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CUME EXAM # 435 WITH SOLUTIONS

This exam is worth 72 points. It is based on the accompanying white paper recently published on the arXiV: Simon et al. (2019), *Outer Solar System Exploration: A Compelling and Unified Dual Mission Decadal Strategy for Exploring Uranus, Neptune, Triton, Dwarf Planets, and Small KBOs and Centaurs*, `https://arxiv.org/pdf/1807.08769.pdf`. A grade of 70% or higher is expected to be a passing grade. **You may use a calculator, but only for algebraic and trigonometric types of calculations – you may NOT use a calculator to store formulae, constants, etc.**

Things You Might Need to Know - also see the following page

Symbol	Value in cgs	Value in SI
c	2.998×10^{10} cm s^{-1}	2.998×10^{8} m s^{-1}
G	6.674×10^{-8} dyn cm^2 g^{-2}	6.674×10^{-11} N m^2 kg^{-2}
k	1.3806×10^{-16} erg K^{-1}	1.3806×10^{-23} J K^{-1}
σ	5.670×10^{-5} erg cm^{-2} K^{-4} s^{-1}	5.670×10^{-8} W m^{-2} K^{-4}
m_p	1.673×10^{-24} g	1.673×10^{-27} kg
1 AU	1.496×10^{13} cm	1.496×10^{11} m
$M_\odot$	1.9891×10^{33} g	1.9891×10^{30} kg
$R_\odot$	6.955×10^{10} cm	6.955×10^{8} m
$L_\odot$	3.827×10^{33} erg s^{-1}	3.827×10^{26} W
$M_\oplus$	5.976×10^{27} g	5.976×10^{24} kg
$R_\oplus$	6.378×10^{8} cm	6.378×10^{6} m
a_{Uranus}	19.22 AU	
$a_{Neptune}$	30.11 AU	
$R_{Neptune}$	24,622 km	
a_{Quaoar}	43.69 AU	
m_{Quaoar}	19.3	

Additional Instructions

- Start each question on a new page and then staple your packet of pages together at the end. Put your name on every page.
- Write legibly! If I cannot read your writing, you likely will not receive as much credit as you deserve because I won't understand what you are trying to convey.
- The topics of the questions and their respective point values are as follows:
  Orbital Mechanics (#1-3): 16 points
  Observations (#4): 12 points
  Ice Giant Science (#5-6): 20 points
  KBO Science (#7): 12 points
  Putting it All Together (#8): 12 points

TABLE 4.1
Mean Specific Heat at Constant Pressure and Specific Gas Constant in Atmospheres

Planet/Satellite	Main Components (Abundance)	$\langle\mu\rangle$ (g mol^{-1})	R_g^* (Jg^{-1} K^{-1})	C_p (J g^{-1} K^{-1})
Venus	CO_2 (0.96) N_2 (0.035)	44.01	0.19	0.85
Earth	N_2 (0.78) O_2 (0.21)	28.97	0.29	1.00
Mars	CO_2 (0.953) N_2 (0.027)	44.01	0.19	0.83
Jupiter[a]	H_2 (0.864) He (0.136)	2.22	3.75	12.36
Saturn[a]	H_2 (0.85) He (0.14)	2.14	3.89	14.01
Titan	N_2 (0.99) CH_4 (0.01)	28.67	0.29	1.04
Uranus[a]	H_2 (0.85) He (0.15)	2.30	3.61	13.01
Neptune[a]	H_2 (0.79) He (0.21)	2.30	3.61	13.01
HD 209458[b]	H_2 (1.0)	2.00	4.16	14.00

Source: From Sánchez-Lavega, A. et al., *Am. J. Phys.*, 72, 767, 2004.

C_p is a function of temperature. A third-order polynomial dependence of the type.
$C_p(T)=a_0+a_1T+a_2T^2$ is usually sufficient for atmospheric calculations within a given temperature range. See Figure 4.21.

[a] For these planets c_p/R_g^* depends on the *ortho–para* hydrogen ratio. See Figure 4.22.

[b] Representative extrasolar planet of the type "Hot Jupiter."

