**Cume # 362**
Jason Jackiewicz
April 16, 2011

This exam covers material related to stellar properties obtained from pulsational characteristics. It is partly based on the attached article, Chaplin *et al.*, "Ensemble Asteroseismology of Solar-Type Stars with the NASA Kepler Mission," *Science*, **332**, 213, (2011). Even though you possibly have not studied much about stellar oscillations, you will be able to rely upon basic principles to work your way through these questions.

The *expected* or *anticipated* passing grade is 75%, or about **49** out of a total of **65 points**.

Show all work clearly and legibly, and if you can't solve something completely, at least give an idea of how you might go about it. Make sure to answer all the sub parts of each question. **DO NOT use your calculators for any formulae or constants, only to calculate.** Remember that $R_\odot = 6.96 \times 10^{10}\,\mathrm{cm}$. Start each numbered problem on a new piece of paper. Take your time, think clearly, *read each sentence carefully*, ask for clarification, and best of luck to you!

# I Qualitative questions (28 points)

1. [**5 points**]. What spectral types of stars are in this catalogue? Discuss some differences between main-sequence and subgiant stars?

2. [**7 points**]. In the second paragraph, the authors say that main-sequence and subgiant stars are unstable to convection. Talk about the convection zones of these types of stars, and describe in some detail what a convective instability is. Try to also express your ideas about convection as an equation, at least an approximate one that illustrates your explanation. What does convection and the convection zone have to do with stellar oscillations?

3. [**6 points**]. In the third paragraph the cadence of Kepler photometry for solar-type stars is discussed. Explain roughly what cadence is minimally necessary for these types of stars to study their pulsations. Based on a cadence you supply, what would the Nyquist frequency be for that time sampling? Explain what the Nyquist frequency tells you in this case.

4. [**6 points**]. What is a power spectrum of stellar oscillations (like the panels in Figure 1 of the paper)? How would they have been computed in this paper (try to use a general formula if possible)? What do you start with? What does it reveal about the star?

5. [**4 points**]. The matrix of power spectra (Figure 1 in the article) shows a trend in frequencies. What is this trend, and how can you connect it to what is stated in the text in various places?

# II A few short calculations (37 points)

1. [**15 points**]. See Figure 1 of this exam. The large frequency separation for acoustic modes of high radial degree on the Sun, $\Delta\nu_\odot$ is equal to the inverse of twice the sound travel time between the stellar surface and core.

   (a) Based on the definition given above, derive a general integral expression for the large frequency separation.

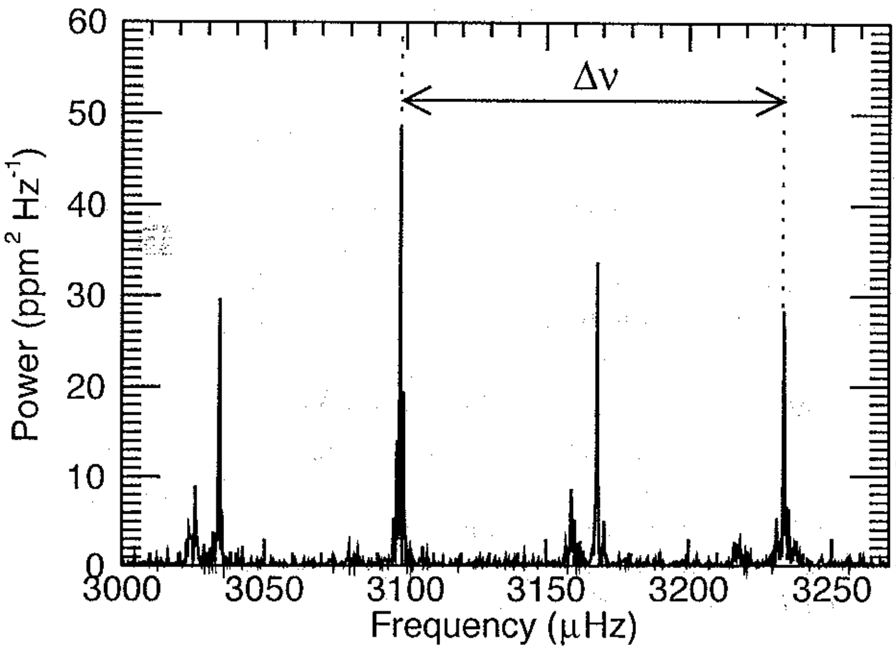


Figure 1: The *large frequency separation* for a solar power spectrum.

