#334

Cume: Cosmology

Passing grade for the cume is 30 points

1. **5pt** Estimate the present day density contribution of radiation Ω_R.
2. **10pt** Assuming that the Universe is flat and that at $z = 0$ the dominant contrubutions are coming from the dark matter $\Omega_M = 0.3$ and from the cosmological constant Ω_Λ, what is the Ω_M at $z = 100$?
3. **5pt** What is the difference in the temperature of CMB at $z = 1000$ between two models: one with the cosmological constant $\Omega_\Lambda = 0.7$ and another without the cosmological constant? All other parameters are the same.
4. **5pt** Assuming that at present $\Omega_R = 10^{-4}$, $\Omega_M = 0.3$, and $\Omega_\Lambda = 0.7$, estimate the redshift of the moment of equality.
5. **10pt** At $a = 0.1$ an observer finds two galaxies almost along the line-of-sight with the relative velocity 1000 km/s. Assuming that the galaxies participate in a perfect Hubble flow, find proper and comoving distance between the galaxies. $\Omega_M = 0.3$, and $\Omega_\Lambda = 0.7$.
6. **10pt** Two flat cosmological models at $a = 1$ have the same Hubble constant and the same spectrum and amplitude of perturbations. One model has a cosmological constant $\Omega_\Lambda = 0.7$, another does not have the cosmological constant ($\Omega_\Lambda = 0$). Which model predicts more quasars at redshift 5?

Relations and constants

- Relation between the expansion parameter a and redshft z: $a = 1/(1+z)$
- Temperature of CMB $T_0 = 2.73$K
- Radiation density constant $a = 7.56 \times 10^{-15} erg\ cm^{-3}\ K^{-4}$
- Speed of light $c = 3 \times 10^{10} cm\ sec^{-1}$
- Solar mass $M_\odot = 2 \times 10^{33} g$
- The Hubble constant at $a = 1$ is $H_0 = 70$km/s/Mpc. The critical density $\rho_{\rm crit} = 9.24 \times 10^{-30} g\ cm^{-3}$
- The Hubble law: $v = Hr$

Natural Dark Energy

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It is now well accepted that both Dark Matter and Dark Energy are required in any successful cosmological model. Although there is ample evidence that both Dark components are necessary, the conventional theories make no prediction for the contributions from each of them. Moreover, there is usually no intrinsic relationship between the two components, and no understanding of the nature of the mysteries of the Dark Sector. Here we suggest that if the Dark Side is so seductive then we should not be restricted to just 2 components. We further suggest that the most natural model has 5 distinct forms of Dark Energy in addition to the usual Dark Matter, each contributing precisely equally to the cosmic energy density budget.

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The idea that most of the matter in the Universe is in a cold, dark, nearly collisionless form has become firmly enshrined as part of the Standard Model of Cosmology. Like the expansion of the Universe in the 1940s or the Big Bang paradigm in the 1970s, the existence of Dark Matter (DM) is now only discounted by a handful of curmudgeons and crackpots [1]. Part of the reason why DM has become so old-hat is that its enigmatic nature has been eclipsed by an even weirder flavour of darkness.

Perhaps the greatest mystery in the whole of modern science is the nature of the so-called Dark Energy (DE) [2]. This bizarre component drives the accelerated expansion of the Universe and dominates the overall energy budget, making the geometry of space very close to flat. The most prosaic explanation for the DE is that it is simply the energy density of the vacuum, and we are then left simply with the problem of why its obvious value (one Planck mass per Planck volume) would be about 10^{120} times bigger than the measurements indicate [3].

There are many theoretical ideas for the physical basis of the DE. However, it is fair to say that none of them are particularly well motivated, and many of them appear like acts of desperation on the part of theorists [4].

