Cume 353: Cosmology

Passing grade for the cume is 70 points.

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- **10pt** Assuming that the fluctuations are still in the linear regime, how should the power spectrum $P(k)$ look at redshift $z = 10$? Assume that at that redshift the growth-rate function of the amplitude of linear perturbations δ is $1/10$ of its present-day value.

- **20pt** On a related issue: the amplitude of perturbation defines the epoch of formation of different astronomical objects. Suppose we have three cosmological models which are normalized in such a way that all of them predict the same abundance of clusters of galaxies at $z = 0$. Also assume that all the models have the same density of dark matter. The models are: (a) the flat universe dominated by dark matter (no cosmological constant or dark energy), (b)the LCDM model: flat cosmology with some mixture of the cosmological constant and dark matter, and (c) the open universe with no cosmological constant. Which models predicts more clusters of galaxies at redshift $z = 3$? Explain your answer.

- **30pt** Position of the peak in the spectrum of fluctuations corresponds to the distance to the horizon at the moment of equality. Find the redshift of the moment of equality assuming the following parameters of the Universe at $z = 0$: $\Omega_{\mathrm{matter}} = 0.3$, $\Omega_{\mathrm{relativistic}} = 10^{-5}$, $h = 0.70$. Here $\Omega_{\mathrm{relativistic}}$ is total contribution of radiation and neutrinos to the density of the Universe.

Page 1

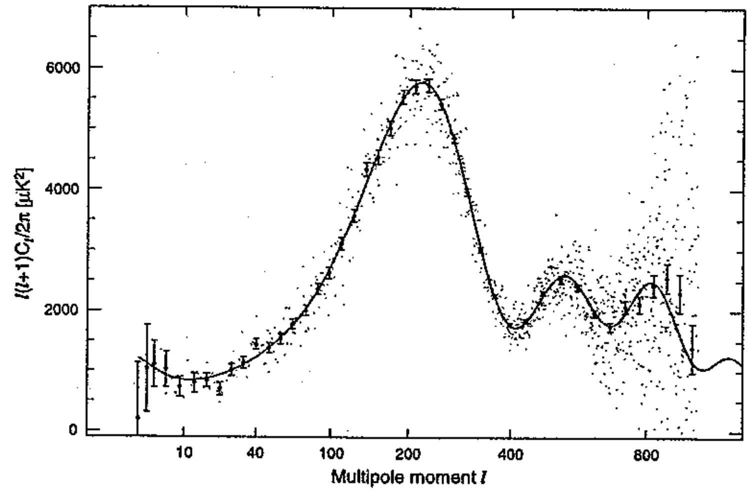

Figure 2. Temperature angular power spectrum corresponding to the *WMAP*-only best-fit ΛCDM model. The gray dots are the unbinned data; the black data points are binned data with 1σ error bars including both noise and cosmic variance computed for the best-fit model.

t al. 2003, 2007), notably
fluctuations, n_s (Figure 3).
ɿd using the Gibbs-cleaned
008) and in Section 2.1.1,
ba⁻ᵈ maps. Their mean
igu 5, and have mean
han the KaQV template-
ndependent method. The
hanged by less than 0.3σ
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ıstraints are little affected
ence in central values from
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ihood Details

smological analysis has a
have been fixed using our
effect of these choices on
first two are the treatment
he treatment of the beam
2009). The multifrequency
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Hinshaw et al. (2007) and
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data, whereas the VW data
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ering A_{ps} to the VW value,
h^2, and σ_8, all within 0.4σ
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K^{2}sr with no point source