(b) Since you don't have the appropriate analytic expression or numerical values to solve this equation, instead estimate the *average* sound speed of the solar interior. Does the number make sense?

(c) Now that you just worked with the large separation, take a look at Figures 2A and 2B in the article. What do you notice that seems a bit strange with the Sun when you compare these two figures? Discuss what you find by relating back to what you now know about $\Delta\nu_\odot$?

2. [**22 points**]. Mass and radius relationships.

(a) Based on discussion in the article for how the frequency of maximum oscillation and the large separation vary with stellar parameters, provide scaled (with respect to the Sun) expressions for these two quantities. For example, for any quantity $q$, provide it as $q/q_\odot$, where $q_\odot$ is the solar variable.

(b) Solve these two equations for the scaled mass and scaled radius.

(c) Using these expressions is referred to as the "direct method"? Explain what is important or unique about this?

(d) This *large separation* is measured between two modes $\nu_{n,\ell}$ and $\nu_{n+1,\ell}$, where $n$ and $\ell$ are the usual labels of spherical harmonics. Firstly, what physically do the $n$ and $\ell$ mean for stellar pulsations (you may and should talk generally about spherical harmonics)? Secondly, since we can measure the individual frequencies quite easily by just fitting the power spectrum, what is the main difficulty in measuring the frequency difference, i.e., the large separation?

(e) The accuracy of the expressions you derived in part (a) could help determine any biases in the mass and radius estimates you derived in part (b). How could we use numerical modeling of stars to help us ascertain this accuracy? Then, given that we don't really know how to simulate very well the excitation and damping of modes in stars, which expression, or parameter (from part (a)), is less amenable to testing with simulations?

# Ensemble Asteroseismology of Solar-Type Stars with the NASA Kepler Mission

W. J. Chaplin,[1*] H. Kjeldsen,[2] J. Christensen-Dalsgaard,[2] S. Basu,[3] A. Miglio,[1,4] T. Appourchaux,[5] T. R. Bedding,[6] Y. Elsworth,[1] R. A. García,[7] R. L. Gilliland,[8] L. Girardi,[9] G. Houdek,[10] C. Karoff,[2] S. D. Kawaler,[11] T. S. Metcalfe,[12] J. Molenda-Żakowicz,[13] M. J. P. F. G. Monteiro,[14] M. J. Thompson,[12] G. A. Verner,[1,15] J. Ballot,[16] A. Bonanno,[17] I. M. Brandão,[14] A.-M. Broomhall,[1] H. Bruntt,[2] T. L. Campante,[2,14] E. Corsaro,[17] O. L. Creevey,[18,19] G. Doğan,[2] L. Esch,[3] N. Gai,[3,20] P. Gaulme,[5] S. J. Hale,[1] R. Handberg,[2] S. Hekker,[1,21] D. Huber,[6] A. Jiménez,[18,19] S. Mathur,[12] A. Mazumdar,[22] B. Mosser,[23] R. New,[24] M. H. Pinsonneault,[25] D. Pricopi,[26] P.-O. Quirion,[27] C. Régulo,[18,19] D. Salabert,[18,19] A. M. Serenelli,[28] V. Silva Aguirre,[29] S. G. Sousa,[14] D. Stello,[6] I. R. Stevens,[1] M. D. Suran,[26] K. Uytterhoeven,[7] T. R. White,[6] W. J. Borucki,[30] T. M. Brown,[31] J. M. Jenkins,[32] K. Kinemuchi,[33] J. Van Cleve,[32] T. C. Klaus[34]

