Instructions

A hearty welcome to Cume #438! This exam will address subjects related to extragalactic astronomy, galaxy formation, and cosmology. It makes frequent reference to the Astrobite summary of the paper by Dachen Ling et al. 2018.

- There are thirteen questions or sub-questions, and 62 points are possible. A score of 47 points is a guaranteed pass.
- Please show all work and use a nice new piece of paper for each section or problem.
- If the exam asks you to obtain a number from the astrobite, then please indicate the number that you obtain and where you obtained it from.
- Be very explicit. For example, if you need to invoke the concept of a "critical density", don't just use that phrase, write down what it actually means.
- Please let me know if you're hopelessly stuck; I may be able to help.

$$\rho_c = \frac{H_0^2}{(8/3)\pi G}$$
$$G = 6.672 \times 10^{-8}\,\text{cm}^3\text{g}^{-1}\text{s}^{-2}$$
$$c = 2.99792458 \times 10^{10}\,\text{cm/s}$$
$$H = 100h\,\text{km s}^{-1}\text{Mpc}^{-1}$$
$$\text{Mpc} = 3.0855 \times 10^{24}\text{cm}$$
$$\pi = 3.14159265359$$
$$M_\odot = 1.989 \times 10^{33}\text{g}$$
$$R_\odot = 6.96 \times 10^{10}\text{cm}$$
$$\text{year} = 3.156 \times 10^{7}\text{s}$$

1 Synopsis

1. (4 points) (Comprehension) In 2–4 sentences, please describe what problem this research is *trying* to solve (don't discuss whether they succeeded yet!).
Solution: Black holes with masses $< 100M_\odot$ and $> 10^6 M_\odot$ are commonly-observed, but BH with masses in between these ranges are not. This has given rise to competing models for how SMBH get started. For example, do they grow all the way from stellar-mass BH up to SMBH scales via Eddington-limited accretion of gas, or do they form at a stroke via "direct-collapse" scenarios? This study is attempting to verify that IMBH exist.Rubric: 2 points for re-stating that IMBH are not observed, 2 points for connection with origin of SMBH.

2. (4 points) (Comprehension) *Now*, in 2–4 sentences, please explain why this study required compiling observations over a 10-year baseline. Solution: Their long time baseline enabled them to establish that the X-ray light originated from Eddington-limited accretion, and the material from the disrupted star required years to finish accreting into the IMBH.Rubric: Four points for some connection between "flaring", time-domain science, or the rarity of these events.

2 General Background

3. (4 points) (Knowledge) The astrobite notes that "most galaxies have a SMBH of the order of 0.1% of their stellar mass". It turns out the scatter in the relationship between black hole mass and the stellar bulge mass is quite small. Why is this (1) surprising and (2) important? (Note: that was two questions!)
Solution: Galaxies' bulges are not sensitive to the central SMBH's gravity, so if the two components formed separately then their masses should not correlate. The strong correlation implies kpc-scale coupling

processes that could involve regulating stellar mass growth.Rubric: 2 points for no gravitational coupling, 2 points for some mention of feedback/coupling.

4. (6 points) (Knowledge) In one sentence each, write down three observational arguments supporting the view that Active Galactic Nuclei are dominated by supermassive black holes.
Solution: Imaging EH of M87; spatially-resolved, rapid proper motion of central gas/stellar populations; Flickering on short timescales; gravitational redshift of emission lines; existence of high proper-motion stars.Rubric: 2 points for each sensible observational argument.

3 Cosmology

5. The paper suggests that black holes may be a component of dark matter. Really?

   (a) (Application) (6 points) Suppose that *all* dark matter is in the form of earth-mass black holes spread uniformly through the Universe. In that case, what is the mean spacing between them? Note: you'll need to assume some cosmological parameters; please be sure I can tell what they are.
   Solution: The DM density is $\Omega_M \rho_c$, where ρ_c is given above. The number density is therefore $n = \Omega_M \rho_c / M_\oplus$, and the mean distance is $n^{-1/3}$. For $H_0 = 70\text{km s}^{-1}\text{Mpc}^{-1}$, $\Omega_M = 0.3$, this works out to about 4 pc.Rubric: 1 point for a reasonable setup or 2 points for a completely correct setup; 1 point each for sensible cosmological parameters; 1 point for a calculation that's close to correct or 2 points for a correct answer to within $2\times$.

