Anatoly

CUME EXAM # - 1 April, 2007. Structure of Galaxies.

325

Passing score for this cume is 66.667 points. Some facts and numbers about our Galaxy and Draco dwarf spheroidal: Distance to Draco is about 80 kpc. Its absolute magnitude is $M_V = -8.6$. The distribution of stellar mass in the galaxy is well approximated as exponential $\rho_* \propto \exp(-r/r_d)$ with the scale length $r_d \approx 0.5$ kpc. Mass of our galaxy inside 80 kpc is $5 \times 10^{11} M_\odot$. Absolute magnitude of the Sun in V band is 4.83. $M_\odot = 2 \times 10^{33} g$. $G = 6.67 \times 10^{-8}$ in cgs units. $pc = 3.08 \times 10^{18}$cm.

1. 15pts Using values presented in the paper, give the mass of *stars* in Draco.
2. 25pts Most of the light of Draco is inside 40 arcmin (see fig 1). Assuming that this is the size of the galaxy, estimate its mass using the virial theorem. (Note that Draco is likely more extended than 40 arcmin, but this is a good start.)
3. 25pts Estimate the tidal radius of Draco. Give its value in kpcs and arcmin.
4. 5pts Explain why data on the motion of stars in Draco require dark matter or MOND.
5. 25pts Modeling of the mass profile in Draco is based on "solutions of the Jeans equations". Let's assume that velocities are isotropic and do not change with distance. Use the Jeans equation (= stellar hydrostatic equilibrium) to find mass inside 40 arcmin. In this case the pressure is equal to $P = \rho_* v_{los}^2$. Assume that the galaxy is spherical (it is very close).
6. 5pts The text says: "we have assumed that the dark matter density profile is given by by a formula with an inner cusp". Please, explain what is a cusp in this context. X

(1) On page 5, the text gives stellar $M/L = 3$ in V.

$$L = L_\odot 10^{-0.4(M-M_\odot)}$$

$$M_* = \left(\frac{M}{L}\right)\cdot L = 3 M_\odot 10^{-0.4(-8.6-4.83)} = 7\cdot 10^5 M_\odot$$

(2) $$M_{vir} = \frac{3 v_{los}^2 r}{G}$$

$40' \rightarrow r = 9.3\cdot 10^{-1}$ kpc

Fig 2 gives $v_{los} \simeq 8$ km/s

$$M_{vir} \simeq 4.1\cdot 10^7 M_\odot$$

Note that this is significantly larger than M_*

(3) The tidal force at distance Δr from the center of Draco is

$$F_{tide} = \frac{\partial}{\partial r}\left(\frac{GM_{galaxy}}{r^2}\right)\Delta r = -\frac{2GM_{gal}}{r^3}\Delta r$$

Here r is Draco's distance to the galactic center

Equating F_{tide} to the gravity force of Draco:

$$\frac{2GM_{gal}}{r^3}\Delta r = \frac{GM_{Draco}}{\Delta r^2} \Rightarrow$$

$$\left(\frac{\Delta r}{r}\right)^3 = \frac{M_{Draco}}{2M_{gal}} \Rightarrow \Delta r = r_{tidal} = 2.8 \text{ kpc}$$

$$r_{tidal} = 120 \text{ arcmin}$$

④ There is a significant discrepancy between the dynamical $M/L \sim 130$ and stellar $M/L \sim 3$. There must be something, which keeps the galaxy from apart. Projection effects, which are called interlopers, cannot explain the differences in M/L. MOND has problems because its constant a_0 is not consistent with other estimates. So, DM is required

⑤ Hydrostatic equilibrium:

$$\frac{1}{\rho}\frac{dP}{dr} = g$$

In our case $g(r) = -\frac{GM(r)}{r^2}$

$P = \rho v^2$, where v is constant

$$\frac{1}{\rho}\frac{d\rho v^2}{dr} = \frac{v^2}{\rho_0 e^{-r/r_d}} \cdot \frac{d\rho_0 e^{-r/r_d}}{dr} = -\frac{v^2}{r_d}$$

$$GM(r) = \frac{v^2}{r_d} \cdot r^2$$

$r_d = 0.5\,\mathrm{kpc}$, $v = 8\,\mathrm{km/s}$, $r = 0.93\,\mathrm{kpc}$

$$M = 2.4\,10^7 M_\odot$$

⑥ Cosmological simulations indicate that density of dark matter in central regions of dark matter halos should increase as

$$\rho_{dm} \propto r^{-1}$$

This divergent behaviour of dark matter is called cusp.