In addition to its search for extrasolar planets, the NASA Kepler mission provides exquisite data on stellar oscillations. We report the detections of oscillations in 500 solar-type stars in the Kepler field of view, an ensemble that is large enough to allow statistical studies of intrinsic stellar properties (such as mass, radius, and age) and to test theories of stellar evolution. We find that the distribution of observed masses of these stars shows intriguing differences to predictions from models of synthetic stellar populations in the Galaxy.

An understanding of stars is of central importance to astrophysics. Uncertainties in stellar physics have a direct impact on fixing the ages of the oldest stellar populations (which place tight constraints on cosmologies) as well as on tracing the chemical evolution of galaxies. Stellar astrophysics also plays a crucial role in the current endeavors to detect habitable planets around other stars (*1*–*5*). Accurate data on the host stars are required to determine the sizes of planets discovered by the transit method, to fix the locations of habitable zones around the stars, and to estimate the ages and to understand the dynamical histories of these stellar systems. Measurements of the levels of stellar activity and their variations over time (*6*) provide insights into planetary habitability, the completeness of the survey for extrasolar planets, and the surface variability shown by our own Sun, which has very recently been in a quiescent state that is unique in the modern satellite era (*7*, *8*).

New insights are being made possible by asteroseismology, the study of stars by observations of their natural, resonant oscillations (*9*, *10*). Stellar oscillations are the visible manifestations of standing waves in the stellar interiors. Main-sequence and subgiant stars whose outer layers are unstable to convection (solar-type stars) display solarlike oscillations that are predominantly acoustic in nature, excited by turbulence in the convective envelopes (*11*, *12*). The dominant oscillation periods are minutes in length and give rise to variations in stellar brightness at levels of typically just a few parts per million. The frequencies of the oscillations depend on the internal structures of the stars, and their rich information content means that the fundamental stellar properties (e.g., mass, radius, and age) can be determined to levels that are difficult to achieve by other means and that the internal structure and dynamics can be investigated in a unique way.

Helioseismology has provided us with an extremely detailed picture of the internal structure and dynamics of the Sun, including tests of basic physics (*13*–*15*). Such investigations are beginning to be possible for other stars. Over the past decade, the quality of seismic observations on other solar-type stars has been improving steadily, from ground-based spectroscopy (*16*–*18*) and the French-led CoRoT (Convection Rotation and Planetary Transits) satellite (*19*, *20*). Now, Kepler is providing ultraprecise observations of variations in stellar brightness (photometry), which are suitable for the study of solarlike oscillations (*21*). During the first 7 months of science operations, more than 2000 stars were selected for observation for 1 month each with a cadence rapid enough to perform an asteroseismic survey of the solar-type population in the Kepler field of view. Here, we report the detection of solarlike oscillations in 500 of those stars. Previously, this type of oscillation had been detected in only about 25 stars.

As is evident from the frequency spectra of the oscillations exhibited by nine stars from the ensemble (Fig. 1), solarlike oscillators present a rich, near-regular pattern of peaks that are the signatures of high-order overtones. The dominant frequency spacing is the so-called large separation, $\Delta\nu$, between consecutive overtones (*22*). The average large separation scales approximately with the square root of the mean density of the star. The observed power in the oscillations is modulated in frequency by a Gaussian-like envelope. The frequency of maximum oscillation power, $\nu_{max}$, scales approximately as $gT_{eff}^{-1/2}$, where $g \propto M/R^2$ is the surface gravity and $T_{eff}$ is the effective temperature of the star (*23*, *24*).

