Cume #470
Spider Pulsars
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April 28, 2023

Please read Clark et al., 2023. This Nature paper studies millisecond pulsars (MSPs). The main text goes from page 451 through 455. Focus on that text. The remaining 7 pages discuss details of the methods, observations, and references. You will only need to consult this material infrequently. You can also ignore the text regarding the significance testing and Monte Carlo simulations, including Figs. 2 and 3.

The *guaranteed passing grade* is 75%.

Please use a new page for each numbered problem. Show all work clearly and please write legibly, and if you can't solve something completely, at least give an idea of how you might go about it. Make sure you are careful to answer ALL parts of each question. Don't spend too much time in the beginning on one question, move on and try them all and then come back if you need to.

This is a remote exam and you are not allowed to use any materials, such as books or websites or any internet (except for emailing a question if needed). You may use a calculator for any arithmetic computations only, not for plotting or algebra or for storing equations.

If any clarification questions are needed, just email or call (505.431.3557). Good luck!

Some numbers and other information you may need:

$$\begin{aligned} h &= 4.135668 \times 10^{-15}\,\mathrm{eV\,s} \\ c &= 3 \times 10^{10}\,\mathrm{cm\,s^{-1}} \\ G &= 6.674 \times 10^{-20}\,\mathrm{km^3\,kg^{-1}\,s^{-2}}; \end{aligned}$$

Error propagation. Let $z = f(x, y, \ldots)$, for some arbitrary function f of one or many *independent* variables, and let Δx, Δy, ... be each variable's uncertainties. Then the uncertainty on the value z can be written as

$$\Delta z = \left[\left(\frac{\partial f}{\partial x}\, \Delta x \right)^2 + \left(\frac{\partial f}{\partial y}\, \Delta y \right)^2 + \ldots \right]^{1/2} . \tag{1}$$

1. (16 points). Understanding the paper.

   (a) (2 points). Pulsars emit radiation in the radio wavelength regime. Describe two insights that this type of radiation gives regarding the systems in this study.

   (b) (2 points). Pulsars can also emit radiation in the gamma part of the spectrum. Describe what the gamma radiation is used for in this paper.

   (c) (2 points). What is the difference between the gamma eclipses and the radio eclipses? I.e., what is causing the radiation to be blocked in each case?

   (d) (3 points). Describe the main scientific goal(s) of this paper and the astrophysical context for what these objects can teach us. Don't describe techniques or methods. Please only use up to 3 sentences.

   (e) (3 points). Now, to address their scientific goals, what experiment do the authors carry out? Please only use up to 3 sentences to describe their strategy.

   (f) (4 points). Calculate the range of wavelengths of the gamma radiation are they using? The "Methods" section describes the energy range of the measurements.