Mass Profiles and Shapes of Cosmological Structures
G. Mamon, F. Combes, C. Deffayet, B. Fort (eds)
EAS Publications Series, Vol. ?, 2005

THE DRACO DWARF IN CDM AND MOND

Łokas, E.[1], Mamon, G.[2] and Prada, F.[3]

Abstract. We present interpretations of the line-of-sight velocity distribution of stars in the Draco dwarf spheroidal galaxy in terms of CDM and MOND assuming constant mass-to-light ratio and anisotropy. We estimate the two parameters by fitting the line-of-sight velocity dispersion and kurtosis profiles for stellar samples differing by a number of stars rejected as interlopers. The results of the fitting procedure for CDM, high mass-to-light ratio (131-141 solar units in V-band) and weakly tangential orbits, are similar for different samples, but the quality of the fit is much worse when fewer interlopers are removed. For MOND, the derived mass-to-light ratio (21 solar units) is too large to be explained by the stellar content of the galaxy.

1 Introduction

The Draco dwarf is a generic example of the class of dwarf spheroidal (dSph) galaxies of the Local Group and as such has been a subject of intensive study in recent years. The object is interesting from the point of view of theories of structure formation due to its large dark matter content and its implications for the missing satellites problem. Dwarf spheroidals are also in the regime of low accelerations and therefore can provide critical tests for the alternatives to cold dark matter (CDM) scenarios, such as the Modified Newtonian Dynamics (MOND). New observations of the object performed recently provided a larger kinematic sample of stellar velocities (Kleyna et al. 2002; Wilkinson et al. 2004) and a better determination of its luminosity profile and shape (Odenkirchen et al. 2001).

This has encouraged us to study the Draco dwarf using a method of velocity moments (Łokas, Mamon & Prada 2005) first applied to the Coma cluster of

[1] Nicolaus Copernicus Astronomical Center, Warsaw, Poland
[2] Institut d'Astrophysique de Paris, France
[3] Instituto de Astrofísica de Andalucia, Granada, Spain

DOI: (will be inserted later)

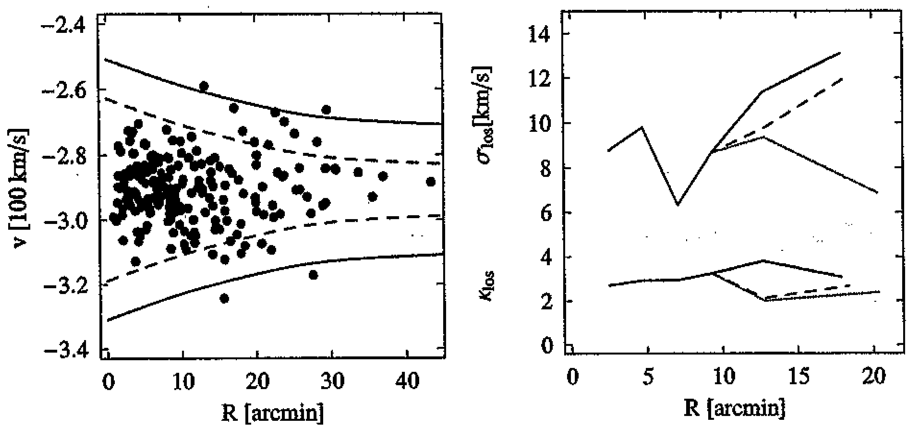

Fig. 1. Left panel: line-of-sight velocities versus distances for 207 stars in the sample of Wilkinson et al. (2004) with lines separating supposed interlopers from members. Right panel: velocity moments calculated from different samples with 0, 4 and 18 interlopers removed (solid, dashed and dotted lines respectively).