Figure 2 shows all the stars on a conventional Hertzsprung-Russell diagram, which plots the luminosities of stars against $T_{eff}$. The temperatures were estimated (*25*) from multicolor photometry

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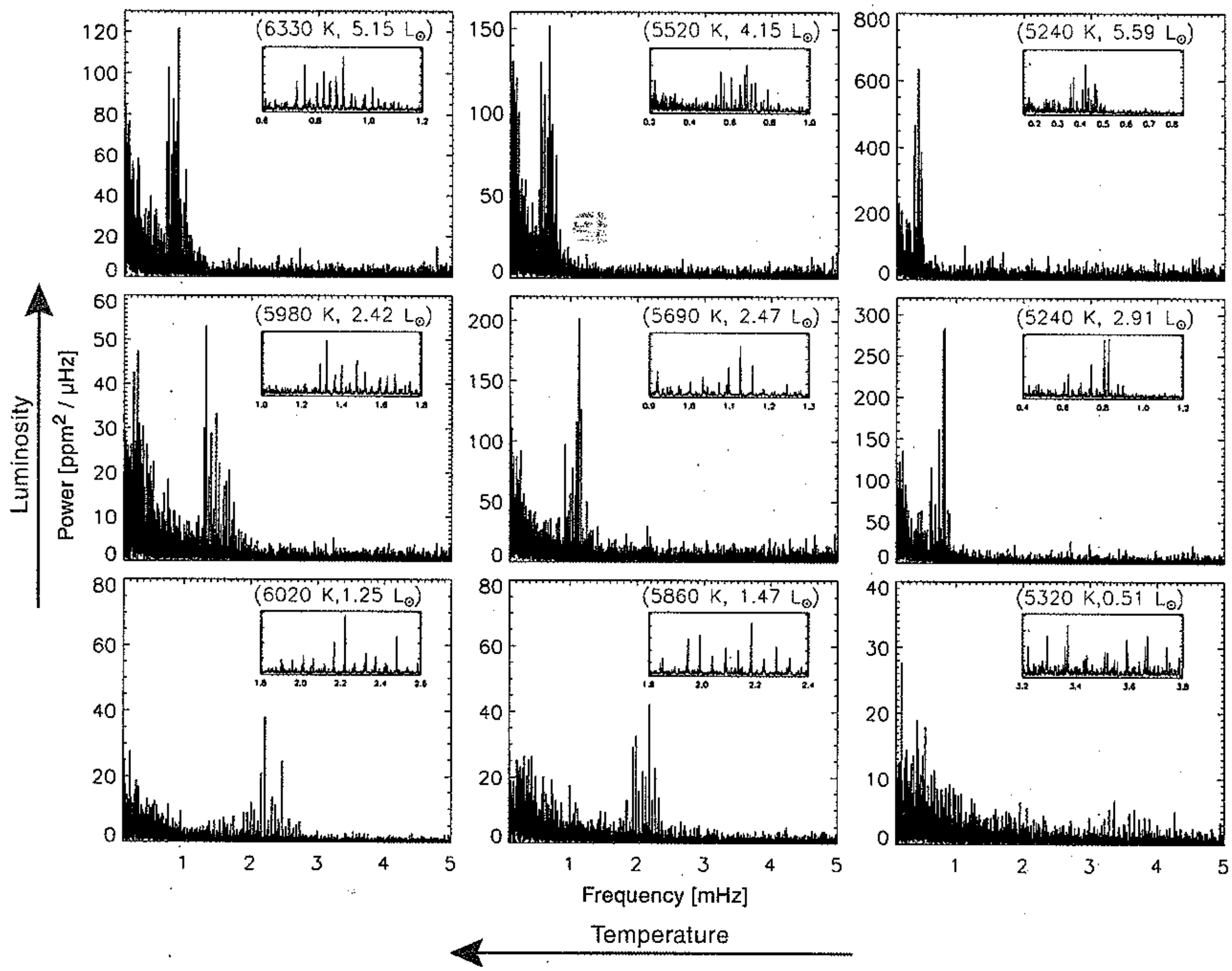


**Fig. 1.** Frequency spectra of the oscillations exhibited by nine stars from the ensemble. Each spectrum shows a prominent Gaussian-shaped excess of power because of the oscillations, centered on the frequency $\nu_{max}$. **(Insets)** Clearer views of the near-regular spacings in frequency between individual modes of oscillation within each spectrum. The stars are arranged by intrinsic brightness [in units of solar luminosity ($L_\odot$)] and temperature, with intrinsically fainter stars showing weaker, less prominent oscillations than their intrinsically brighter cousins. ppm, parts per million.