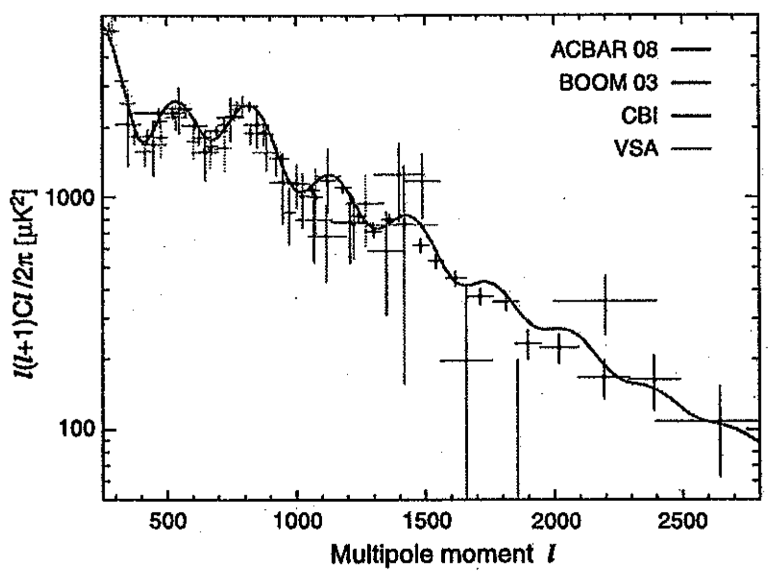

Figure 7. Best-fit temperature angular power spectrum from *WMAP* alone (red), which is consistent with data from recent small-scale CMB experiments: ACBAR, CBI, VSA, and BOOMERANG.

is found to be consistent with the effects of noise, tested with simulations. We also confirm that the effect on parameters is even less for ΛCDM models using *WMAP* with external data, and that the choice of mask has only a small effect on the tensor amplitude, raising the 95% confidence level by $\sim 5\%$.

Finally, we test the effect on parameters of varying aspects of the low-ℓ TT treatment. These are discussed in Appendix B, and in summary we find the same parameter results for the pixel-based likelihood code compared to the Gibbs code, when both use $\ell \leqslant 32$. Changing the mask at low-ℓ to KQ80, or using the Gibbs code up to $\ell \leqslant 51$, instead of $\ell \leqslant 32$, has a negligible effect on parameters.

3.2. *Consistency of the ΛCDM Model with Other Datasets*

the Abell/ACO cluster (Miller & Batuski
uster (Tadros, Efstathiou, & Dalton 1998), and
(HTP00), already concluded that their data
model power spectra with baryonic oscillations
hout. However, the oscillations at large scales
re 36, notably the dip at $k \sim 0.035\ h\ \mathrm{Mpc}^{-1}$ and
.05 $h\ \mathrm{Mpc}^{-1}$, are substantially larger than pre-
an 1 ΛCDM concordance model illustrated in
onfirmed, such a feature would challenge the
dance model with scale-invariant initial con-
eference for a large baryon fraction is also seen
letails below), which, however, shows that our
nonetheless perfectly consistent with the con-
on fraction—about one-sixth of the 100,000
ave a baryon fraction below the *WMAP* value

so illustrates why galaxy clustering data is so
to CMB measurements. The 100,000 gray
a Monte Carlo Markov chain analysis of the
le flat scalar adiabatic models parameterized by
dark energy, dark matter, and baryonic matter,
ex and amplitude, and the reionization optical
hasized by Eisenstein et al. (1999) and Bridle
MAP alone cannot determine Ω_m to better than a
so because of a strong degeneracy with other
rtunately, the *WMAP* degeneracy banana in
n to be almost orthogonal to the SDSS degen-
means that combining the two measurements
htens the constraints on all the parameters in-
generacy—notably Ω_m, h, and σ_8.
SDSS results in a larger context, Figure 38
wi other measurements of the matter power