Orbital Mechanics

1. **Upon launch, interplanetary spacecraft generally go into a temporary "parking" orbit around Earth and then the third rocket stage is fired in order to send it on its way.**

   a) What is orbital velocity of a 400 kg satellite orbiting the Earth at altitude of 300 km above the surface? (4 points)

   The orbital velocity is obtained by equating the centripetal force felt by the spacecraft ($F_c = mv^2/R$) and the gravitational force it feels due to the Earth ($F_g = GM_{\oplus}m/R^2$). Solving for v we get

   $$v_{orb} = \sqrt{\frac{GM_{\oplus}}{R}} \tag{1}$$

   Note that the above expression is independent of the mass of the satellite! Plugging in numbers (R = 6378 km + 300 km = 6678 km) we get v_{orb} = 7.73 km/s.

   b) From this orbit, how much delta-v has to be added (by firing the third rocket stage) in order to escape the Earth? (4 points)

   We must first compute the escape velocity of the Earth. To do so, we equate kinetic energy ($\frac{1}{2}$ mv^2) to potential energy ($\frac{GM_{\oplus}m}{r}$) and solve for v to get

   $$v_{esc} = \sqrt{\frac{2GM_{\oplus}}{R}} \tag{2}$$

   After plugging in the numbers, v_{esc} for Earth (from the 300 km altitude orbit) is 10.93 km/s. Thus, we would need to add another 3.2 km/s to the velocity of the spacecraft at a 300 km altitude orbit in order for it to be able to escape Earth's gravity.

2. **Assume that a Uranus-bound spacecraft leaves the Earth system with velocity of 16 km/s. It will pass by Jupiter for a gravity assist en route to Uranus. How long does it take to get to Jupiter? State whatever assumptions you make in doing this calculation. (4 points)**

   There is more than one way to do this problem depending on the nature of the simplifying assumptions you made. If you assumed that the spacecraft traveled in a straight line from Earth to Jupiter (whose orbital semimajor axis is 5.2 AU, call it 5 AU), then it traveled a distance of 4 AU. If traveling at 16 km/s, that means it would have taken the spacecraft 434 days, or 1.2 years, to reach Jupiter. [Note that this is essentially what *New Horizons* did. It was launched in Jan. 2006 with an orbital velocity of 16.26 km/s, making it the "fastest man-made object ever launched from Earth," and it headed straight to Jupiter, arriving there in Feb. 2007.]

You could have also used a Hohmann transfer orbit to answer this question, and that would also have been an acceptable way to approach this problem.

3. Given the positions of the planets during the Voyager era (Fig. 1), sketch on the figure where Uranus and Neptune will be in 2035. (4 points)

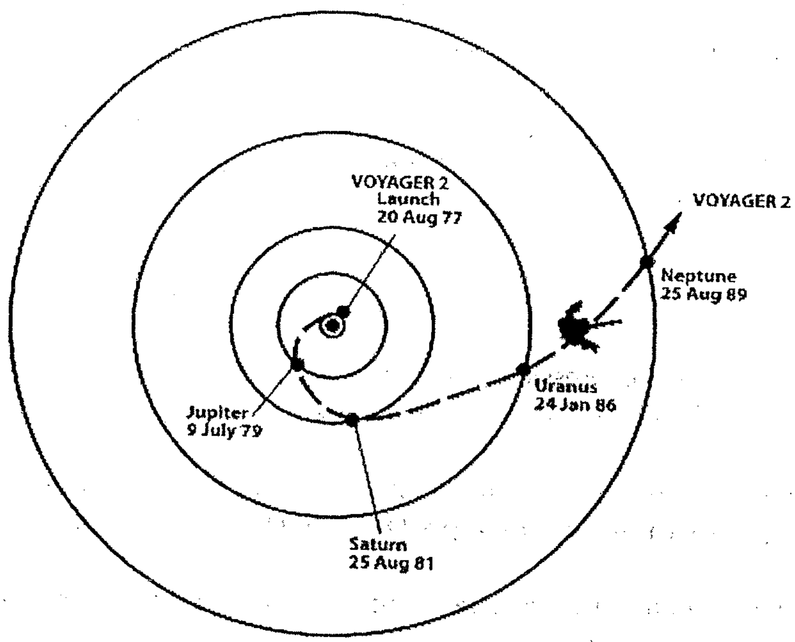

Figure 1: Voyager 2 trajectory.