   (b) (Synthesis) (4 points) *Describe* an experiment that could test this idea observationally.
   Solution: Microlensing surveys. By measuring the observed fluxes of a large number of stars repeatedly and waiting for them to brighten and fade, one can derive statistical constraints on the contribution to the Milky Way of faint, low-mass objects, some of which may be black holes.Rubric: Two points for mentioning a viable experiment, two points for a convincing description.

6. (Application) (5 points) How far away from Earth is the lenticular galaxy Gal2?
Solution: Figures 1 and 2 indicate that 10" = 10.7 kpc. This implies a distance of 10.7kpc/10" = 220Mpc. This is a cosmological distance so in reality we should refer to this as the object's *angular diameter distance*.Rubric: 1 point for pulling numbers from the astrobite, 2 points for a good setup, 2 points for the correct answer.

4 Compact Objects

7. Let's review the Eddington Limit (also known as the Eddington luminosity).

   (a) (2 points) (Knowledge) In 1–2 sentences, what is the Eddington limit physically?
   Solution: This is the luminosity for which an object's inward gravitational pull is balanced by the outward force from its radiation.Rubric: Two points for a sensible answer.

   (b) (4 points) (Application) Derive a symbolic expression for the Eddington limit as a function of the central object's mass.
   Solution: In classrooms, one considers a pure-hydrogen plasma and balances the gravitational attraction on the proton with Thomson scattering on the (free!) electron. Writing this down gives

$$\frac{GM_{\rm BH} m_p}{r^2} = \frac{1}{c} \frac{L}{4\pi r^2} \sigma_T$$

   solving for L gives the correct answer. In practice, the gas is not a pure hydrogen plasma and there are other radiation coupling processes such as scattering by electrons in ions. This limit is relevant to accretion by any object or to stellar phenomena such as radiation-driven expansion of atmospheres, novae, and X-ray flares.Rubric: 2 points for writing down something sensible, 2 points for a correct answer.

8. (5 points) (Application) Compute the radius of the event horizon for the IMBH that they discover. (Note that you don't need to know a formula for the event horizon if you recall what it is physically. Assume that the IMBH does not rotate.). Solution: To compute the *Schwarzschild radius*, just set the escape velocity of a test particle to c:
$$\frac{GM_{\rm BH}m}{R_{\rm S}} = \frac{1}{2}mc^2$$
Solving for $R_{\rm S}$ gives $2GM_{\rm BH}/c^2$. Plugging in the mass of the IMBH gives about 300,000 km, or less than the Earth-Moon distance.Rubric: 2 points for setting up a calculation for $R_{\rm S}$; 1 point for grabbing the IMBH mass; 2 points for computing $R_{\rm S}$ correctly.

9. (4 points) (Application) Estimate the orbital radius at which a star of mass $1M_\odot$ is disrupted by the IMBH reported in this paper. Solution: The star disrupts when it reaches the *Roche limit*. To compute the Roche limit, set the tidal gravitational force from the IMBH between the star's center and its surface to the star's gravity:
$$2\frac{GM_{\rm BH}m}{r^3}R_* = \frac{GM_*m}{R_*^2}$$
Solving for r gives $R_*(2M_{\rm BH}/M_*)^{1/3}$, and plugging numbers gives about 4×10^{12}cm. This is small in the sense that it's about 40% larger than Mercury's orbit, but still about 100$\times$ larger than $R_{\rm S}$.Rubric: 2 points for setting up a calculation for the Roche Limit, 2 points for a correct answer.

5 Analysis

10. (8 points) (Analysis) The authors use HST measurements along with some modeling in order to constrain the stellar mass of the cluster hosting the IMBH candidate. List three sources of uncertainty affecting this measurement. Choose one source of uncertainty and explain it in 2–3 sentences.
Solution: The distance to the system; measurement uncertainty; the assumed stellar population (IMF, binarity, metallicity); the assumption that the cluster is an SSP.Rubric: Two points each for listing sources of uncertainty, two points for a sensible explanation of one of them.

11. (6 points) (Evaluation) You are a scientific editor at *Nature*. Do you feel that this result merits publication? Answer with specific reference to at least two strengths and two weaknesses. (Note: Answers that are more critical are not necessarily more correct.)Rubric: One point each for sensible reference to strengths/weaknesses; two points for a clear argument as to whether the article is publishable.