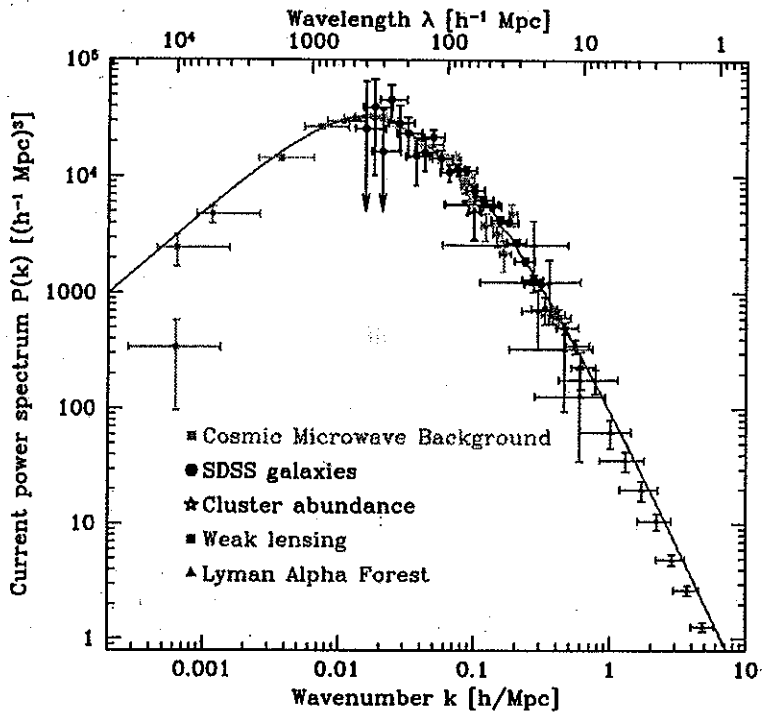

FIG. 38.—Comparison of our results with other $P(k)$ constraints. The location of CMB, cluster, lensing, and Lyα forest points in this plane depends on the cosmic matter budget (and, for the CMB, on the reionization optical depth τ), so requiring consistency with SDSS constrains these cosmological parameters without assumptions about the primordial power spectrum. This figure is for the case of a vanilla flat scalar scale-invariant model with $\Omega_m = 0.28$, $h = 0.72$, and $\Omega_b/\Omega_m = 0.16$, $\tau = 0.17$ (Spergel et al. 2003; Verde et al. 2003; Tegmark et al. 2003), assuming $b_* = 0.92$ for the SDSS galaxies. [*See the electronic edition of the Journal for a color version of this figure.*]

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APPENDIX

97. Models and notation

This book deals with departures from an ideal homogeneous and isotropic Friedmann-Lemaître cosmological model. The homogeneity and isotropy of the background model imply that all physical variables can be expressed as functions of the proper cosmic time t kept by an observer at rest in a patch of fluid, the standard convention being that $t = 0$ at the singular epoch of the big bang. The proper distance between two chosen particles in the model, reckoned along a hypersurface of fixed t, must scale with time as

$$r(t) = a(t)x, \tag{97.1}$$

where x is a constant for the pair and $a(t)$ is the universal expansion parameter. The wavelength of a free photon stretches with other lengths as $a(t)$, so the wavelength λ_0 observed now, epoch t_0, of radiation emitted at epoch t at wavelength λ by an object comoving with the fluid is

$$\lambda_0 = \lambda a(t_0)/a(t) = \lambda(1 + Z), \tag{97.2}$$

where Z is the cosmological redshift. The redshift often is used as a label of an epoch.

The rate of increase of proper separation of a pair of particles in the model is

$$\frac{dr}{dt} = \frac{\dot{a}}{a} r = H(t)r. \tag{97.3}$$

The dot means the derivative of a with respect to cosmic time. $H(t)$ is Hubble's constant at epoch t: the present value is written as

$$H_0 = 100\, h \text{ km s}^{-1} \text{ Mpc}^{-1},$$

$$H_0^{-1} \sim 3 \times 10^{17}\, h^{-1}\text{s} \sim 1 \times 10^{10}\, h^{-1} \text{ y}. \tag{97.4}$$

The dimensionless parameter h reflects the uncertainty in this important quantity. It is generally thought to be in the range

$$0.5 \lesssim h \lesssim 1. \tag{97.5}$$

Equation (97.3) gives the redshift of an object at proper distance $r \ll cH_0^{-1}$:

$$Z \simeq H_0 r/c. \tag{97.6}$$

This is a good approximation at $Z \ll 1$. A convenient measure of the distance to the horizon is the Hubble length

$$r_H = cH_0^{-1} = 3000\, h^{-1}\,\mathrm{Mpc}. \tag{97.7}$$

This is about the distance free radiation has travelled since the big bang.