The evidence for DE comes from measurements like those of distant supernovae, with the effects of the DE coming through the evolution of the background cosmology via appropriate integrals over the reciprocal of the Hubble expansion rate. The Friedmann equation can be written in terms of the Hubble parameter as

$$H(z) = H_0\big\{\Omega_{\rm R}(1+z)^4 + \Omega_{\rm M}(1+z)^3 + \Omega_{\rm K}(1+z)^2 + \qquad + \Omega_\Lambda(1+z)^0\big\}^{1/2},$$

supplemented with the constraint equation

$$\Omega_{\rm R} + \Omega_{\rm M} + \Omega_{\rm K} + \Omega_\Lambda = 1.$$

Here $\Omega_{\rm R}, \Omega_{\rm M}, \Omega_{\rm K}$ and Ω_Λ represent the fraction of the critical energy density $(3H^2c^2/8\pi G)$ in the form of radiation, matter, curvature and vacuum, respectively. Each density component has an associated pressure, which is what makes each evolve differently, with energy conservation leading to $\rho \propto (1+z)^{3(1+w)}$. The 'equation of state' parameter $w \equiv p/\rho c^2$, is thus $\frac{1}{3}$, 0, $-\frac{1}{3}$ and -1 for radiation, matter, curvature and vacuum, respectively.

One of the motivations for exploring more general cosmological components was the simple observation that there's something missing from the Friedmann equation – in other words, cosmologists asked themselves: 'How come there's no $(1+z)^1$ term?' As we shall propose below, this term is not really 'missing', it is just 'dark'.

This realization, coupled with the fact that $w \leq -\frac{1}{3}$ is required for acceleration, led to the generalization of the cosmological constant to the so-called 'quintessence' concept, i.e. a component which is bracketed by the properties of curvature and vacuum (and we can regard $w = -\frac{1}{3}$ as simply another fluid, even within a flat Universe).

Measurements of the temperature and blackbody spectrum of the Cosmic Microwave Background show that the photon energy density is very small in today's Universe [5]. We also know that the photon's less spin-challenged cousin, the graviton, makes a negligible contribution. Hence we can drop the $\Omega_{\rm R}$ component in the Friedmann equation. The same would be true of any putative new fluid with $w = \frac{2}{3}$, etc., since these would decay even more rapidly as the Universe expands.

However, at the other end of the Friedmann equation we have components which actually *increase* as the Universe expands, so-called 'phantom' DE [6]. This allows us to explore components with $w \leq -1$, which will make the Universe end with what has been called 'the Big R.I.P.'

Since there are many possibilities for dark energy, cosmologists often use historical or philosophical principles to navigate among the choices. One particularly useful idea was espoused by Walter of Ockham [7] in the 14th century, when he said '*entia sunt multiplicanda praeter necessitatem*', which is loosely translated as 'you can't

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get enough of a good thing'. In other words, if dark energy is so appealing, then let's have more of it!

One of the so-called 'problems' with the Standard Model of Cosmology [8] is that there are several apparent coincidences with no obvious physical explanation [9]. However, in this paper we wish to stress the inverse of this, namely that the *most natural* solution is that there are multiple forms of DE [10].

We suggest here that the most reasonable cosmological model should have each of the major constituents contributing equally to the cosmic energy budget of a spatially flat Universe. In other words we expect $\Omega_{D1} = \Omega_{D2} = \Omega_{D3} = \ldots = \frac{1}{N}$, with N being the total number of kinds of Darkness [11].

The model with the optimal level of naturalness has 6 Dark flavours, each contributing $\frac{1}{6}$ to the total energy density. Thus, in additional to the common-or-garden Dark Matter, we also have components which scale as $(1+z)^2$, $(1+z)^1$, $(1+z)^0$, $(1+z)^{-1}$ and $(1+z)^{-2}$, and the individual values of w are quantized in units of $\frac{1}{3}$ [12].

This combination yields a current equation of state for the DE (i.e. non-material Darkness) which is precisely -1, just as in the more traditional model. The value of Ω_{DM} in the natural model is consistent with current Supernovae and other cosmological data [13].

However, the model has a quite different evolution of $w(z)$. Hence even if it turns out to be difficult to precisely measure the value of $\dot{w}$ in the past, all we have to do is wait for it to change in the future! In fact, since the naturalness of this model is only transient, then there is a very strong prediction: sentient observers and the cosmologists who exist alongside them, are fated not to last into future eras when the model loses its naturalness. Some theoretical constructs are sometimes criticized for a perceived lack of predictability – however, no such criticism can be levelled against the Natural Dark Energy Hypothesis, since it predicts nothing less than the end of civilization as we know it.

