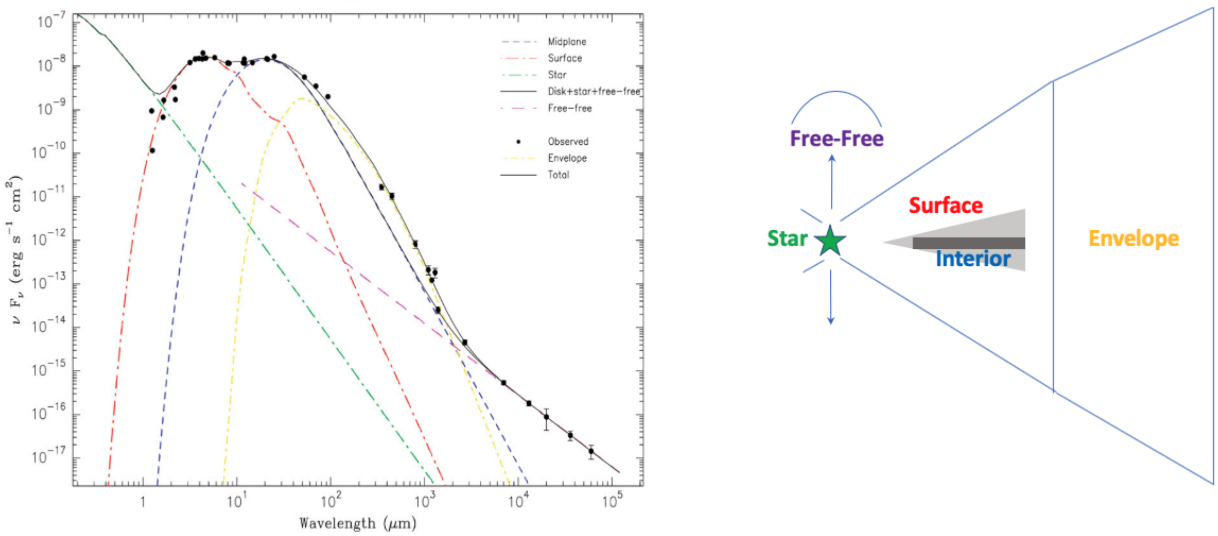

Figure 2: Disk structure and SED. Showing the solution of the labels, and color-coding the figure in the right-hand-side.

rate is high and also means youth.

Diagnostics

3. Figure 2 shows the observed spectrum of a class I source, closely resembling HL Tau. The plot on the right hand side shows different regions of the disk and the plot on the left hand side shows a fit to the observed SED summing the contributions from each of these regions.

   (a) (6 pts) The figure shows a region of "flaring", which means that the aspect ratio (H/r) increases with radius r. Based on the pressure scale height you found in Eq (1), explain why disks should be flared. In a few sentences, explain what is the consequence for the temperature structure of disks if they are flared. (Knowledge/Application)

   Disks will flare if $dh/dr > 0$, where $h = H/r$, meaning that the aspect ratio increases with distance. Given $H = c_s\Omega^{-1}$, if the temperature falls as $T \propto r^{-q}$, then the dependence of H on distance is $H \propto r^{(3-q)/2}$, and the aspect ratio thus $h \propto r^{(1-q)/2}$. If the temperature decreases more weakly than $q = 1$ (i.e., $T \propto 1/r$), disks should flare. A flared disk intercepts more starlight, and thus flared disks should be hotter).

   2 pt: Identify that the temperature must increase weakly with distance, so Ω dominates (less gravity, more puff up). 2 pt: for the mathematical detail. 2 pt:

explaining the effect of insolation. [update: in view of canceling Q2a (they did not find $H = \Omega/c_s$ in that question, the rubric is changed to 3 pts for each of the other two concepts answered correctly.]

