Instructions

A hearty welcome to Cume #471! This exam will drill a subset of the topics that Nancy's welcome letter encourges all incoming graduate students to review. There is no associated paper.

- There are 19 questions and 52 total points are possible. A score of 39 points is a guaranteed pass.
- Please show all work, use a nice new piece of paper for each section or problem, and submit work in problem order.
- Be very explicit. For example, if you need to invoke the concept of a "critical density", don't just use that phrase, write down what it actually means for full credit.
- If you cannot obtain an answer to one question that is necessary for a subsequent question, just assume something for the upstream problem and then move on as you will be graded on your method.
- You may not use *any* resources to complete this cume other than your memory and what is provided in the cume itself. Relatedly, science recommends putting your phone behind a zipper and powering down computers, tablets, smartwatches, smartpencils, smartcoffeemugs, or any other circuitry that you weren't born with.
- I will be available throughout the exam period in my personal Zoom room
(`https://nmsu.zoom.us/j/6118396439`
and by email (finlator AT nmsu.edu).

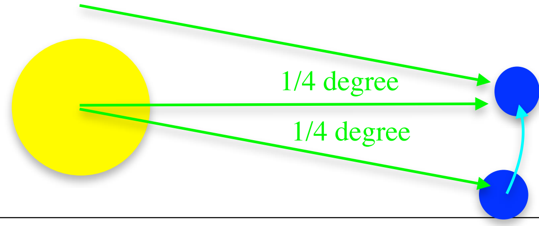
$$
\begin{aligned}
G &= 6.672 \times 10^{-8}\,\mathrm{cm}^3\mathrm{g}^{-1}\mathrm{s}^{-2} \\
c &= 2.99792458 \times 10^{10}\,\mathrm{cm/s} \\
\mathrm{Mpc} &= 3.0855 \times 10^{24}\mathrm{cm} \\
\pi &= 3.14159265359 \\
M_\odot &= 1.989 \times 10^{33}\mathrm{g} \\
M_\oplus &= 5.98 \times 10^{27}\mathrm{g} \\
R_\oplus &= 6.38 \times 10^{8}\mathrm{cm} \\
1\mathrm{erg} &= 6.242 \times 10^{11}\mathrm{eV}
\end{aligned}
$$

The Sun's absolute magnitude is 4.83. The electrostatic force between two charges is $f = \frac{q_1 q_2}{r^2}$ where the charges q_1 and q_2 are in esu and one electron has a charge of -4.8×10^{-10} esu.

1 The Celestial Sphere

1. (2 points) Define the "Ecliptic" in words and using a sketch. (Knowledge) The ecliptic is the projection onto the sky of the Sun's equatorial plane or, equivalently, of the plane that planets orbit in.
2. (2 points) From what latitudes on Earth can you see Polaris? (Knowledge) Anywhere in the Northern hemisphere.
3. (2 points) From what latitudes on Earth can you see stars on the Celestial Equator? (Knowledge) All of them.
4. (4 points) A "sidereal day" is the amount of time that passes for a star other than the Sun to rise twice. A "solar day" is the amount of time that passes for the Sun to rise twice. According to the Giant Impact Hypothesis, Earth's days were once six hours long. What was the difference in duration between a solar day and a sidereal day at that time? Assume that the definitions of "seconds" and "minutes" do not change, and that Earth's orbit about the Sun has not changed. (Application) The angle that Earth's orbit swept through during one Solar day was

$$\frac{360 \text{ degrees}}{1 \text{ year}} \frac{1 \text{ year}}{365 \times 24 \text{ hours}} \frac{6 \text{ hours}}{1 \text{ solar day}} \approx \frac{1}{4} \text{ degree solar day}^{-1}$$



This is the same as the extra angle that Earth swept through to go from completing a sidereal to a Solar day. The rotation frequency was therefore

$$\frac{360 \text{ minutes}}{1 \text{ Solar day}} \frac{1 \text{ Solar day}}{360 + \frac{1}{4} \text{ degrees}} \approx 1 \text{ minutes degree}^{-1}$$

Multiplying these two gives the desired result, or roughly 15 seconds (a more accurate answer gives 14.78 seconds).
A quicker way to do this is to realize that there has to be one more sidereal day than Solar day in a year. If the length of a year is T and the periods of sidereal and solar days are $t_{\rm sid}$ and $t_{\rm sol}$, then this gives

$$\frac{T}{t_{\rm sid}} = 1 + \frac{T}{t_{\rm sol}}$$

Substituting in $T = 365 \times 24$ hours/year and $t_{\rm sol} = 6$ hours gives the same answer as the above.

