

Instructions

A hearty welcome to Cume #480! This exam will drill concepts associated with extragalactic astronomy and cosmology. The associated paper is “Unveiling galaxy chemical enrichment mechanisms out to cosmic dawn from direct determination of O & Ar abundances from JWST/NIRSPEC spectroscopy,” by Bhattacharya et al. (<https://arxiv.org/pdf/2408.13396>)

- There are 14 questions and 51 total points are possible. A score of 36 points is a guaranteed pass.
- Please show all work, use a nice new piece of paper for each section or problem, and submit work in problem order.
- Be very explicit. For example, if you need to invoke the concept of a “critical density”, don’t just use that phrase, write down what it actually means for full credit.
- If you cannot obtain an answer to one question that is necessary for a subsequent question, just assume something for the upstream problem and then move on as you will be graded on your method.
- You may not use *any* resources to complete this cume other than your memory, a scientific calculator, and what is provided in the cume itself. Relatedly, science recommends putting your phone behind a zipper and powering down computers, tablets, smartwatches, smartpencils, smartcoffeemugs, or any other circuitry that you weren’t born with.
- I will be available throughout the exam period in my office (AY 104).

$$G = 6.672 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}$$

$$c = 2.99792458 \times 10^{10} \text{ cm/s}$$

$$\text{Mpc} = 3.0855 \times 10^{24} \text{ cm}$$

$$\pi = 3.14159265359$$

$$M_{\odot} = 1.989 \times 10^{33} \text{ g}$$

$$M_{\oplus} = 5.98 \times 10^{27} \text{ g}$$

$$R_{\oplus} = 6.38 \times 10^8 \text{ cm}$$

$$1 \text{ erg} = 6.242 \times 10^{11} \text{ eV}$$

$$H^2(z) = H_0^2 [\Omega_M(1+z)^3 + \Omega_{\kappa}(1+z)^2 + \Omega_r(1+z)^4 + \Omega_{\Lambda}]$$

1 Key Concepts

- (2 points) What is an α -element (half-credit for just listing examples)?
An element[, one isotope of] which has a nucleus that be constructed by merging enough [α particles / He nuclei] [onto a ^{12}C nucleus].
- (2 points) What electronic transition gives an $\text{H}\beta$ photon, and what kind of atom produces it?
A photon produced by a neutral hydrogen atom whose electron decays from $n = 4 \rightarrow 2$.
- (4 points) Column 3 of Table 1 gives the “Balmer decrement,” which is referred to as $c(\beta)$ in the text of Section 2.3. (i) What is the Balmer decrement and (ii) what is it used for?
(i) The Balmer decrement is the observed ratio of the $\text{H}\alpha$ and $\text{H}\beta$ emission line fluxes. It is used to infer the amount of dust reddening affecting nebular line emission from star-forming galaxies. This is important because the intrinsic and measured flux ratios of widely-separated emission lines are modulated by dust.
- (2 points) In one sentence, distinguish physically between the likely progenitors of core-collapse supernovae (CCSNe) and Type Ia supernovae.
CCSNe occur when a massive star’s core that has fused all the way up to the iron peak collapses to a neutron star or black hole. A Type Ia SN occurs when an accreting or merging white dwarf becomes too massive for electron degeneracy pressure to balance gravity, collapses, and fuses entirely up to the iron peak.

5. (3 points) In 1–2 sentences, what physical considerations determine the Chandrasekhar Mass? What is that mass in $M_{\odot}$, roughly?
A balance between electron degeneracy pressure and gravity. It is reached around $1.4M_{\odot}$.
6. (2 points) In 1–2 sentences, what is the nucleosynthetic *yield* of an element (such as oxygen), conceptually?
This has multiple definitions, but the classic one in Tinsley 1980 is the ratio of the total mass of new metals released in CCSNe by an IMF-averaged stellar population to the total mass of long-lived stars formed.