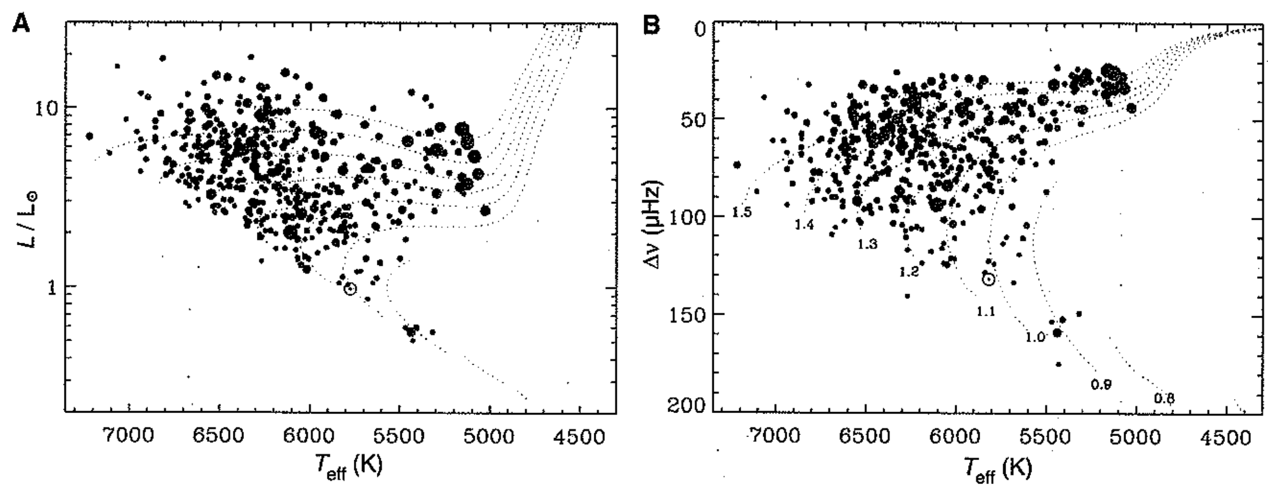


**Fig. 2. (A)** Estimates of the luminosities of the stars (in units of the solar luminosity) of the ensemble of Kepler stars showing detected solarlike oscillations, plotted as a function of effective temperature. Stars from Fig. 1 are plotted with red symbols. **(B)** Average large frequency separations, $\Delta\nu$, against effective temperature. The symbol sizes are directly proportional to the prominence of the detected oscillations (i.e., the signal-to-noise ratios). These ratios depend both on stellar properties (e.g., the photometric amplitudes shown by the oscillations and the intrinsic stellar backgrounds from convection) and the apparent brightness of the stars. The dotted lines show predicted evolutionary tracks (*33*) for models of different stellar mass (0.8 to 1.5 solar masses, in steps of 0.1). The Sun is marked with a solar symbol (⊙).