Since our hypothesis elegantly avoids any of the so-called 'coincidence' problems, we do not need to rely on any Anthropic reasoning to explain the near-equality of the Ωs. However, there is nevertheless one nagging issue, and that is: why is Ω_R so very much smaller than the other components? We offer 4 possible non-Anthropic resolutions of this 'photon anomaly': (1) the radiation is mainly composed of photons, which are particles of light, and hence not bound by the rules which fix the behaviour of the Dark sector [14]; (2) perhaps one could invoke the 'tired light' hypothesis, in which the photons are partially absorbed by all the darkness which they have to travel through [15]; (3) the dark part of the radiation could in fact *be* one of the Dark Energy components; or (4) *if* the radiation were now a significant part of the cosmic energy budget, then the Universe would be a very much hotter and more hostile place, and hence we might not exist to observe it.

In this natural DE picture the different components presumably come out of a complete fundamental theory, and hence it is also reasonable to expect that the components could be coupled. Exploration of *interacting* parts of the Dark Side may help resolve other astrophysical puzzles. As well as solving the Dark Matter and Dark Energy enigmas, we have every reason to believe that our model will be just as successful at solving the mysteries of the Cuspy Halo Problem, Gamma-Ray Bursts, Ultra-High Energy Cosmic Rays, Baryon Asymmetry, Primordial Magnetic Fields, the low CMB quadrupole, etc. [16]

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[1] Citations removed following legal advice.

[2] E.g. Albrecht A., et al., 2006, Report of the Dark Energy Task Force, astro-ph:0609.591.

[3] If this was written instead as 66^{66} then it would appear less mysterious, since there would be only one number to explain, rather than 2.

[4] For example: Einstein A., 1917, Sitzungsber. Preuss. Akad. Wiss., 142; Weinberg S., 1987, PRL, 59, 2607; Battye R.A. & Moss A., 2007, astro-ph:0703.744.

[5] Which is lucky, since otherwise we would all be cooked.

[6] Caldwell R., 2002, Phys. Lett., B545, 23; Lucas G., et al., 1999, SW, Ep. 1.

[7] His older brother William also said some other things, but those are not relevant here. This principle, that if something has strong merit then you want to make it as powerful as possible, is sometimes called 'Occam's Laser'.

[8] For a particularly serious overview, see Scott D., 2006, Can. J. Phys., 84, 419, astro-ph:0510.731.

[9] Scott D. & Frolop A., 2006, astro-ph:0604.011.

[10] We call these sextessence, septessence, octessence etc. The ninth component would be nonessence, which clearly does not exist.

[11] This may seem alarming to people who are afraid of the Dark. However, we *already* know that the Universe is entering a period when the Dark Sector comes to dominate. This is our Hour of Darkness, so we should just let it be.

[12] Suggesting a connection with quark charges.

[13] Of course there are several other possibilities, which, although having slightly less naturalness, nevertheless may turn out to have to be considered because they are a better fit to the data. Other examples have: 5 components with $\Omega_{Di} = 0.2$ and $w = -\frac{5}{6}$ or -1; or 4 components with $\Omega_{Di} = 0.25$, e.g. with the Dark Energy having terms of the form $(1+z)^{3/2}$, $(1+z)^0$ and $(1+z)^{-3/2}$, so that $w = -1$ still.

[14] There may be a form of Dark Entropy which dominates over the normal radiation, perhaps stopping us from reading the full information content of the CMB; see Scott D. & Zibin J.P., astro-ph:0511.135.

[15] Terry Pratchett wrote 'Light thinks it travels faster than anything but it is wrong. No matter how fast it travels, it finds the darkness has always got there first, and is waiting for it.'

[16] Further suggestions should be sent to Dr. Frolop.

Cume #554 – Sebastian Trujillo (Perfect score)

1. Friedmann Eq: $$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_m}{a^3} + \frac{\Omega_R}{a^4} + \Omega_\Lambda + \frac{(1-\Omega_{tot})}{a^2}$$

And $\Omega_R(t)$:

$$\Omega_R(t) = \frac{\rho_R(t)}{\rho_{crit}} = \frac{8\pi G}{3H^2}\rho_R(t) = \frac{8\pi G}{3H^2}\frac{\rho_R}{a^4} = \frac{H_0^2}{H^2}\frac{\Omega_R}{a^4}$$

$$\Rightarrow \Omega_R(t) = \frac{1}{a^4}\frac{\Omega_R}{\frac{\Omega_m}{a^3} + \frac{\Omega_R}{a^4} + \Omega_\Lambda + \frac{(1-\Omega_{tot})}{a^2}}$$

$$= \frac{\Omega_R}{\Omega_m a + \Omega_R + \Omega_\Lambda a^4 + (1-\Omega_{tot})a^2}$$