2 Reading Comprehension

7. (4 points) In at most two sentences, how does $[\alpha/\text{Fe}]$ constrain a galaxy's star formation history? Put differently, what do you know about a galaxy with a low versus a high $[\alpha/\text{Fe}]$?
The $[\alpha/\text{Fe}]$ ratio roughly measures the ratio of the output from CCSNe, which occur quasi-instantaneously when stars form, with Fe, whose presence is dominated by Type Ia SNe. As the latter take much longer than the former, a high $[\alpha/\text{Fe}]$ indicates a short star formation timescale; that is, that most of the metals were formed over a short time after star formation began.
8. (2 points) In at most one sentence, what is the primary advantage of the current work over Rogers 2024 and Welch 2024 according to the Introduction?
Large sample size and wider redshift range.
9. (8 points) I have annotated the “intermittent starburst scenario” model in Figure 3b to indicate four regimes (I, II, III, and IV). Describe how the model gives rise to the trajectory at each of these four points along the yellow/orange curve. Your answer should refer explicitly to both of the peaks in the inset star formation history and include mention of why the slope evolves as it does.
(I) Both CCSNe and Type Ia contribute, but delayed onset of Type Ia means that (O/Ar) slowly decreases below CCSNe yield ratios. This is the initial peak; (II) rapid inflow dilutes metallicity without changing ratios; this is the beginning of the second peak; (III) CCSNe increase (O/Ar) back close to CCSNe yield ratio during the second SF peak; (IV) Type Ia set in again, slowly suppressing (O/Ar) as before.

3 Using Metallicities

10. (4 points) Galaxies' metallicities are often quoted in units $12+\log(X/H)$ where X is the element of interest. This really means, “twelve plus the base-ten logarithm of the ratio of the number of atoms of X to the number of atoms of H.” Compute the argon mass fraction of the galaxy ERO 10612. By “argon mass fraction” is meant the ratio of the argon mass to the mass in H and He. Indicate clearly what datum you adopt from the paper. You will need to know that the atomic mass of argon is 39.95 AMU.
Adopting a value of $12+\log(\text{Ar}/\text{H}) = 5.82$ and approximating the total mass as the mass of hydrogen and helium, this is

$$\begin{aligned}
 Z_{\text{Ar}} &= \frac{M_{\text{Ar}}}{M} \\
 &= \frac{N_{\text{Ar}}m_{\text{Ar}}}{N_{\text{H}}m_{\text{H}}/X_{\text{H}}}, \text{ where} \\
 \frac{N_{\text{Ar}}}{N_{\text{H}}} &= 10^{5.82-12}
 \end{aligned}$$

and X_{H} is the cosmic hydrogen mass fraction, roughly 0.76 (an astro PhD should know this number). This comes to 2×10^{-5} .

11. (6 points) From Table 2, calculate the mean uncertainty in their measured $\log(\text{O}/\text{Ar})$. Then, calculate the standard deviation in the distribution of $\log(\text{O}/\text{Ar})$ values. Using the result, speculate as to whether the observed scatter is consistent with being driven by the redshift dependence, observational uncertainties, or some other factor.
Calculation the mean uncertainty and standard deviation in the usual way gives a mean uncertainty of 0.21 and a standard deviation in the values of 0.37. The fact that the scatter is larger than the formal uncertainties indicates that redshift evolution or intrinsic variation are implicated.

12. Suppose that the nucleosynthetic yields for oxygen and argon are y_{O} and y_{Ar} , respectively. Suppose further that a “closed-box” galaxy began with mass M , and at some later time has formed a mass M_* in long-lived stars. In a closed-box model, no gas or stars enter or leave the galaxy.
- (a) (1 point) Argue that the mass remaining in gas is $M - M_*$.
 If the system is a closed box, then the mass in gas must be the initial mass M minus the mass M_* that has condensed into stars.
- (b) (2 points) Write down an expression for the gas-phase oxygen metallicity (that is, the mass fraction of gas that is oxygen) Z_{O} .

$$\begin{aligned} Z_{\text{O}} &= M_{\text{O}}/M_{\text{g}} \\ &= \frac{y_{\text{O}} M_*}{M - M_*} \end{aligned}$$

- (c) (2 points) Write down an expression for the ratio of the gas-phase oxygen to argon metallicities $Z_{\text{O}}/Z_{\text{Ar}}$.
 $Z_{\text{O}}/Z_{\text{Ar}} = y_{\text{O}}/y_{\text{Ar}}$

4 Cosmology

13. (4 points) Derive an expression for the age of the Universe at $z = 6$, then substitute in numbers to get a numerical answer in years. (Hint: You may assume an Einstein-de Sitter cosmology.)
 From the Friedman Equation given at the top of the exam and setting $\Omega_M = 1$ and all other Ω s to zero:

$$\frac{\dot{a}}{a} = \frac{1}{H_0} \sqrt{(1+z)^3}$$

Substituting $a = (1+z)^{-1}$ and integrating leads to

$$t = \frac{2}{3H_0} a^{3/2}$$

Students must *know* $H_0 \approx 70$ km/s/Mpc. Substituting gives roughly 0.5 Gyr.

5 Analysis

14. (3 points) List three sources of uncertainty affecting the inferred argon metallicity of CEERS 1536.
 Possibilities include uncertain continuum correction, observational noise, ionization correction (ICF), dust condensation, dust reddening.