Cume #445
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This exam is based in part on the paper "The hot limit of solar-like oscillations from *Kepler* photometry" by Luis Balona (2020). You might have noticed that this paper was discussed in astro-ph a week ago. The exam covers some of the physics of stars and their interiors, a little bit about Kepler data, and some of the methods of signal analysis. The *anticipated passing grade* is 75%.

Please use a new page for each numbered problem. Show all work clearly and please write legibly, and if you can't solve something completely, at least give an idea of how you might go about it. Make sure you are careful to answer ALL parts of each question. Don't spend too much time in the beginning on one question, move on and try them all and then come back if you need to.

This is a remote exam and you are not allowed to use any materials, such as books or websites or your computer (except for emailing a question if needed). You may use a calculator for any arithmetic computations only, not for plotting or algebra or for storing equations.

If any clarification questions are needed, just call me. Good luck!

Here are a few useful things you may need:

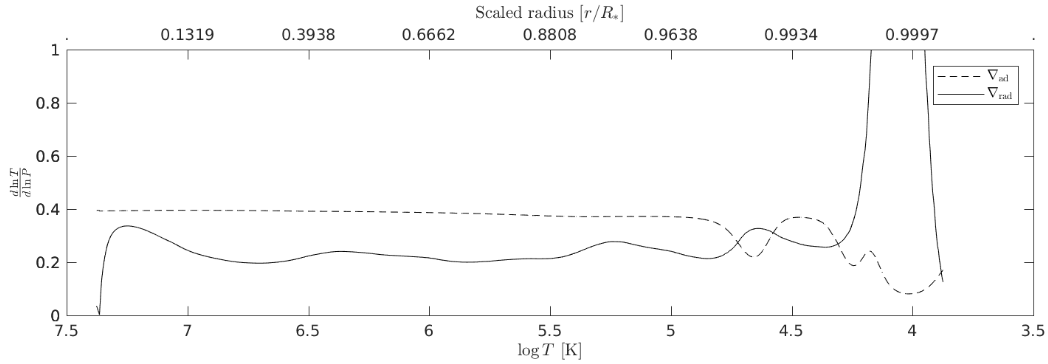
- Radiative temperature gradient (using ideal gas equation of state, neglecting radiation):

$$\nabla_{\rm rad} \equiv \left(\frac{{\rm d}\ln T}{{\rm d}\ln P}\right)_{\rm rad} = \frac{3k_{\rm B}}{16\pi acGm_{\rm u}}\frac{\kappa}{\mu}\frac{L}{m}\frac{\rho}{T^3}.$$

  All symbols should be recognized, with κ the opacity, and μ the mean molecular weight.
- Adiabatic temperature gradient:

$$\nabla_{\rm ad} \equiv \left(\frac{{\rm d}\ln T}{{\rm d}\ln P}\right)_{\rm ad} = 1 - \frac{1}{\Gamma}.$$

- The Γ is known as the adiabatic exponent, and is also the ratio of specific heats: $\Gamma = C_{\rm P}/C_{\rm V}$. For an ideal gas, $C_{\rm P} = 5R_g/(2\mu) = 5/3 \cdot C_{\rm V}$, where R_g is ideal gas constant.
- The angular frequency of a periodic signal ω is related to the cyclic frequency f by $\omega = 2\pi\nu$.
- The power spectrum of a time-varying signal $x(t)$ is $P(\omega) = x^*(\omega)x(\omega)$, where $x(\omega)$ is the Fourier transform of $x(t)$. To obtain the true amplitude of any periodicities in the signal, the amplitude spectrum is simply $\sqrt{P(\omega)}$.
- Trig identity: $\sin^2(x) = 0.5\,[1-\cos(2x)]$.