(b) (12 pts) The labels are omitted in the legend on the left plot. Identify which curves correspond to "star", "surface", and "interior", and explain your reasoning. (Knowledge/Analysis)

Star: green. Surface: red: Interior: blue.
The star is in optical (blocked by envelope), the hottest blackbody.
The surface is emission from super-heated dust that absorbs the optical radiation from the star and re-radiates in infrared. Its height is the optical depth of 1 in visible.
The interior of the disk is reprocessed radiation from the surface. Half of the flux from the surface is radiated to space and half is radiated down, heating up the interior. As such, it will be colder than the surface.

3x (2 pts for each identification, 2 pts for the explanation)

(c) (4 pts) Class I sources have an extended envelope in addition to "surface" and "interior". The model shown in the figure also requires free-free emission to explain the spectrum. Identify these two extra curves and explain your reasoning. (Analysis)

Envelope: yellow; magenta: free-free. The envelope should be the coldest blackbody. The free-free emission is a bremsstrahlung spectrum (independent of frequency in flux, linear with frequency in νF_ν), not a blackbody.
2x(1 pt for identification, 1 pt for explanation)

(d) (4 pts) The article states that if line emission is optically thick the resulting mass is underestimated, and that optically thin lines are a better tracer of disk mass. Explain why, based on Eq (1). (Application)

Since the density distribution is Gaussian, the mass is by and large confined around the midplane, with $\pm$2H confining 95% of the mass. Optically thick lines will be emitted from high up at the disk atmosphere, where the gas is too thin and dynamically irrelevant. Extrapolations from this layer to the total mass will be prone to large errors. In optically thin lines on the other hand the radiation produced may be coming from the massive layers below and possibly probing the midplane, where planet formation occurs.

1 pt if identifyting that optically thick lines trace the atmosphere. 2 pt is noticing that due to the Gaussian distribution most of the mass is in a thin slab around the midplane; 1 pt if pointing out that optically thin can trace these massive layers.

Results

4. Although self-gravity of protoplanetary disks is very often negligible, the paper claims that their results would render vast swaths of the disk of HL Tau gravitationally unstable. This is quantified by the "Toomre Q" parameter, quoted in Eq (4) of the paper.

$$Q = \frac{c_s \Omega}{\pi \Sigma G} \tag{3}$$

where c_s is the sound speed, Ω is the Keplerian frequency, Σ is the column density and G is the gravitational constant. The disk is prone to gravitational instability if $Q < 1$.

(6 pts) Consider the radial dependency of the various parameters to explain the near-parabolic shape of Q in Figure 3. Specifically, why is there a "sweet spot" around 50-100 AU? (Application)

c_s, Ω , and Σ, all fall with distance with different rates. The shape of Q implies that the product $c_s\Omega$ falls with distance faster than Σ does. At the inner disk the disk is hot and the Keplerian frequency is fast: pressure and shear stabilize against the gas selfgravity. As we move outwards the disk cools down and the orbital periods are larger: the minimum of Q is achieved. Even further out, the column density is too thin. There is not enough mass and Q starts to rise again as self-gravity becomes less important.

2 pt for identifying that c_s, Ω, and Σ fall with distance. 2 pt for realizing $c_s\Omega$ must fall faster than Σ. 2 pt for noticing that in the outer disk there is not enough mass.

Conclusion

5. As implications for planet formation, the authors state that the presence of a gap between 65 and 74 AU, right where they predict Q to be near or below 1, is an indication that if a planet is responsible for the gap, it must have been formed by gravitational instability.

(4 pts) The authors strengthen their conclusion by citing a work that measured a spiral feature in HL Tau, a feature that is expected from marginally gravitationally unstable disks. Considering that the spiral is detected in HCO+, an optically thick line, and that HL Tau is a class I source, what else could be causing the spiral? (Analysis)

The shear will turn any localized event into a spiral. That HCO+ is optically thick means that the spiral is seen very high up in the disk surface. It could be caused by any event that alters the shape of the disk surface, such as infall, or the tides from a companion, something that is common in a cluster environment.

4 pt for any plausible possibility, 2pt for an interesting idea.