galaxies by Łokas & Mamon (2003) and tested with N-body simulations by Sanchis, Łokas & Mamon (2004) and Wojtak et al. (2005). The method relies on joint fitting the velocity dispersion and kurtosis profiles which allows us to break the degeneracy between the mass distribution and velocity anisotropy. We have assumed that the dark matter density profile is given by a formula with an inner cusp and exponential cut-off at large distances recently proposed by Kazantzidis et al. (2004) as a result of their simulations of tidal stripping of dwarfs by the potential of a Milky Way size galaxy. We found that the results depend dramatically on the sample of stars under consideration and concluded that the larger samples are probably contaminated by unbound stars because the velocity moments constructed from them are inconsistent.

Since this dark matter profile is not very different in shape from the distribution of stars given by the Sersic profile, in this contribution we reanalyze the same data assuming a simpler hypothesis that mass traces light and using different binning. This case is directly comparable to the alternative interpretation of the data in terms of MOND (Sanders & McGaugh 2004) which also assumes constant mass-to-light ratio but modifies the gravitational acceleration. We examine the two hypotheses subsequently in the next two sections.

2 Cold Dark Matter

The kinematic data we have used are plotted in the left panel of Fig. 1 which shows the line-of-sight velocities versus distances of 207 stars counted as members

Table 1. Fitted parameters in the case of CDM

number of interlopers	fitting σ_{los} M/L_V	β	χ^2/N	fitting σ_{los} and κ_{los} M/L_V	β	χ^2/N
18	131	-0.44	11.1/4	131	-0.34	15.4/10
4	147	-1.24	21.7/4	141	-0.50	27.6/10
0	156	-1.86	25.9/4	141	-0.19	39.1/10

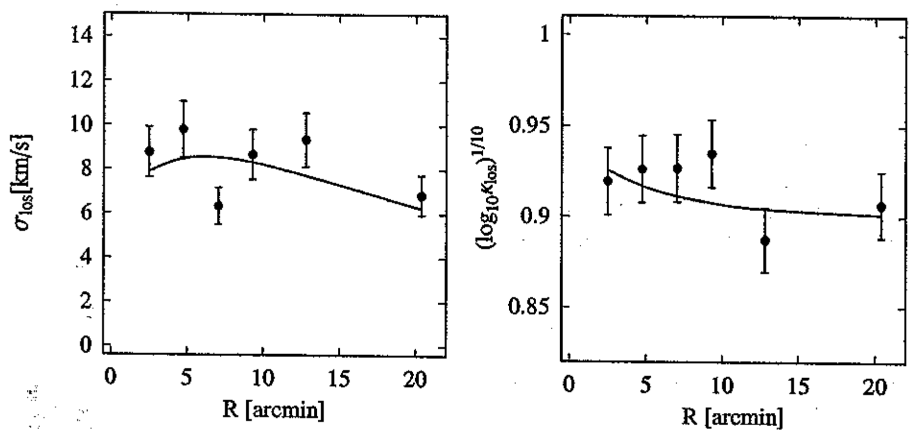

Fig. 2. Velocity moments for the most restrictive sample (with 18 interlopers removed) together with the best-fitting model in the case of CDM.

of Draco by Wilkinson et al. (2004). Since some of the stars clearly are discrepant from the main body of the galaxy we further exclude some of them as interlopers. The selection is indicated by the solid and dashed lines intended to follow the overall shape of such diagrams for gravitationally bound object. The line-of-sight velocity moments, dispersion σ_{los} and kurtosis κ_{los}, calculated from the different samples thus obtained are plotted in the right panel of the Figure. In each case we divide the data into 6 radial bins with 30 stars. We see that the results in the outer bins from which the interlopers are removed are affected dramatically and both moments are significantly reduced in value. We have assumed that the binary fraction in the stellar sample is small and its effect on the moment negligible in comparison with other uncertainties (but see the discussion in Łokas et al. 2005).