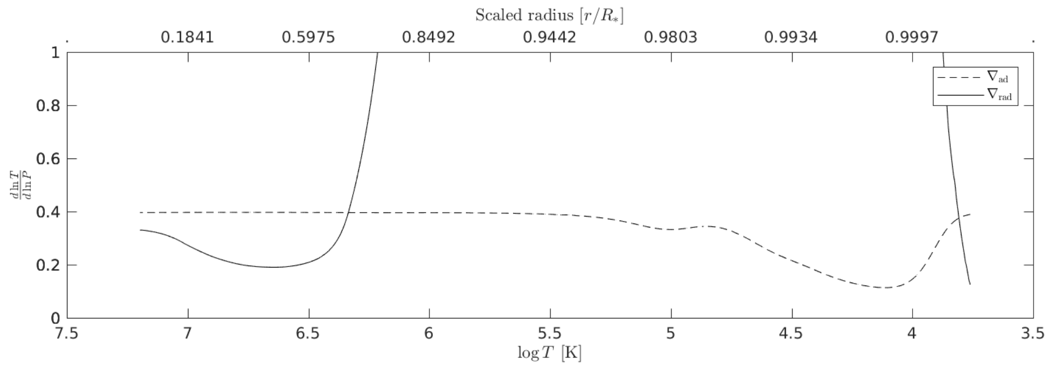


Figure 1: Each panel shows the adiabatic (ad) and radiative (rad) temperature gradients of a zero-age main-sequence stellar model. The mass of the stellar model is different in each panel. The bottom x-axis is the log of the temperature over the entire model. Also shown on the top of each panel is the corresponding radial coordinate from $r = 0$ to $r = R_*$. Note that the y-axis is truncated.

1. (20 points). Classical pulsations, such as those observed in δ Scuti, γ Doradus, Cepheids, RR Lyrae, etc., are those driven by opacity mechanisms. Solar-like oscillations in stars, on the other hand, are stochastically driven by near-surface turbulent convection. This process mainly occurs in the Sun and other stars with certain similar properties to the Sun. This paper explores the possibility of both types being present (simultaneously or not) in some hot main-sequence stars.

   (a) (3 points). See Figure 1 of the paper. In which *evolutionary states* (not spectral types) of stars are solar-like oscillations primarily found to be observed? Name 3 of these states.

   (b) (2 points). Physically speaking, describe what the radiative temperature gradient is for a stellar interior.

   (c) (3 points). What are the energy transport mechanisms in the interior regions where $\nabla_{\rm rad} > \nabla_{\rm ad}$, and also when $\nabla_{\rm ad} > \nabla_{\rm rad}$?

   (d) (3 points). See Figure 1 of the exam. Which panel, top or bottom, is the higher-mass model? Give a specific reason why you made this choice.

   (e) (2 points). Pick one of the two models. Describe one possible reason for the large radiative gradient increase?

   (f) (4 points). Describe *quantitatively* the behavior of the adiabatic gradient in either model, both in the deep interior (why 0.4?) and in the outer layers.

   (g) (3 points). The paper mentions a "hot edge" for solar like oscillations to exist in main-sequence stars. Why do you think there must be such a limit?

2. (9 points). In time-domain astronomy, one samples a signal coming from some object at regular intervals, say, with a time cadence $\mathrm{d}t$. If there are periodicities in the signal, the Nyquist frequency (let's denote it ν_N) is the maximum frequency at which you can retrieve information. It is equal to half the sampling frequency. Let's denote the sampling frequency ν_S.

   (a) (2 points). Firstly, how would you express ν_N in terms of ν_S? Then, how would you express ν_S in terms of $\mathrm{d}t$?

   (b) (3 points). Now, show that the temporal Nyquist frequency of Kepler long-cadence data is about 24 cycles per day, as is mentioned in the paper.

   (c) (2 points). What is the Nyquist frequency (in cycles per day) of Kepler short-cadence data, which is obtained every $\mathrm{d}t = 1$ minute? Show your work.

   (d) (2 points). The Sun acoustic oscillations have 5-minute periods, and we been observing these oscillations with various instrumentation for 60 years. What is the maximum time cadence that any of these instruments can have to be able to observe these modes and satisfy the Nyquist criterion?

3. (13 points). Let's try to understand the autocorrelation of a signal a little better, which is an important quantity in time-domain astronomy. We'll consider a continuous signal (y) over a finite interval ($\pm 2\pi$):

$$C(\tau) = \int_{-2\pi}^{+2\pi} y(t)y(t+\tau)\,\mathrm{d}t, \tag{1}$$

where τ is the time lag. To keep things really simple, consider a simple sinusiodal signal $y(t) = \sin(\omega t)$, with angular frequency $\omega = 1\,\mathrm{rad\,s^{-1}}$.

(a) (3 points). What is the cyclic frequency of this signal? What is its period?

(b) (5 points). The autocorrelation is always maximal at a time lag $\tau = 0$. First, draw the integrand of $C(\tau = 0)$ with proper axes over the interval. Then compute the numerical value for $C(\tau = 0)$. Show all work.

(c) (5 points). What is the autocorrelation value at a lag of $+\pi/2$? Again, sketch this scenario, and then show your work to derive and/or argue the value.

4. (13 points). The paper uses some confusing language describing its use of autocorrelations and power spectra. Let's use a simpler example than a real star to learn a bit more about signals buried in the data. See Figure 2 of the exam. A signal with 5 "solar-like oscillations" and some random noise was simulated (top panel). The signal obeys the equation in the paper (Section 3) for the asymptotic limit. Imagine this is a light curve of a star. Various quantities are then calculated from that light curve, given by the other panels. The title shows what quantity the panel contains. Use the figure to answer the following questions.