We need to figure out what percentage of their orbits Uranus and Neptune have traveled since the time of the Voyager encounters. We can use the simplified version of Kepler's Third Law for this: $p^2 = a^3$, where p is the orbital period in years and a is the orbital semi-major axis in AU.

For Uranus, the orbital semimajor axis is 19.22 AU, so $p_{Ura} = 84.26$ yr. From 1986 to 2035, 49 years will have elapsed, which is 0.58 p_{Ura}. So it will be a little more than halfway around from where it was during the Voyager encounter.

For Neptune, the orbital semimajor axis is 30.11 AU, so $p_{Nep} = 165.22$ yr. From 1989 to 2035, 46 years will have elapsed, which is 0.28 p_{Nep}. So it will be a little less than a third of the way around from where it was during the Voyager encounter.

Observations

4. The late 2020s will usher in a new era of extremely large ground-based telescopes, with the European Extremely Large Telescope (39 m diameter) coming online in 2025 and the Giant Magellan Telescope (24.5 m) and the

Thirty Meter Telescope coming online in 2027. Given these new ground-based assets, it will be important to convince NASA that a spacecraft mission to the outer solar system is still required to achieve the proposed science objectives.

a) Compute the angular resolution of Hubble Space Telescope, James Webb Space Telescope, and the TMT at relevant wavelengths and compare this to the angular size of both Neptune and Quaoar. Discuss our ability to characterize discrete atmospheric storms on Neptune and surface features on Quaoar with each of the three telescopes. (8 points)

We use the general formula for angular resolution: $\theta = 1.22\ \lambda/D$. Note that θ is in radians. We need to know the diameters of all three telescopes: HST (2.4 m), JWST (6.5 m), and TMT (30 m). As for "relevant wavelengths," HST is optimized for NUV, optical, and NIR observations. JWST is optimized for infrared, while TMT is optical/NIR. So let's choose 0.7 μm for HST and TMT, and 10 μm for JWST. The calculations thus become

$$\theta_{HST} = 1.22(0.7 \times 10^{-6})/2.4\ m = 3.56 \times 10^{-7}\ rad = 0.073'' \tag{3}$$

$$\theta_{JWST} = 1.22(10 \times 10^{-6})/6.5\ m = 1.88 \times 10^{-6}\ rad = 0.39'' \tag{4}$$

$$\theta_{TMT} = 1.22(0.7 \times 10^{-6})/30\ m = 2.85 \times 10^{-8}\ rad = 0.0059'' \tag{5}$$

[The Earth's atmosphere is going to limit the angular resolution of the TMT unless adaptive optics are used (they will be)].

The angular size of Neptune (assuming we observe it at or near opposition) is $\theta \sim$ (2 R_{Nep})/29 AU = 1.132 $\times$ 10^{-5} rad =2.3". Thus Neptune will be resolved by all three telescopes. If a large storm on Neptune subtends roughly a tenth of the disk, it could still be resolved by all three telescopes (barely by JWST).

The angular size of Quaoar (assuming we observe it at or near opposition) is $\theta \sim$ (2 R_Q)/43 AU = 3.1 $\times$ 10^{-7} rad =0.06". Get the diameter of Quaoar ($\sim$ 1000 km) from the diagram on page 6 of the paper. Thus Quaoar will not resolved by HST or JWST, although it will be resolved by TMT.

b) Assume that a Quaoar-like object (same size and composition) is discovered with the TMT and it has an apparent magnitude of 24.1. What is its distance? (4 points)

We are given the apparent magnitudes of two objects, Quaoar (m_1 = 19.3) and the mystery object (m_2 = 24.1). This can be related to the fluxes of the two objects as follows:

$$m_1 - m_2 = -2.5\ log_{10}\left(\frac{F_1}{F_2}\right) \tag{6}$$

Plugging in the numbers, we get $F_1/F_2 = 83.2$. We also know that flux goes as $1/d^2$, so we can rewrite F_1/F_2 as

$$\frac{F_1}{F_2} = \left(\frac{d_2^2}{d_1^2}\right) \rightarrow \frac{d_2}{d_1} = \sqrt{83.2} = 9.12. \tag{7}$$

So the mystery object is 9.12 times further than Quaoar, or at a distance of 9.12×43.7 AU = 398.6 AU.