The evolution of the model is determined by the dynamic equation (eq. 7.13)

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho_b + 3p_b/c^2) + \Lambda/3, \tag{97.8}$$

with the energy equation

$$d\rho_b/dt = -3(\rho_b + p_b/c^2)\,\dot{a}/a. \tag{97.9}$$

The mass density is ρ, the pressure is p, and the subscripts refer to the background model.

Equations (97.8) and (97.9) can be integrated once:

$$\frac{\dot{a}^2}{a^2} = \frac{8}{3}\pi G\rho_b + \frac{\Lambda}{3} - \frac{R^{-2}}{a^2}. \tag{97.10}$$

The constant of integration R^{-2} appears in the expression for the line element,

$$ds^2 = g_{ij}\,dx^i dx^j = c^2dt^2 - \frac{a^2dx^2}{1 - x^2/R^2c^2} - a^2x^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{97.11}$$

The coordinates x, θ, ϕ are comoving, fixed to fluid elements. To simplify equations, I have given R units of time rather than length. The proper radius of curvature of the hypersurface t = constant is $a|R|c$. In a closed

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  - (a) $l < 100$: fluctuations in the gravitational potential at the surface of last scattering and the gravitational redshift. Photons, that move from low grav.potential lose energy and, thus, make slightly colder spots on the sky. Photons from high grav.potential make positive ΔT.
  - (b) $l = 100 - 1000$: The Doppler shift due to velocities in acoustic oscillations: $\Delta\nu/\nu = \Delta V/c$.
  - (c) $l > 1000$: Acoustic peaks are dumped by a combination of the Silk effect (diffusion of photons) and effects of finite thickness of the recombination.

The plot on Page 2 shows the power spectrum of density fluctuations in the Universe $P(k)$. The full curve is the LCDM theory and different symbols are observations. The plot is for redshift $z = 0$

- **20pt** Note that the axes have funny units. The comoving wave number k is in units of h/Mpc and the power spectrum is in units of $(h^{-1}Mpc)^{-3}$. Masses are typically presented in units of mass $h^{-1}M_{\odot}$. Here h is the Hubble constant in units of 100 km/s/Mpc. Explain why each of the measured quantities of $P(k)$, mass, and wavenumbers scale with the Hubble constant in this way.

  Answer: This is all about measuring distances using the Hubble flow: $V = Hr$. The recession velocity is measured observationally, and it does depend on H. We then get the distance $R = V/H \propto h^{-1}$. If mass of an astronomical object is measured dynamically, it scales as $M \propto v^2R/G$, where v is a measure of random velocities in the object and R is its radius. Then v is measured observationally (e.g., los velocities in groups of galaxies, or temperature of gas in clusters of galaxies).Then, $M \propto R \propto h^{-1}$. Wavenumber is $k = 2\pi/\lambda \propto h$. The power spectrum $P(k)$ is defined in units of inverse volume. Thus, the scaling is $P \propto h^3$

- **10pt** Assuming that the fluctuations are still in the linear regime, how should the power spectrum $P(k)$ look at redshift $z = 10$? Assume that at that redshift the growth-rate function of the amplitude of linear perturbations δ is 1/10 of its present-day value.

  Answer: In the linear regime all waves grow with the same rate: $P(k, z) = \delta^2 P(k, z = 0)$

- **20pt** On a related issue: the amplitude of perturbation defines the epoch of formation of different astronomical objects. Suppose we have three cosmological models which are normalized in such a way that all of them predict the same abundance of clusters of galaxies at $z = 0$. Also assume that all the models have the same density of dark matter. The models are: (a) the flat universe dominated by dark matter (no cosmological constant or dark energy), (b)the LCDM model: flat cosmology with some mixture of the cosmological constant and dark matter, and (c) the open universe with no cosmological constant. Which models predicts more clusters of galaxies at redshift $z = 3$? Explain your answer.