Assigning sampling errors to the moments we fit them with the models based on solutions of the Jeans equations characterized by only two constant parameters, the mass-to-light ratio in V-band, M/L_V, and the anisotropy parameter $\beta = 1-\sigma_\theta^2/\sigma_r^2$. The remaining assumptions and adopted parameters are as in Łokas et al. (2005). The results for different samples are listed in Table 1.

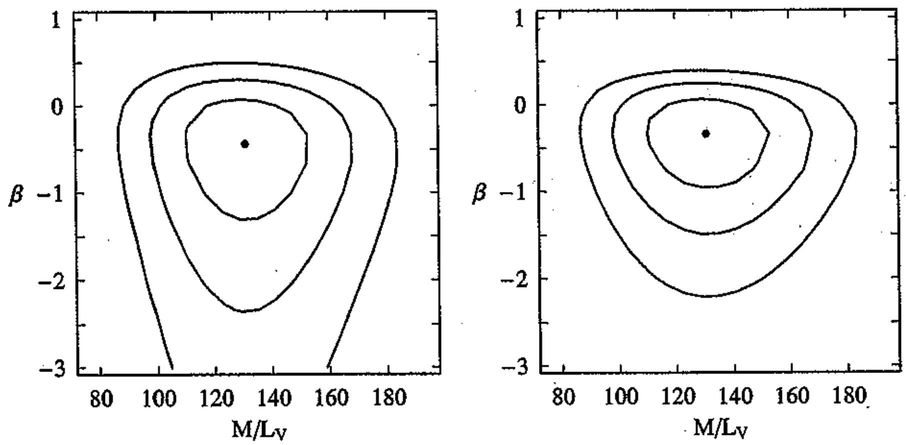

Fig. 3. The 1σ, 2σ and 3σ confidence regions in the parameter plane for the most restrictive sample (with 18 interlopers removed) obtained from fitting velocity dispersion alone (left panel) and both moments (right panel) in the case of CDM.

Fitting first the velocity dispersion alone we find a stronger preference for tangential orbits in the case of samples with smaller number of interlopers removed. This is understandable: since the shape of the mass profile is constrained to be strongly decreasing by the assumption of constant M/L_V the strongly increasing dispersion profile induces tangential orbits. The tendency disappears, however, once the kurtosis is included in the analysis; then all samples yield similar best-fitting parameters although the quality of the fit is much worse for the less restrictive samples. The reason for this is the fact that for these samples the moments seem inconsistent: if the increasing dispersion profile is due to tangential orbits then this should result in decreasing instead of increasing kurtosis profile (see the discussion in Łokas et al. 2005).

The velocity moments together with the best-fitting models for the case of the most restrictive sample (with 18 interlopers) are shown in Fig. 2. Fig. 3 illustrates the benefit from including the kurtosis in the analysis for the same sample. While the best-fitting parameters are similar in both cases, the confidence limits for β one can read from the contours shown in the Figure are much more constrained from fitting both moments (right panel) than from fitting the dispersion alone (left panel).

The mass-to-light ratio $M/L_V = 131 M_\odot/L_\odot$ found here for the most restrictive sample is similar to the minimum value of 134 reached at 10 arcmin for the corresponding sample of 189 stars considered by Łokas et al. (2005) (see their Fig. 6). Note however, that the quality of the fit is somewhat worse here because of a smaller number of parameters.