   (a) (5 points). What are frequencies ν_i and amplitudes A_i for the 5 oscillations? Order them in increasing frequency. Approximate values are fine, so just eyeball it.

   (b) (4 points). What is the approximate value of the large frequency spacing $\Delta\nu$? Explain how you obtained this value (which numbers and/or which plots?).

   (c) (2 points). What is the significance of the vertical line in the bottom panel? Explain why it has the value it has?

   (d) (2 points). Does this simulated light curve more closely resemble that of a δ Scuti or of a γ Doradus star? Why?

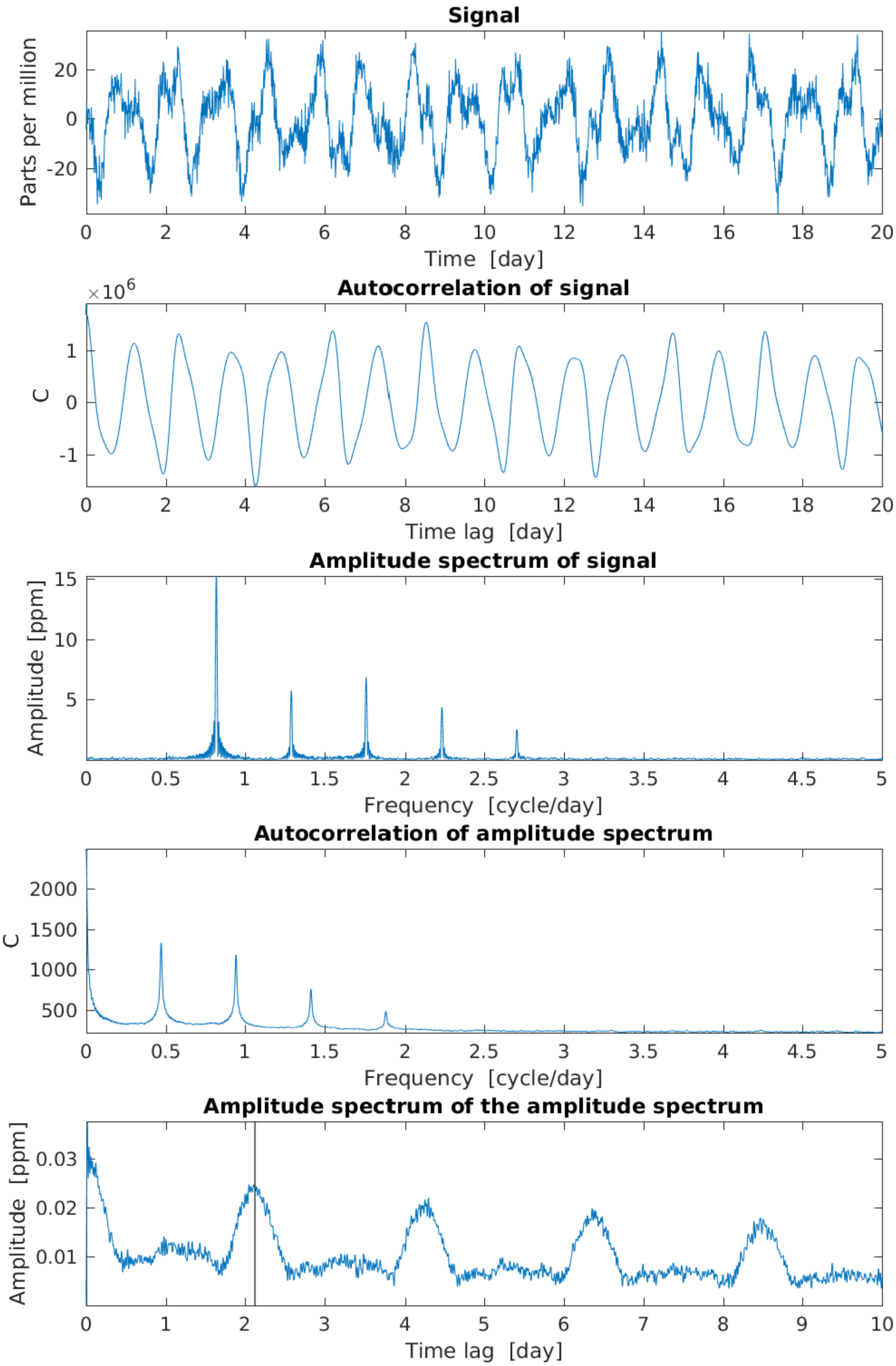


Figure 2: The top panel is a simulated light curve with 5 frequencies. The remaining panels are computations from the signal, given in the title. See question 4.