2 Parallax

5. (2 point) Define "parsec" in words and using a sketch. (Knowledge) A parsec ("parallax second") is the distance to star that appears to oscillate between two points spaced 2 arcseconds apart owing to Earth's orbital motion about the Sun.
6. (2 points) The galaxy M31 is roughly 765 kpc away. What is its parallax? (Application) Parallax varies inversely with distance, and an object parsec away has a parallax of 1 arcsecond. Therefore, M31's parallax motion is 1/765,000 arcseconds (or 1.3 microarcseconds.)
7. (4 points) You wish to observe M31's parallax using a space based, diffraction-limited telescope at visual wavelengths. How large does your primary mirror need to be so that an individual bright star's parallax motion can be detected? Assume that you can determine the star's position to 10% of the size of its Airy disk. Does this mirror size seem feasible? (Hint: You will need to combine the parallax in the previous problem with the wavelength of visible light and the formula for diffraction limit). (Knowledge + Application) Solve this problem by setting the parallax equal to one tenth the size of the Airy disk:

$$\begin{aligned} 1.3\mu\text{as} &= 0.1\tfrac{1.22\times 5000\mathring{A}}{D} && (1)\\ D &= 9 - 10\text{km} && (2) \end{aligned}$$

8. (2 points) In not more than four sentences, describe an alternative experiment (as realistic as possible) for enabling astronomers to measure the parallax motion of M31. (Synthesis) There are lots of possibilities including larger telescope, using multiple smaller telescopes and performing interferometry, going to higher energy (such as UV or x-ray), going to longer baseline, or switching to secular parallax.

3 Apparent and Absolute Magnitudes

To answer these questions, you must know that (i) the distance to Proxima Centauri is 1.288 pc; and (ii) the faintest star that a human eye can see has an apparent magnitude of about 6 (*thanks, Hipparchus*).

9. (4 points) If you lived on Proxima Centauri, could you see the Sun with an unaided eye? Please *compute* the Sun's apparent magnitude as viewed from Proxima Centauri and *use the result* to answer whether the Sun would be visible. (Application) The Sun's absolute magnitude M and apparent magnitude m as viewed from Proxima Centauri are

$$\begin{aligned} M &= -2.5\log\tfrac{L}{10\text{pc}^2} + C && (3)\\ m &= -2.5\log\tfrac{L}{1.288\text{pc}^2} + C && (4)\\ & && (5) \end{aligned}$$

Combining these gives $m - M = 5\log(10/1.288)$. Substituting in $M = 4.83$ gives $m = 0.36$, which is almost as bright as Vega. So, yes, aliens in orbit around Proxima Centauri should easily see the Sun.

10. (4 points) You observe a galaxy with an absolute magnitude of -21.5. Assume for simplicity that all of its stars are G-stars. What is its stellar mass? (Application) A formal way to solve this is to write down the absolute magnitudes of the Sun and the galaxy:

$$M_\odot = -2.5\log\frac{L_\odot}{10\mathrm{pc}^2} + C = 4.83 \tag{6}$$
$$M_{\mathrm{Gal}} = -2.5\log\frac{AL_\odot}{10\mathrm{pc}^2} + C = -21.5 \tag{7}$$
$$\tag{8}$$

Here, A is what we are solving for; it comes to $10^{10.532}$ or $3.4\times10^{10}M_\odot$.

11. (2 points) Describe an additional measurement that you could use to improve the previous estimate. (Synthesis) There are lots of possibilities. Measurements of photomeric color and emission line strengths would constrain age, metallicity, and dust reddening, all of which will impact the stellar mass estimate; going to near-infrared fluxes would yield an estimate that is more robust to these uncertainties. Relatedly, accounting for a realistic IMF would also help.

4 Classical Mechanics

12. (2 points) In what way is Kepler's First Law known to be incomplete or incorrect? (Knowledge) The Sun is not fixed to a focus; it actually orbits about the Solar Systems center of mass.
13. (4 points) Jupiter's moon Europa has an orbital semimajor axis of 9.59 R_J ($R_J \equiv$ Jupiter's mean radius) and an orbital period of 3.5255 days. Meanwhile, its moon Asre has a semimajor axis of 162 R_J. Compute Asre's orbital period about Jupiter. (Hint: You don't need to know Jupiter's mass). (Application) Kepler's third law $T^2 \propto R^3$ applies to any system. In this case,

$$\frac{T^2}{3.5255^2} = \left(\frac{162}{9.59}\right)^3 \tag{9}$$
$$T = 245 \text{ days} \tag{10}$$

Note that Asre is "Ersa" backwards; Ersa is a real moon that was only discovered in 2018.