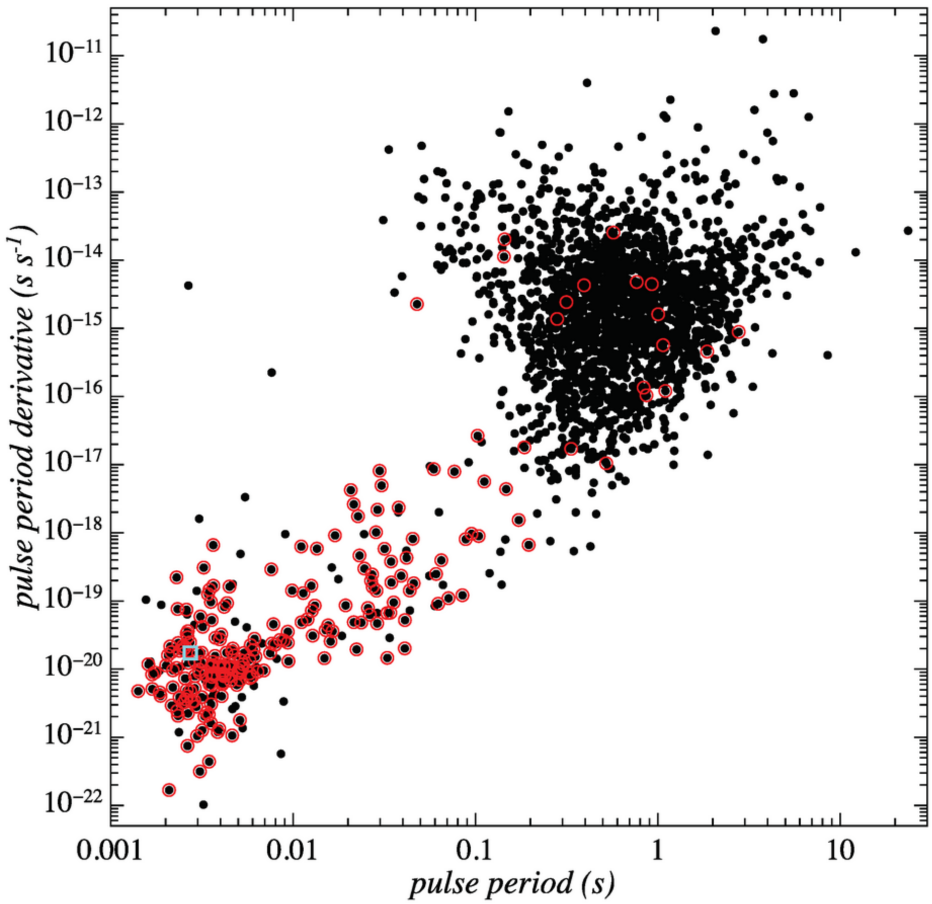

Figure 1: Pulsar populations in terms of their pulse period and its time rate of change. Ignore the colors and any of the highlighted points. From Wex and Kramer (2020).

2. (10 points). Figure 1 shows the known population of pulsars in the period-period derivative phase space. The period derivative, or $\dot{P}$, is better thought of as the spin-down rate, and describes the rate at which the pulse period P is changing. Combining these two quantities in the right way gives the spin age τ, which is a good estimate for the age of the neutron star.

   (a) (4 points). MSPs are defined with periods $P \leq 30\,\mathrm{ms}$. For a typical MSP in the figure, what is its spin age τ? Give your answer in years.

   (b) (4 points). For a typical "regular" (non MSP) neutron star, what is τ?

   (c) (2 points). The pulsar in the Crab Nebula has a spin period $P = 0.0331\,\mathrm{s}$ and $\dot{P} = 8.4 \times 10^{-13}\,\mathrm{s\,s^{-1}}$ (it is one of the dots in the figure). From your historical knowledge, is this pulsar's spin age reasonable?

3. (14 points). Neutron stars.

   (a) (3 points). What is the approximate mass range that stars are born with that might eventually become neutron stars?

   (b) (5 points). Describe qualitatively, in four sentences or less, the stellar evolution channel to a neutron star. Start from the main sequence. You don't need to be too detailed here, just focus primarily on the stellar core and any major evolutionary events that occur.

   (c) (2 points). What is the expected mass range of neutron stars once they reach this final state?

   (d) (4 points). Estimate the surface velocity for fastest rotator shown in Figure 1. Put your answer in units of the speed of light c. You will need to provide a typical radius for a neutron star.

4. (13 points). Mass function and binaries.

(a) (5 points). The paper mentions how strongly the mass estimate of a pulsar in a binary depends on the inclination angle of the system, since $M \propto 1/\sin^3 i$. From error analysis, show that if the true inclination is 90° (orbit seen edge on), but is mistakenly measured as 70°, then the **relative uncertainty** on the estimated mass of the pulsar $\Delta M/M = (M_{\rm est} - M_{\rm true})/M_{\rm true}$ is about 40%. In other words, you've overestimated the mass by that much.

(b) (5 points). The mass functions of the pulsar binaries are given in Eqs. (4) and (5) in the paper. Describe how the 5 unique orbital parameters on the right-hand-sides in those two equations can be obtained to estimate each star's mass, and the observations needed to get the parameters in an ideal situation. Be as specific as possible.

(c) (3 points). Imagine that for one of the systems described in the paper, the companion star has filled it Roche lobe. Make a simple top-down perspective (i.e., looking down onto the orbital plane) sketch of this binary configuration, with appropriate labels. Make sure to indicate the Roche lobes.