

Cume # 366 – Solutions

Jason Jackiewicz

December 3, 2011

This exam covers material related to the Sun and stars and focuses on stellar evolution and abundances. The *anticipated* passing grade is 75%, or about 50 out of a total of 66 points.

Show all work clearly and legibly, and if you can't solve something completely, at least give an idea of how you might go about it. Make sure to answer all the sub parts of each question. **DO NOT use your calculators for any formulae or constants, only to calculate.** Start each numbered problem on a new piece of paper. Take your time, think clearly, *read each sentence carefully*, ask for clarification, and best of luck to you!

A few things you may or may not need:

- Solar radius: $R_{\odot} = 6.96 \times 10^{10}$ cm
- Solar mass: $M_{\odot} = 2.0 \times 10^{33}$ g
- Mass of hydrogen nucleus: 1.6726×10^{-24} g
- Mass of helium nucleus: 6.6465×10^{-24} g
- Stefan-Boltzmann constant: $\sigma = 5.68 \times 10^{-5}$ erg cm⁻² s⁻¹ K⁻⁴
- Ideal gas law: $P = \rho RT/\mu$, where P is pressure, ρ is density, R is the gas constant, T is temperature, and μ is the mean molecular weight.

I The evolving Sun (38 points)

1. [4 points]. Assume the solar hydrogen abundance by number is 92.5% and helium is 7.5%. What are the approximate respective abundances by mass?

ANSWER: 1 H is about 1 proton mass, while 1 He is about 4 proton masses (2 protons + 2 neutrons). So take the total mass as 1, then $0.925 m_p + 0.075 \cdot 4 m_p = 1$. Thus, $m_p = 0.8163$, and so if H is 92.5% it is 75.5% of the mass, while He is 24.5%. This works out to about 1.51×10^{33} g of H and 4.9×10^{32} g of He.

2. Answer the following [19 points].

- (a) [2 points]. Very briefly describe what it means for a star to be on the main sequence.

ANSWER: The main sequence is the period in a star's lifetime when it is stably fusing hydrogen into helium in its core.

- (b) [8 points]. Estimate the total main-sequence lifetime of the Sun (in years), using its energy generation and energy output rates.

ANSWER: First, it's important to realize the energy is generated by fusion and leaves by luminosity. The timescale is $t_{\text{MS}} = E/L$. We can compute L from the well-known

$$L_{\odot} = 4\pi\sigma R_{\odot}^2 T_{\text{eff}}^4 = 3.86 \times 10^{33} \text{ erg s}^{-1}, \quad (1)$$

where $T_{\text{eff}} = 5780 \text{ K}$. The energy generated is due to the mass difference between 4 hydrogen nuclei and 1 helium nucleus. It must also be recognized that not all of the hydrogen will be converted to helium, only about 10-20%. In the conversion, about 0.7% of the mass is turned into energy. So collecting all contributions, the total energy that will be generated by the Sun is

$$E = mc^2 \approx (0.15) \cdot (0.007) \cdot (0.755) \cdot (2 \times 10^{33} \text{ g}) \cdot (3 \times 10^{10} \text{ cm}^2 \text{ s}^{-2}) = 1.34 \times 10^{51} \text{ erg.} \quad (2)$$

So dividing by the luminosity gives about 10.1 billion years. There is an acceptable range here based on the other estimates.

- (c) [5 points]. Using your expression from the previous problem, assume a mass-luminosity relationship power law, and derive a very approximate proportionality relation for the main-sequence lifetime of any star of mass M (ignore any constants unrelated to M). What might be an appropriate range of values for the exponent in your resulting expression for a main-sequence star, and why?

ANSWER: The equation implied above essentially scales like $t_{\text{MS}} \sim M/L$. If the mass-luminosity generically goes like $L \sim M^\nu$, then $t_{\text{MS}} \sim M^{1-\nu}$. We know for sure that more massive stars have shorter main-sequence lifetimes, so the exponent ν should be greater than 1 to provide an inverse relationship. In fact, a range of ν from 3-5 is more suitable for most stars.