  Answer: The number-density of clusters of galaxies depends on the amplitude of perturbations on cluster scales: the larger the amplitude, the larger is the number of clusters to form. Thus, the model, which predicts the largest amplitude of perturbations, is the model with more clusters. In turn, the growth-rate of perturbations $\delta(a)$ depends on the density of matter $\Omega_{\rm matter}(a)$, where $a = 1(1+z)$ is the expansion parameter. For the plain-vanilla cosmological model (a) $\delta(a) \propto a$. This is the fastest growth possible. In the LCDM the fraction of mass in matter was declining with time resulting in slowing down the growth of perturbations. Effect is even stronger for the open model. Once all of them are normalized to the same amplitude of perturbations at $z = 0$, the open model will have the largest $\Omega_{\rm matter}$ at $z = 3$, the largest amplitude of perturbations, and, thus, the largest abundance of clusters of galaxies.

- **30pt** Position of the peak in the spectrum of fluctuations corresponds to the distance to the horizon at the moment of equality. Find the redshift of the moment of equality assuming the following parameters of the Universe at $z = 0$: $\Omega_{\rm matter} = 0.3$, $\Omega_{\rm relativistic} = 10^{-5}$, $h = 0.70$. Here $\Omega_{\rm relativistic}$ is total contribution of radiation and neutrinos to the density of the Universe.

  Answer: The moment of equality is the time when the density of the non-relativistic matter (dark matter plus baryons) is equal to the density of relativistic particles: $\rho_{\rm dm+baryons} = \rho_{\gamma+\rm neutrinos}$. The density of normal matter changes with the expansion parameter as $\rho_{\rm matter}(a) = \rho_{0,\rm matter}/a^3$ (mass in a comoving volume is preserved). For the relativistic particles there is another effect: energy of each particle (photon or neutrino) declines as $E = h\nu \propto a^{-1}$. Thus, $\rho_{rmrelativistic}(a) = \rho_0/a^4$. Densities at present are $\rho_{\rm whatever} = \Omega_{\rm whatever}\rho_{\rm crit}$

  Equate $\rho_{\rm dm+baryons}(a) = \rho_{\gamma+\rm neutrinos}(a)$ and get the moment of equality:

  $$a_{eq} = \Omega_{\rm relativistic,0}/\Omega_{\rm matter,0} = 3.3 \times 10^{-5}. \qquad (1)$$

  Then, $(1+z)_{eq} = 1/a_{eq} = 30000$.

KLYPIN SOLUTION

(a) $dF_G = F_G$ (there is no "d" in this)

$$\vec{F}_G = -\frac{GM(r)\,dm}{r^3}\cdot\vec{r}$$

$$\frac{dU}{dm} = -\frac{GM(r)}{r} - \int_r^\infty \frac{4\pi G\rho(y)y^2dy}{y}$$

$$= -\frac{GM(r)}{r} - \int_r^\infty \frac{G\,dm}{y}$$

(b) In general,
+ (d)

$$U = +\frac{1}{2}\int \rho\, U\, dV = \frac{1}{2}\int U\, dm$$

For a spherical system:

$$U = +\int_0^R 4\pi\rho(r)r^2dr\cdot\left[-\frac{GM(r)}{r} - \int_r^\infty \frac{4\pi G\rho(y)y^2dy}{y}\right]$$

For constant-density

$$\rho(r) = \text{Const} = \rho_0 = \frac{3M}{4\pi R^3}$$

$$\int_r^\infty 4\pi G\rho(y)y\,dy = 4\pi G\rho_0\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$$

$$\frac{GM(r)}{r} = \frac{4\pi}{3}G\rho_0 r^2$$

$$U = -\frac{(4\pi\rho_0)^2}{2}G\int_0^R r^2dr\left\{\frac{R^2}{2} - \frac{r^2}{6}\right\} = -\frac{(4\pi\rho_0)^2}{2}G\,R^5\frac{2}{3\cdot 5}$$

Thus,

$$U = -\frac{3}{5}\frac{GM^2}{R}$$