5 Bohr's Model of the Hydrogen Atom

14. (2 point) Identify one key assumption that Bohr's model of the hydrogen atom uses to explain the discrete nature of hydrogen's emission spectrum (there are two assumptions; you only need identify one of these). (Knowledge) The first assumption is that electrons orbit in circles about the nucleus. The second is that orbital angular momentum is quantized; that is, electrons can only revolve around the proton with certain specific values to their angular momentum. The third assumption is that total orbital energy is quantized and electrons can only "leap" from one state to another, often by absorbing or emitting light.
15. (4 points) The Bohr radius of a ground-state He+ ion (that is, a helium atom that is missing one electron) is 26.45pm (1 pm = 10^{-12} m). Starting from the formula for the electrostatic force between two charges given above, compute the ionization potential (the amount of energy necessary to ionize) in ergs (or eV, if you prefer) of a ground-state He+ ion. (Application) The electrostatic energy gained by an electron in moving from ∞ to distance r is the integral over the force:

$$E_{\mathrm{bind}} = \int_\infty^R \frac{q_1 q_2}{r^2}dr \tag{11}$$
$$= \frac{q_1 q_2}{R} \tag{12}$$

Substituting in $q_1 = -1$ esu, $q_2 = 2$ esu, and $r = 26.45\times10^{-10}$ cm, I get -1.7×10^{-10} ergs or -109 eV. This is not actually the ionization potential because it does not factor in the electron's kinetic energy K. In Bohr's model, that can be derived from the virial theorem. $U + 2K = 0$ implies that the total energy $U + K = U/2 =$ -K. In this case, the ionization potential is 54.4 eV, which is what you learned in 535.
16. (2 points) What kind of light would be needed to *just barely* ionize a He+ ion? (radio, gamma-ray, infrared, etc) (Application or Knowledge) UV

6 Hydrostatic Equilibrium

The equation of hydrostatic equilibrium is

$$\frac{dp}{dz} = -g\rho$$

17. (2 points) In the case of a stellar atmosphere, what two forces are balanced for a gas that is in hydrostatic equilibrium? (Knowledge) Gravity versus pressure gradient. Strictly speaking, neither pressure nor a pressure gradient is a "force." To obtain the force associated with a pressure gradient acting on a gas parcel of surface area A and vertical height Δr, one has to multiply them by the pressure gradient $f = dp/dr A\Delta r$.
18. (4 points) By integrating the hydrostatic equilibrium equation from the surface to the core, derive the expression for the pressure at the center of an isoTHermal, constant-densIty, self-gravitatiNG sphErE (THINGEE). (Application) First, re-write the equation of hydrostatic equilibrium appropriate to a spherical shell about an enclosed mass M_r:

$$\begin{aligned}\frac{dP}{dr} &= -\frac{GM_r}{r^2}\rho \\ &= -\frac{4}{3}\pi G\rho^2 r\end{aligned}$$

Integrating this from the surface to the core gives

$$P_{\mathrm{surface}} - P_{\mathrm{core}} = -\frac{4}{6}\pi G\rho^2 R^2$$

Substituting in for Earth ($\rho \approx 5$ g cm^{-3}, $R = 6 \times 10^8$ cm, $\mathrm{P}_{\mathrm{surface}} \approx 0$) yields a central pressure of 1.4×10^{12} dynes cm^{-2}, which is less than a factor of three off from the accepted value of 3.62×10^{12}.
19. (2 points) Terrestrial planets are approximately THINGEEs. However, they are *differentiated*, meaning that, within their molten interiors, heavier elements sank to the core. Qualitatively, how does this change the estimate for the central pressure that you would obtain from the previous calculation? (Only partial credit will be awarded for simply stating the correct result.) The central pressure would increase. To see this formally, substitute in for the density in the previous result. This gives $P \propto M^2/R^4$. Decreasing R increases P. Intuitively, a test-volume of one cm^{-3} has more "stuff" packed on top of it if the same total planetary mass is packed into a smaller volume.