- (d) [4 points]. How much mass has the Sun lost over its current lifetime assuming a constant rate of energy output and no loss due to flares, CMEs, or the solar wind (show computations)? What is this percentage in terms of the current mass of the Sun?

ANSWER: The Sun is about 4.57 billion years old. As from before, the mass is $m = E/c^2$, where $E = Lt$, thus $m = Lt/c^2$. So

$$m = (3.86 \times 10^{33}) \cdot (4.57 \times 10^9) \cdot (365) \cdot (24) \cdot (60) \cdot (60) \cdot (9 \times 10^{-20}) = 6.18 \times 10^{29} \text{ g,} \quad (3)$$

or, about 0.03% of the current mass of the Sun. That total mass loss is about 100 Earth masses over the solar lifetime.

3. [4 points]. Please see Figure 1. What is so special about the luminosity, how has it changed? What has caused this change based on your knowledge of stellar evolution and stellar processes? Use the figure profiles to explain your answer.

ANSWER: According to the figure, the luminosity used to be about 70% percent of its current value. Note the decrease in central hydrogen abundance as the luminosity increases. As hydrogen is converted to helium, the mean molecular weight μ of the core increases. Since the gas pressure has to be enough to support the overlying material, and the ideal gas law $P = \rho RT/\mu$ holds, ρT must increase to account for the increasing μ . The core contracts and ρ increases, as seen in the figure. The temperature also modestly increases. If the temperature increases, the radiative flux of energy increases, as can also be seen by the decrease in the opacity in the figure. Thus, the luminosity of the Sun increases. In another sense, since the opacity of the core decreases, more flux can escape, increasing the luminosity.

4. [4 points]. What is the current central hydrogen abundance percentage (by mass) given an initial value of 70%? Does this agree with your estimates of its main sequence lifetime and its current age? Explain.

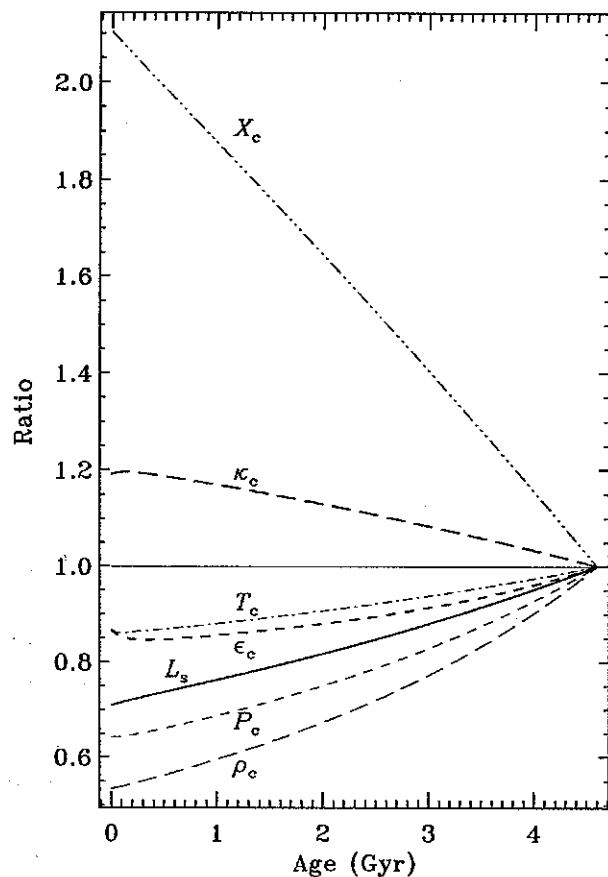


Figure 1: Variations of the properties at the solar core (denoted by subscripts “c”), and the solar surface luminosity L_s , over the evolution of a solar model. All parameters are scaled by their current values. The central parameters are: X_c : hydrogen abundance; κ_c : opacity; T_c : temperature; ϵ_c : energy generation rate; P_c : pressure; ρ_c : density. Adapted from Christensen-Dalsgaard (2008).