~~at present $H = H_0$, $a = 1$;~~

$$\cancel{\Omega_R(a=1) = \frac{\Omega_R}{\Omega_m + \Omega_R + \Omega_\Lambda + (1-\Omega_{tot})}}$$

$\Rightarrow$ Model universe as blackbody with $T_0 = 2.73\,K$:

Yes

$$\rho_R = \frac{aT_0^4}{c^2} \Rightarrow \Omega_R = \frac{\rho_R}{\rho_{crit}} = \frac{aT_0^4/c^2}{9.24\times10^{-30}\,gm/cm^3}$$

stefan-boltzmann const.

$$\Rightarrow \Omega_R \approx 5.05\times10^{-5}$$

Correct

$5+10+5+5+10+10 = \underline{45 pts}$

② Flat: $\Omega_{tot} = 1 \Rightarrow \Omega_m = 0.3$, $\Omega_\Lambda = 0.7$

$$\Omega_m(t) = \frac{\Omega_m}{\Omega_m + \frac{\Omega_R}{a} + \Omega_\Lambda a^3} \checkmark = \frac{\Omega_m}{\Omega_m + \underbrace{\Omega_R(1+z)}_{\approx 0} + \Omega_\Lambda(1+z)^{-3}}$$

$$\Rightarrow \Omega_m(z=100) = \frac{0.3}{0.3 + 0.7(101)^{-3}} = 0.999$$

$\underline{\underline{0.999998}}$

Correct

10pts

(3) since $\rho_R = \rho_0 / a^4 \Rightarrow T = \frac{T_0}{a}$ for radiation

this argument is based solely on kinematics of the expansion so it is model independent.

Yes

$T(z=1000) = T_0(1+1000) = 2.73\,K\,(1001) = 2732\,K$ for both models.

Correct. 5pts

(4) equality: $\Omega_m(t_{eq}) = \Omega_R(t_{eq})$

$$\Rightarrow \frac{\Omega_m}{\Omega_m + \frac{\Omega_R}{a} + \Omega_\Lambda a^3 + \underbrace{(1-\Omega_{tot})}_{0} a} = \frac{\Omega_R}{\Omega_m a + \Omega_R + \Omega_\Lambda a^4 + \underbrace{(1-\Omega_{tot})}_{0} a}$$

$$\Rightarrow \Omega_m(\Omega_m a + \cancel{\Omega_R} + \cancel{\Omega_\Lambda a^4}) = \Omega_R\left(\cancel{\Omega_m} + \frac{\Omega_R}{a} + \Omega_\Lambda a^3\right)$$

$$\Omega_m^2 a + \Omega_m \Omega_\Lambda a^4 - \underbrace{\frac{\Omega_R^2}{a}}_{\approx 0} + \Omega_R \Omega_\Lambda a^3 = 0$$

$$\Omega_m \Omega_\Lambda a^3 + \Omega_R \Omega_\Lambda a^2 + \Omega_m^2 = 0$$

equality: $\frac{\Omega_R(t_{eq})}{\Omega_m(t_{eq})} = 1$ ✓ ; $\rho_m(a) = \frac{\rho_m}{a^3}$ ✓ ; $\rho_R(a) = \frac{\rho_R}{a^4}$ ✓