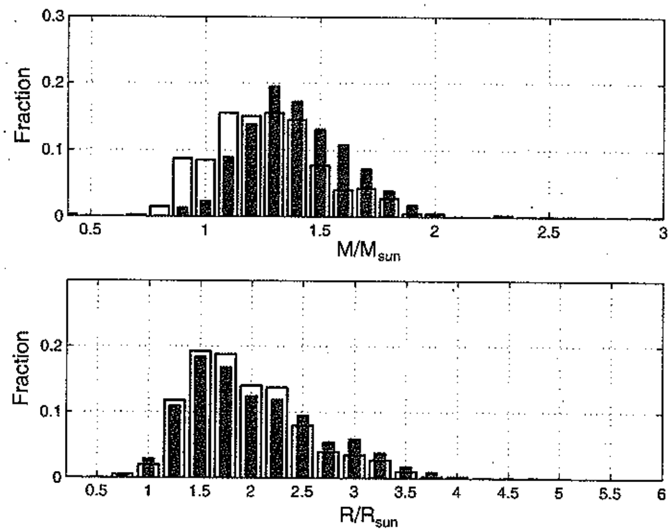


**Fig. 3.** Black lines show histograms of the observed distribution of masses (**top**) and radii (**bottom**) of the Kepler ensemble (*27*). In red, the predicted distributions from population synthesis modeling, after correction for the effects of detection bias (*27*). The population modeling was performed by using the TRILEGAL code (*34, 35*).

available in the Kepler Input Catalog (*26*). Luminosities were estimated from the temperatures and the seismically estimated radii [see below and (*27*)]. We also plot $\Delta\nu$ against temperature, and, just like the conventional diagram, this asteroseismic version delineates different types of stars and different evolutionary states (the $\nu_{max}$ version is similar). Main-sequence stars, burning hydrogen into helium in their cores, lie in a diagonal swathe (from the lower right to top left) on each diagram. Both asteroseismic parameters, $\Delta\nu$ and $\nu_{max}$, decrease along the main sequence toward hotter solar-type stars, where surface gravities and mean densities are lower than in cooler stars (and luminosities are higher). After exhaustion of the core hydrogen, stars eventually follow nearly horizontal paths in the luminosity plot toward lower temperatures as they evolve as subgiants, before turning sharply upward to become red giants (*28*, *29*). The values of $\Delta\nu$ and $\nu_{max}$ decrease comparatively rapidly through the subgiant phase. Detailed information on the physics of the interiors of these stars is emerging from analysis of Kepler data (*30*).

We have detected solarlike oscillations in relatively few stars that have $\Delta\nu$ and $\nu_{max}$ larger than the solar values. These stars are intrinsically fainter and less massive than the Sun, and we see fewer detections because the intrinsic oscillation amplitudes are lower than in the hotter main-sequence and evolved subgiant stars. This detection bias means that the most populous cohort in the ensemble is that comprising subgiants. Subgiants have more complicated oscillation spectra than main-sequence stars. The details of the spectra depend on how, for example, various elements are mixed both within and between different layers inside the stars. Seismic analysis of the Sun has already shown that merely reproducing the luminosity and temperature of a star will not guarantee that the internal structure, and hence the underlying physics, is correct. This inspired the inclusion of additional physics, such as the settling over time of chemical elements because of gravity, in stellar models (*13*). The Sun is a relatively simple star compared with some of the solar-type stars observed by Kepler.

We made use of the $\Delta\nu$ and $\nu_{max}$ of the ensemble together with photometric estimates of the temperatures to estimate the masses and radii of the stars in a way that is independent of stellar evolutionary models—by using the so-called direct-method of estimation (*27*)—and then compared the observed distributions with those predicted from synthetic stellar populations (Fig. 3). The synthetic populations were calculated by modeling the formation and evolution of stars in the Kepler field of view, which lies in the Cygnus region of the Orion arm of our Galaxy, the Milky Way (*27*). This modeling requires descriptions of, for example, the star-formation history (including the frequency of occurrence of stars with various masses), the spatial density of stars in the disc of the Milky Way, and the rate at which the Galaxy is chemically enriched by stellar evolution (*31*).

Previous population studies have been hampered by not having robust mass estimates on individual stars (*31*). Precise estimates of masses of solar-type stars had been limited principally to stars in eclipsing binaries (*32*). The Kepler estimates add substantially to this total and in numbers that are large enough to do statistical population tests by using direct mass estimates, which had not been possible before.