ANSWER: $(1/2.1) \times 0.7 = 0.33$, or about 33%. The Sun has burned about half ($0.33/0.70$) of its H, and that is why it is middle aged ($\sim 4.57/10$).

5. [5 points]. If we approximate the Earth’s temperature assuming it absorbs all incoming energy and radiates it as a blackbody, the equilibrium temperature can be approximated as

$$T_E = \left(\frac{L_\odot}{16\pi d^2 \sigma} \right)^{1/4}. \quad (4)$$

What was the temperature of the Earth long ago at formation? What were the implications of this for Earth?

ANSWER: Just by ratios, if the luminosity is 70% of the current value, then the fourth root of 0.7 is about 0.915, or a temperature 8.5% lower, or about 25 K less, than it is today at ~ 290 K. This implies the Earth was frozen, since 273 K is the freezing point, all things being equal. If it were frozen at 265 K, it would even reflect more solar radiation, thus cooling it even further.

6. [2 points]. This is known as the “faint young Sun paradox.” Of course, the Earth is not as simple as a blackbody. What is one reasonable possibility that can accommodate a less luminous Sun long ago without having large ramifications for Earth’s climate at the time?

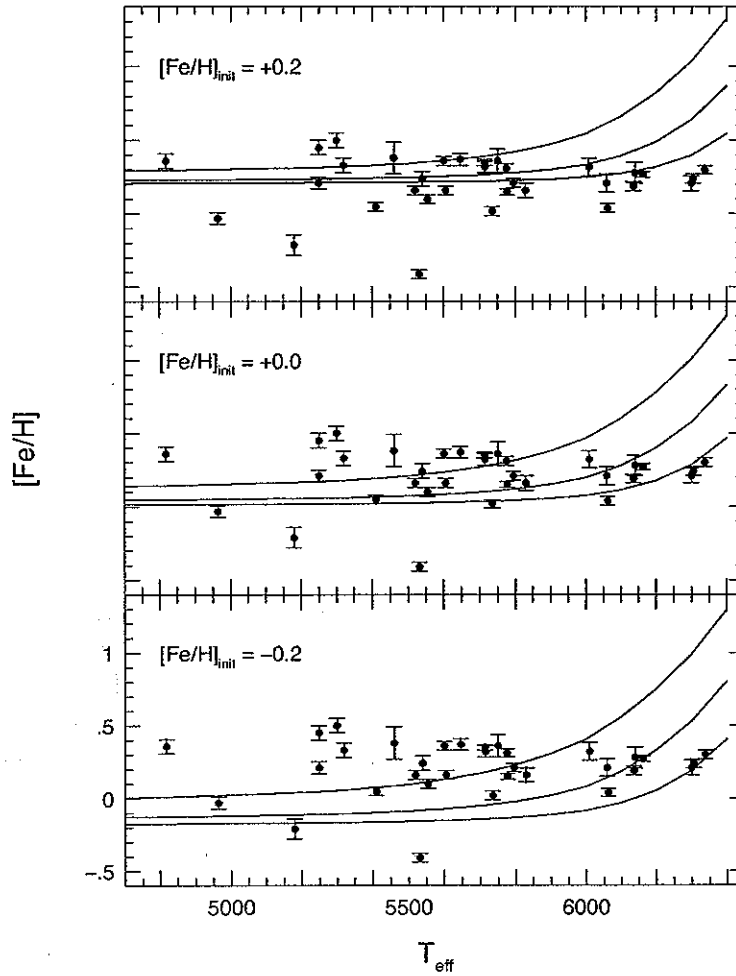


Figure 2: Effective temperatures and $[\text{Fe}/\text{H}]$ derived from spectra for 33 planet-hosting stars (as of 2001), along with uncertainties. The lines denote the effect of adding 1, 3, and 10 Earth masses $M_\oplus$ of Fe to main-sequence stellar models to contaminate their convection zones (all models have convective envelopes). Each panel denotes the initial value of $[\text{Fe}/\text{H}]$ of the model before contamination, as labeled. The lines are **not** fits to the data points. From Pinsonneault et al. (2001).