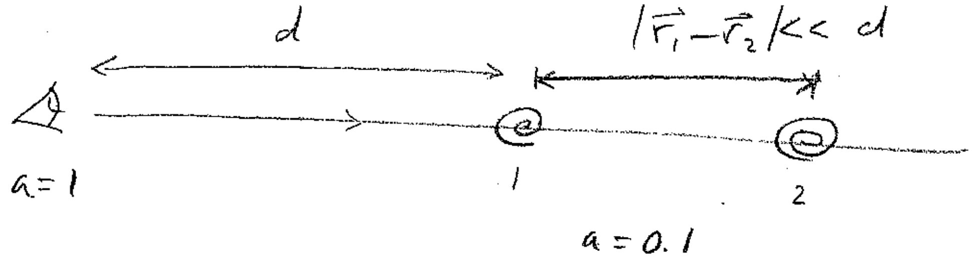
$$\frac{\Omega_R(t)}{\Omega_m(t)} = \frac{\rho_R(a)}{\rho_m(a)} \frac{\cancel{\rho_{crit}}}{\cancel{\rho_{crit}}} = \frac{\rho_R}{\rho_m}\frac{a^3}{a^4} = \frac{8\pi G}{3H_0^2}\frac{3H_0^2}{8\pi G}\frac{\Omega_R}{\Omega_m}\frac{1}{a}$$

$$= \frac{\Omega_R}{\Omega_m}(1+z) \Rightarrow 1 = \frac{\Omega_R}{\Omega_m}(1+z_{eq}) \Rightarrow z_{eq} = \frac{\Omega_m}{\Omega_R} - 1$$

$$\Rightarrow z_{eq} = 2999$$ ✓

Correct 5pts

(5)



$$\vec{r} = a\vec{x} \quad \Rightarrow \quad \dot{\vec{r}} = \dot{a}\vec{x} + a\dot{\vec{x}} = \frac{\dot{a}}{a}(a\vec{x}) + a\dot{\vec{x}}$$

proper — comoving

$$\Rightarrow \vec{v} = H\vec{r} + \cancel{\vec{v}_{pec}}^{\to 0} \quad \text{for perfect Hubble flow}$$

$$\vec{v}_1 = H\vec{x}_1, \quad \vec{v}_2 = H\vec{x}_2$$

$$\vec{v}_1 - \vec{v}_2 = 1000\ \mathrm{km/s} = H(\vec{r}_1 - \vec{r}_2) \checkmark$$

find H for $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$:

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_m}{a^3} + \cancel{\frac{\Omega_R}{a^4}}^{\approx 0} + \Omega_\Lambda + \cancel{\frac{(1-\Omega_{tot})}{a^2}}^{\to 0}$$

$$\Rightarrow H = H_0\left[\frac{\Omega_m}{a^3} + \Omega_\Lambda\right]^{1/2} \Rightarrow H(a=0.1) = 70\ \frac{\mathrm{km}}{\mathrm{s\,Mpc}}\left[\frac{0.3}{0.1^3} + 0.7\right]^{1/2}$$

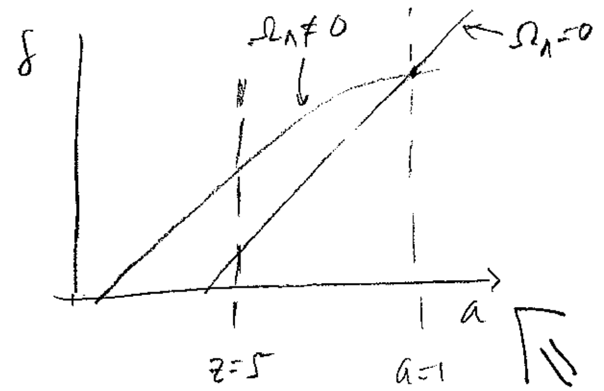
$$\Rightarrow H(a=0.1) = 1213\ \mathrm{km/s\,Mpc} \checkmark$$

$$\Rightarrow |\vec{r}_1 - \vec{r}_2| = \frac{1000\ \mathrm{km/s}}{1213\ \mathrm{km/s\,Mpc}} = 0.82\ \mathrm{Mpc} = \underline{820\ \mathrm{kpc}\ (\text{proper})}$$

also $$|\vec{x}_1 - \vec{x}_2| = \frac{|\vec{r}_1 - \vec{r}_2|}{a} = \underline{8.2\ \mathrm{Mpc}\ (\text{comoving})}$$

Yes 10pts

⑥ relative amplitude of density perturbations evolves differently in the presence of Λ:



in the presence of Ω_Λ, density perturbations grow very slowly after a certain moment before a=1

Yes.

Yes. This is correct diagram.

If we match the amplitudes at present (a=1) then the amplitude of the Λ model will be larger at z=5. This implies that perturbations will collapse sooner in the model with Λ, producing more quasars at z=5 than the $\Lambda = 0$ counterpart.

10pts