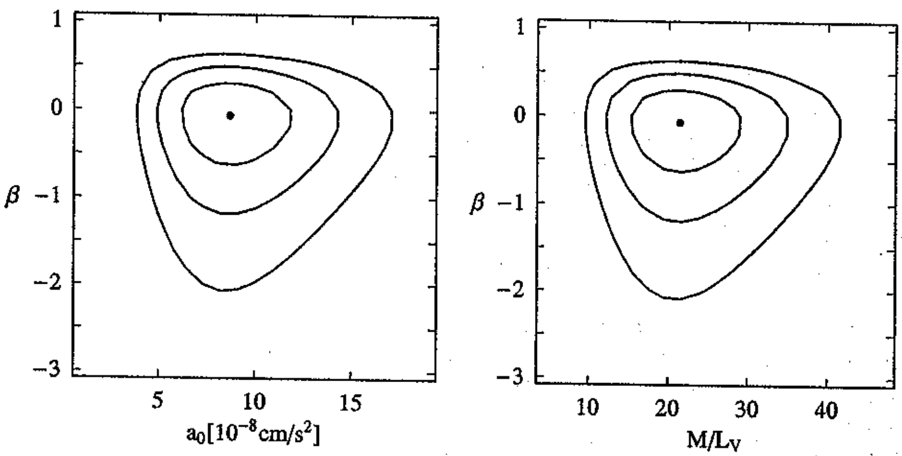

Fig. 4. The 1σ, 2σ and 3σ confidence regions in the parameter plane for the most restrictive sample (with 18 interlopers removed) obtained from fitting velocity dispersion in the case of MOND assuming standard M/L_V (left panel) and a_0 (right panel).

3 Modified Newtonian Dynamics

The lowest order Jeans equation which models the velocity dispersion can be simply modified in the case of MOND by replacing the Newtonian gravitational acceleration g_N with g_M related by $g_N = \mu(g_M/a_0)g_M$ where the function μ is chosen as $\mu(x) = x(1+x^2)^{-1/2}$ (Milgrom 1983) and a_0 is the characteristic acceleration scale of MOND supposed to be universal. The value found to fit well most rotation curve data for spiral galaxies is $a_0 = 1.2 \times 10^{-8}$ cm s^{-2} (Sanders & McGaugh 2004).

We have fitted the velocity dispersion data for the most restrictive sample, first assuming $M/L_V = 3\mathrm{M}_\odot/\mathrm{L}_\odot$, a value characteristic of the Draco stellar sample, and finding the best-fitting parameters a_0 and β. Next, we adopted the canonical value $a_0 = 1.2 \times 10^{-8}$ cm s^{-2} and fitted M/L_V and β. The results, in terms of best-fitting parameters and confidence regions are shown in Fig. 4. The quality of the fit, $\chi^2/N = 9.1/4$, is the same for both cases.

In agreement with the earlier findings of Łokas (2001, 2002) using older kinematic data, we conclude that the velocity dispersion profile of Draco cannot be well reproduced with reasonable values of either a_0 or M/L_V. The best-fitting $a_0 = 8.7 \times 10^{-8}$ cm s^{-2} and $M/L_V = 21.4\ \mathrm{M}_\odot/\mathrm{L}_\odot$ are about an order of magnitude larger than expected and the confidence regions are rather narrow excluding the standard values at more than 3σ level. These values are even higher (while orbits more tangential and quality of the fits worse) for the other samples with less interlopers removed.

4 Conclusion

We conclude from this study that the constant mass-to-light ratio models provide a reasonably good fit (with $\chi^2/N = 15.4/10$) to the kinematic data for Draco, but only for the most restrictive sample (with 18 interlopers removed). For other samples, although similar best-fitting parameters are found from the joint fitting of both moments, the fits are much worse ($\chi^2/N = 27.6/10$ and $\chi^2/N = 39.1/10$ respectively for samples with 4 and 0 interlopers removed). This result is in qualitative agreement with the conclusion reached by Łokas et al. (2005) who considered a wider class of dark matter distributions.

The velocity dispersion data are also poorly fitted by MOND. The best-fitting parameters for the most restrictive sample, either a_0 or M/L_V are found to be about an order of magnitude larger than expected. However, before we conclude that the case of Draco really falsifies MOND other possibilities have to be considered. The present results might mean that a_0 is not really a universal constant in MOND as previously claimed and takes different values for different classes of objects. Another possibility to be explored is that in modifying the gravitational potential according to MOND, the influence of the Milky Way (here neglected) has to be taken into account. In addition, since in MOND the mass is proportional to the fourth power of velocity dispersion any interlopers still present in our sample may artificially inflate the mass more strongly than in the case of CDM.

The most interesting issue to address in the future research on dSph galaxies is the determination of their dark matter content without the uncertainties due to unbound stars. This could be done either by modelling more distant dwarfs which are not under direct influence of the giant galaxies of the Local Group or by careful removal of interlopers aided perhaps by simulations of their origin.

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