ANSWER: The Earth certainly has had an atmosphere, and it is possible there was more CO_2 than there is today, thus increasing its greenhouse effect and keeping the planet unfrozen.

II Stars with planets and associated composition (28 points)

It has been observed that planet-hosting stars generally seem to have more heavy elements than stars not hosting planets. One idea is that the stars accreted metal-rich material from swallowing planets or planetary material. This section explores that idea.

1. [5 points]. See Figure 2. What does the y label $[\text{Fe}/\text{H}]$ mean? Be specific, qualitatively and quantitatively. If $[\text{Fe}/\text{H}] = 1$, for example, what does that suggest about a star (give a numerical answer)?

ANSWER: $[\text{Fe}/\text{H}]$ is a generic term to mean metallicity. It is really the abundance of heavy elements with respect to hydrogen in a star with respect to solar values. In other words,

$$[\text{Fe}/\text{H}] = \log_{10} \left[\left(\frac{N_{\text{Fe}}}{N_{\text{H}}} \right)_{\text{star}} / \left(\frac{N_{\text{Fe}}}{N_{\text{H}}} \right)_{\text{Sun}} \right]. \quad (5)$$

Thus, if $[\text{Fe}/\text{H}] = 1$, that means some star has a metallicity of 10 times the solar value.

2. [4 points]. If the Sun has 2 percent of its mass in elements heavier than helium, about how much would the star in the figure with the highest $[\text{Fe}/\text{H}]$ have?

ANSWER: From the figure, we see the highest value is $[\text{Fe}/\text{H}] = 0.5$, or about 3.16 times the solar value. Thus its metallicity is about 6.3%.

3. [5 points]. Is it plausible from Figure 2 that planet-hosting stars are metal rich because of accretion processes? Please explain your thinking.

ANSWER: No, not really. The models clearly predict that accretion of metals would be evident as an increasing trend in the metallicity of hotter stars. That is not observed, the data are flat. The hotter stars do not seem to be more metal rich than the cooler stars. The fact that model lines go through some of the data points means nothing at all; if cool stars with planets are metal rich from accretion then hotter stars must be as well.

4. [2 points]. Why might the hotter star models show such an increase in apparent $[\text{Fe}/\text{H}]$ when large amounts of Fe are added to them?

ANSWER: They are more sensitive to metals because they have shallower, and thus less massive, convection zones than cooler stars. Mixing is less effective, and metallicity has a stronger influence.

5. [3 points]. See Figure 2. For stars cooler than the Sun and with $[\text{Fe}/\text{H}] \gtrsim 0.3$, what is the minimum amount of Fe they would have needed to accrete if they began their main-sequence lives with $[\text{Fe}/\text{H}] \sim -0.2 - 0.0$?

ANSWER: They would have needed to accrete at least $10 M_{\oplus}$ to match the models.

6. [6 points]. On the other hand, for stars hotter than this value and with initial $[\text{Fe}/\text{H}] \sim -0.2 - 0.0$, how much Fe would they have needed to accrete to match the models? If this is the case, what would the necessary initial $[\text{Fe}/\text{H}]$ of cooler stars need to be to match the models? Comment on the likelihood of this for the solar neighborhood.

ANSWER: The hot stars would only have needed to accrete about $1 M_{\oplus}$ to match the models. However, if this is the case, the cooler stars would need an initial $[\text{Fe}/\text{H}]$ of about 0.2 (top panel) to match the models reasonably. This is inconsistent with the solar neighborhood, where $[\text{Fe}/\text{H}] \sim 0.0$. That would imply a metallicity of nearby stars of about 1.58 the solar value, which is possible but unlikely.

7. [3 points]. Why would it be hard to have planets around stars of very low $[\text{Fe}/\text{H}]$?

ANSWER: There needs to be rocky material for terrestrial planets, and probably gas planets too, and rocks are more massive than H or He!