Whereas the distributions of stellar radii in Fig. 3 are similar, the same cannot be said for the mass distributions. We have quantified the significance of the differences by using statistical tests. Differences in radius were judged to be marginally significant at best. In contrast, those in mass were found to be highly significant (>99.99%) (*27*). The observed distribution of masses is wider at its peak than the modeled distribution and is offset toward slightly lower masses.

Tests suggest that, for the bulk of the stars, bias in the estimated masses and radii is no larger than the estimated uncertainties (*27*). On the assumption that the observed masses and radii are robust, this result may have implications for both the star-formation rate and the initial mass function of stars. Mixing or overshooting of material between different layers (including stellar cores) and the choice of the so-called mixing length parameter, which measures the typical length scale of the convection and is one of the few free parameters in stellar evolution theory, may also be relevant. It is yet to be tested whether the expected small fraction of unresolved binaries could have contributed to the mass discrepancy.

### References and Notes

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Cume # 362 – Solutions
Jason Jackiewicz
April 16, 2011

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The *expected* or *anticipated* passing grade is 75%, or about **49** out of a total of **65 points**.

Show all work clearly and legibly, and if you can't solve something completely, at least give an idea of how you might go about it. Make sure to answer all the sub parts of each question. **DO NOT use your calculators for any formulae or constants, only to calculate.** Remember that $R_\odot = 6.96 \times 10^{10}$ cm. Start each numbered problem on a new piece of paper. Take your time, think clearly, *read each sentence carefully*, ask for clarification, and best of luck to you!

# I Qualitative questions (28 points)

1. **[5 points]. What spectral types of stars are in this catalogue? Discuss some differences between main-sequence and subgiant stars?**

   *ANSWER: We have (a few K), G, and F type stars ($\sim 5000 - 7200$ K), both main sequence and subgiant, using the HR diagram (2A) in the paper. Subgiants are slightly brighter than main-sequence stars, and a bit bigger too. They are likely fusing hydrogen in a shell around the core since the core contracted and heated up.*

2. **[7 points]. In the second paragraph, the authors say that main-sequence and subgiant stars are unstable to convection. Talk about the convection zones of these types of stars, and describe in some detail what a convective instability is. Try to also express your ideas about convection as an equation, at least an approximate one that illustrates your explanation. What does convection and the convection zone have to do with stellar oscillations?**

   *ANSWER: The convection zones of some of these types of stars make up the outer envelope where heat gets transported by convection to the surface. Since some stars in the sample are more massive than our Sun (as shown in Figure 3A), it's likely that some of these stars have convective cores too. A convective instability may occur inside a star when a parcel of gas at a particular density is displaced adiabatically to a slightly shallower depth. If the parcel has a lighter density than its new surroundings, it continues its march upwards. This is a convective instability. More technically, it occurs when the local temperature gradient is larger (steeper) than the adiabatic temperature gradient ($\mathrm{d}T/\mathrm{d}r > |\mathrm{d}T/\mathrm{d}r|_{\mathrm{ad}}$). The convective and turbulent motions in stars excites waves, and can also damp them and pump energy into them. Convection is, therefore, thought to play a major role in driving solar-like oscillations.*

3. **[6 points]. In the third paragraph the cadence of Kepler photometry for solar-type stars is discussed. Explain roughly what cadence is minimally necessary for these types of stars to study their pulsations. Based on a cadence you supply, what would**

*because models can pretty easily reproduce solar effective temperatures and the luminosity. However, when plotted with the large frequency separation as the dependent variable in 2B, the Sun does not necessarily fall on a model curve at the right place for its temperature. Since the large separation depends on the sound speed throughout the interior as just computed, the models don't get that sound speed absolutely correct (likely because of the core region and the convection boundary).*

2. [22 points]. Mass and radius relationships.

