

Instructions

A hearty welcome to Cume #471! This exam will drill a subset of the topics that Nancy's welcome letter encourages all incoming graduate students to review. There is no associated paper.

- There are 19 questions and 52 total points are possible. A score of 39 points is a guaranteed pass.
- Please show all work, use a nice new piece of paper for each section or problem, and submit work in problem order.
- Be very explicit. For example, if you need to invoke the concept of a “critical density”, don't just use that phrase, write down what it actually means for full credit.
- If you cannot obtain an answer to one question that is necessary for a subsequent question, just assume something for the upstream problem and then move on as you will be graded on your method.
- You may not use *any* resources to complete this cume other than your memory and what is provided in the cume itself. Relatedly, science recommends putting your phone behind a zipper and powering down computers, tablets, smartwatches, smartpencils, smartcoffeemugs, or any other circuitry that you weren't born with.
- I will be available throughout the exam period in my personal Zoom room (<https://nmsu.zoom.us/j/6118396439>) and by email (finlator AT nmsu.edu).

$$\begin{aligned}
 G &= 6.672 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2} \\
 c &= 2.99792458 \times 10^{10} \text{ cm/s} \\
 \text{Mpc} &= 3.0855 \times 10^{24} \text{ cm} \\
 \pi &= 3.14159265359 \\
 M_{\odot} &= 1.989 \times 10^{33} \text{ g} \\
 M_{\oplus} &= 5.98 \times 10^{27} \text{ g} \\
 R_{\oplus} &= 6.38 \times 10^8 \text{ cm} \\
 1 \text{ erg} &= 6.242 \times 10^{11} \text{ eV}
 \end{aligned}$$

The Sun's absolute magnitude is 4.83. The electrostatic force between two charges is $f = \frac{q_1 q_2}{r^2}$ where the charges q_1 and q_2 are in esu and one electron has a charge of -4.8×10^{-10} esu.

1 The Celestial Sphere

1. (2 points) Define the “Ecliptic” in words and using a sketch.
2. (2 points) From what latitudes on Earth can you see Polaris?
3. (2 points) From what latitudes on Earth can you see stars on the Celestial Equator?
4. (4 points) A “sidereal day” is the amount of time that passes for a star other than the Sun to rise twice. A “solar day” is the amount of time that passes for the Sun to rise twice. According to the Giant Impact Hypothesis, Earth's days were once six hours long. What was the difference in duration between a solar day and a sidereal day at that time? Assume that the definitions of “seconds” and “minutes” do not change, and that Earth's orbit about the Sun has not changed.

2 Parallax

5. (2 point) Define “parsec” in words and using a sketch.
6. (2 points) The galaxy M31 is roughly 765 kpc away. What is its parallax?
7. (4 points) You wish to observe M31's parallax using a space based, diffraction-limited telescope at visual wavelengths. How large does your primary mirror need to be so that an individual bright star's parallax

motion can be detected? Assume that you can determine the star's position to 10% of the size of its Airy disk. Does this mirror size seem feasible? (Hint: You will need to combine the parallax in the previous problem with the wavelength of visible light and the formula for diffraction limit).

8. (2 points) In not more than four sentences, describe an alternative experiment (as realistic as possible) for enabling astronomers to measure the parallax motion of M31.

3 Apparent and Absolute Magnitudes

To answer these questions, you must know that (i) the distance to Proxima Centauri is 1.288 pc; and (ii) the faintest star that a human eye can see has an apparent magnitude of about 6 (*thanks, Hipparchus*).

9. (4 points) If you lived on Proxima Centauri, could you see the Sun with an unaided eye? Please *compute* the Sun's apparent magnitude as viewed from Proxima Centauri and *use the result* to answer whether the Sun would be visible.
10. (4 points) You observe a galaxy with an absolute magnitude of -21.5. Assume for simplicity that all of its stars are G-stars. What is its stellar mass?
11. (2 points) Describe an additional measurement that you could use to improve the previous estimate.

4 Classical Mechanics

12. (2 points) In what way is Kepler's First Law known to be incomplete or incorrect?
13. (4 points) Jupiter's moon Europa has an orbital semimajor axis of $9.59 R_J$ ($R_J \equiv$ Jupiter's mean radius) and an orbital period of 3.5255 days. Meanwhile, its moon Asre has a semimajor axis of $162 R_J$. Compute Asre's orbital period about Jupiter. (Hint: You don't need to know Jupiter's mass).

5 Bohr's Model of the Hydrogen Atom

14. (2 point) Identify one key assumption that Bohr's model of the hydrogen atom uses to explain the discrete nature of hydrogen's emission spectrum (there are two assumptions; you only need identify one of these).
15. (4 points) The Bohr radius of a ground-state He⁺ ion (that is, a helium atom that is missing one electron) is 26.45pm (1 pm = 10^{-12} m). Starting from the formula for the electrostatic force between two charges given above, compute the ionization potential (the amount of energy necessary to ionize) in ergs (or eV, if you prefer) of a ground-state He⁺ ion.
16. (2 points) What kind of light would be needed to *just barely* ionize a He⁺ ion? (radio, gamma-ray, infrared, etc)

6 Hydrostatic Equilibrium

The equation of hydrostatic equilibrium is

$$\frac{dp}{dz} = -g\rho$$

17. (2 points) In the case of a stellar atmosphere, what two forces are balanced for a gas that is in hydrostatic equilibrium?
18. (4 points) By integrating the hydrostatic equilibrium equation from the surface to the core, derive the expression for the pressure at the center of an isoTHERmal, constant-densItY, self-gravitatiNG sphErE (THINGEE).
19. (2 points) Terrestrial planets are approximately THINGEES. However, they are *differentiated*, meaning that, within their molten interiors, heavier elements sank to the core. Qualitatively, how does this change the estimate for the central pressure that you would obtain from the previous calculation? (Only partial credit will be awarded for simply stating the correct result.)