Read the paper "Systematic variation of the stellar initial mass function in early-type galaxies" by Cappellari et al. Nature 484, 487 (2012).

I expect a passing grade will be 70-75 percent.

1 solar mass = 2×10^{33} gm. 1 solar luminosity = $3.8 \times 10^{33} ergs/s$. 1 pc = 3.086×10^{18} cm. $G = 6.67 \times 10^{-8} cm^3 gm^{-1} s^{-1}$.

1. (20 points) Initial mass function

- (a) Make a plot that shows both a Salpeter IMF and a Kroupa IMF, roughly normalized so that the populations they represent would have the same total amount of *light*. Label the axes with names and numbers. Note the limits on the x-axis.
 - Plot should be log-log with mass and relative number. X-axis should range from 0.1 to 100 Msun. Salpeter IMF should have slope -1.35 (in log-log). Kroupa IMF should be similar at high mass end to provide same amount of light, but flatter at low mass end (Kroupa has significantly less mass); actually is a 3-segment power law.
- (b) In a Salpeter IMF with a lower mass cutoff of 0.1 solar masses and a upper mass cutoff of 100 solar masses, at what mass does one have half the mass below and half the mass above?

Integrate MdN/dM to find half-mass mass at around 0.68 solar masses

- (c) In the solar neighborhood, why is there a difference between the present day mass function and the initial mass function? Make a sketch showing qualitatively how the two might compare, again labelling the axes.
 - In the solar neighborhood, there is a spread of stellar ages, so there are relatively more low mass stars than high mass stars compared with the IMF, since more massive older stars have evolved. IMF and PDMF should agree below 1 solar mass where evolution over a Hubble time is unimportant.
- (d) The techniques in this paper constrain the total mass in a population, but not the shape of the IMF. What different techniques/observations might you use to go about measuring the shape of the IMF? What objects would you look at? What are some of the issues/pitfalls you might encounter?

One would look at a resolved stellar population and would count the relative numbers of stars of different brightnesses and convert these to masses. Ideally, work at a fixed age, which would give the IMF in a mass range depending on the age; however, in clusters need to be careful of dynamical effects. In field, need some information about star formation history. In both cases, need to be careful of unresolved binaries.

2. (35 points) Mass-to-light ratios

- (a) Using normal convention, what is the mass-to-light ratio of the Sun? one, because the normal convention if to express M/L in solar units
- (b) Quantitatively estimate the mass-to-light ratios of an O star and an M star. If $L \propto M^{3.3}$, then $M/L \propto M^{-2.3}$. For 30 Msun, $M/L \sim$ 0.0004, for 0.2 Msun, $M/L \sim$ 40. Other masses and luminosities could be chosen ...

(c) Which would you expect to be larger, the stellar mass-to-light ratio of a typical spiral galaxy or a typical elliptical galaxy? Why? Roughly what are the expected mass-to-light ratios?

Typical spiral has younger stellar population so lower M/L. Spirals perhaps around 1-2 (they are not exclusively young!), ellipticals perhaps 3-8.

(d) Normally, the stellar mass-to-light ratio is thought to be larger for a steeper IMF slope. However, the paper suggests that their inferred high M/L could come from either a significantly steeper or a significantly flatter slope. How can this be?

For a flat slope in an old population, there are a lot of dead stars that leave dark remnants, leading to a high M/L.

(e) Why is it very difficult to tell just from the spectrum of an elliptical galaxy what the stellar mass-to-light ratio is?

Because the light all comes from the most luminous stars, which in an older population, are almost all of the same mass.

(f) (10 points) Most of the light in the Milky Way comes from a region within 15 kpc from the center. If the rotation curve is flat at 220 km/s out to this distance, estimate the fraction of dark matter within 15 kpc, assuming a stellar M/L ratio of 2. Assume that the Milky Way has a absolute visual magnitude of $M_V = -20.5$, with the Sun having $M_V = 4.8$.

Calculate a dynamical mass from $M = v^2R/G = 1.6e11$ solar masses. Absolute mags give luminosity of 1.3e10 solar luminosities; with M/L=2, this gives stellar mass of 2.6e10 solar masses. Could add in a fraction of this for gas. Combine to find dark matter fraction around 0.83 (without any gas correction).

3. (15 points) Dynamical models

(a) (5 points) Of the five galaxies in Figure 1, which one do you think is most likely to have the largest amount of rotational support? Why? NGC 4638, the one with low V_{rms} in the center.

(b) (10 points) In the paper, one of the models does not require the presence of dark matter at all. Which model is that? Given what you know about ellipitical galaxies, is this a plausible model? Referring to other data that you might be familiar with, why or why not? What additional stellar dynamical data would you want to obtain in order to rule out this model?

Model A. Probably not plausible because E's most likely have extended dark matter halos. This is demonstrated by more distant tracers, e.g. satellite galaxies, also weak lensing. To rule this out, one would want dynamical data at larger radii.

4. (25 points) Elliptical galaxies and galaxy formation

(a) (10 points) Describe some of the variations seen in parameters that describe elliptical galaxies, including surface brightness profiles, isophotal shapes, dynamical properties, and line strengths, and discuss correlations between these parameters. Surface brightness profiles generally represented by Sersic indices, but vary from $n \sim 4$ to $n \sim 1$ as a function of luminosity. Isophotal shapes range from disky to boxy, also correlated with luminosity: more luminous Es tend to be boxy. v/σ varies from 0 to ~ 1 . Line strengths, e.g. Mg, are larger at larger luminosity. Luminosity, velocity dispersion, effective radius coupled by fundamental plane relation.

(b) (5 points) Merging has been suggested as an important path to get to present day elliptical galaxies. Are all mergers the same? Describe some of the variations in possible merger scenarios.

Various mass ratios and orbital trajectories. Also mergers with and without gas and/or stars.

(c) (10 points) State in your own words the paper's primary conclusion regarding IMF variations in ellipticals: how do they infer them and what do they claim they depend on? Discuss the implications of this result for the merger hypothesis for the formation of luminous ellipticals.

Based on dynamical modelling, the IMF varies systematically in ellipticals, with higher mass galaxies having larger M/L. If so, can't form luminous ellipticals with gas-poor mergers!