   (a) **Based on discussion in the article for how the frequency of maximum oscillation and the large separation vary with stellar parameters, provide scaled (with respect to the Sun) expressions for these two quantities. For example, for any quantity $q$, provide it as $q/q_\odot$, where $q_\odot$ is the solar variable.**

   *ANSWER: It states that the large separation scales with the square root of the mean density. Therefore, $\Delta\nu \sim M^{0.5}R^{-1.5}$. The frequency at maximum amplitude scales as $\nu_{\rm max} \sim MR^{-2}T_{\rm eff}^{-0.5}$. Scaled relations would eliminate any messy constants in these equations and would look like:*

$$\frac{\Delta\nu}{\Delta\nu_\odot} = \left(\frac{M}{M_\odot}\right)^{0.5}\left(\frac{R}{R_\odot}\right)^{-1.5} \tag{6}$$
$$\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}} = \left(\frac{M}{M_\odot}\right)\left(\frac{R}{R_\odot}\right)^{-2}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{-0.5}. \tag{7}$$

   (b) **Solve these two equations for the scaled mass and scaled radius.**

   *ANSWER: By eliminating variables, one can show*

$$\left(\frac{R}{R_\odot}\right) = \left(\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}}\right)\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-2}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{0.5}, \tag{8}$$
$$\left(\frac{M}{M_\odot}\right) = \left(\frac{\nu_{\rm max}}{\nu_{\rm max,\odot}}\right)^{3}\left(\frac{\Delta\nu}{\Delta\nu_\odot}\right)^{-4}\left(\frac{T_{\rm eff}}{T_{\rm eff,\odot}}\right)^{1.5}. \tag{9}$$

   (c) **Using these expressions is referred to as the "direct method"? Explain what is important or unique about this?**

   *ANSWER: It shows we can estimate the mass and radius without use of models, just observations of pulsation frequencies, and temperature. This can be quite powerful, especially given the high accuracy from which we can measure frequencies from Kepler.*

   (d) **This *large separation* is measured between two modes $\nu_{n,\ell}$ and $\nu_{n+1,\ell}$, where $n$ and $\ell$ are the usual labels of spherical harmonics. Firstly, what physically do the $n$ and $\ell$ mean for stellar pulsations (you may and should talk generally about spherical harmonics)? Secondly, since we can measure the individual frequencies quite easily by just fitting the power spectrum, what is the main difficulty in measuring the frequency difference, i.e., the large separation?**

   *ANSWER: Any oscillation of a spherical star can be represented in a spherical harmonic expansion. The $n$ label of the function denotes the number of zeros the function has in the radial direction. The $\ell$ label tells us how many total zeros are on the surface. We'll ignore $m$ for now. The main difficulty in measuring this separation is that you need to know that you have modes of $\nu_{n,\ell}$ and $\nu_{n+1,\ell}$, i.e., you have to identify these modes. That is not always easy, and must usually be done with modeling. Also, since we can't resolve stellar surfaces,* *the number of nodes along the surface, $\ell$, usually cancel each other out, limiting us to only the first few low $\ell$ values.*

(e) **The accuracy of the expressions you derived in part (a) could help determine any biases in the mass and radius estimates you derived in part (b). How could we use numerical modeling of stars to help us ascertain this accuracy? Then, given that we don't really know how to simulate very well the excitation and damping of modes in stars, which expression, or parameter (from part (a)), is less amenable to testing with simulations?**

*ANSWER: With stellar modeling we may compare the values of, for example, $\Delta\nu$ computed for a model of mass $M$ and radius $R$, with the large separation that the model provides in its simulated frequencies. We can do the same for $\nu_{\max}$ given the mass, radius and temperature of the model, and compare with the model's frequencies of maximum amplitude. However, since we don't have a good handle on the damping and excitation for solar-like stars, it is extremely unlikely the mode amplitudes will be accurate, which depends highly on these things. Therefore, doing this experiment with $\nu_{\max}$ will not be successful.*