Ice Giant Science

5. One component of the proposed mission architecture is a Neptune atmospheric probe. If such a probe is designed to withstand pressures up to 10 bars, how many pressure scale heights would this probe be able to study before its demise? (6 points)

The expression for a pressure scale height, H, or the height over which the pressure falls by $1/e$, comes from the following:

$$p(z) = p(z_o)exp\left[-\frac{(z-z_o)}{H}\right], \tag{8}$$

where the scale height $H = \frac{kT}{\mu g}$. The value for H in Neptune's atmosphere is about 20 km. You would get this by plugging in values for T ($\sim$ 60 K), μ ($2.3 \times m_H$), and g ($\sim$ 11 m/s^2). If you assume that the probe turned on at about P $\sim 10^{-6}$ bar and it ended at 10 bars, then plugging in the numbers we get that the probe sampled approximately 16 scale heights through Neptune's atmosphere.

6. Suppose the Neptune probe measured a pressure-temperature profile *for the location that it entered the atmosphere* that is consistent with the red line in Fig. 2. The black dashed line in Fig. 2 represents the dry adiabatic lapse rate.

a) Label the axes in Fig. 2 including the parameters that are being plotted, the directions in which they are increasing, and ballpark numbers for those parameters. Label the vertical region(s) or layer(s) that the probe would be sensitive to. (4 points)

The x-axis is temperature, and it increases to the right. The minimum value in the atmospheres of Uranus and Neptune is about 50 K. The y-axis corresponds to pressure, and it increases downward. The minimum value in the atmospheres of Uranus and Neptune is about 0.1 bar. For reference, example P-T profiles for several atmospheres in our solar system are shown in Fig. 3. The probe will go from the top of the atmosphere to 10 bars, so it will be sensitive to the stratosphere and the troposphere.

b) What is the definition of the dry adiabatic lapse rate? (4 points)

The dry adiabatic lapse rate is the rate of change in the temperature of an air parcel with height, assuming a dry atmosphere (no latent heat of vaporization involved). The expression for the dry adiabatic lapse rate is

$$\Gamma_d = -\frac{dT}{dz} = \frac{g}{c_p}, \tag{9}$$

where c_p is the specific heat at constant pressure.

c) Is the probe entry location statically stable? In other words, what will happen to a Neptune air parcel if it is released at the location of the green star (towards the bottom of the graph)? Explain your answer. (6 points)

The actual (environmental) temperature profile for Neptune is given by the red line in Fig. 2, while the DALR is represented by the black line. At the probe location (green star), the slope of the temperature profile is less than Γ_d, meaning that the decrease in temperature with altitude is *greater* than adiabatic, *i.e.* it is "superadiabatic." This means that the atmosphere is unstable to vertical displacement, so an air parcel released at that location would continue to rise.

KBO Science

7. If Fig. 4 represents the best surface map of the target KBO available from the ground-based ELTs and the rotation period of the KBO is 14 hours, draw a rotational light curve that would be measured by the spacecraft as it approached the target. Label your axes! (4 points)

An example light curve is shown below in Fig. 5. You could have put time on the x-axis instead of longitude; in that case your time would run from 0 to 14 hours.

8. Suppose that the KBO flyby is carrying along a projectile impactor (sort of like the *Deep Impact* mission) to determine some fundamental properties of the KBO. Assume that the projectile has a mass of 50 kg and is traveling at the speed of 10 km/s when it impacts, resulting in a crater of 150 m in diameter. Using crude scaling relations (estimate if you don't know or remember), what is the density of the KBO that was impacted? State whatever assumptions you make in answering this question. (8 points)

To answer this you would need to use the empirical Gault relation (although I did not

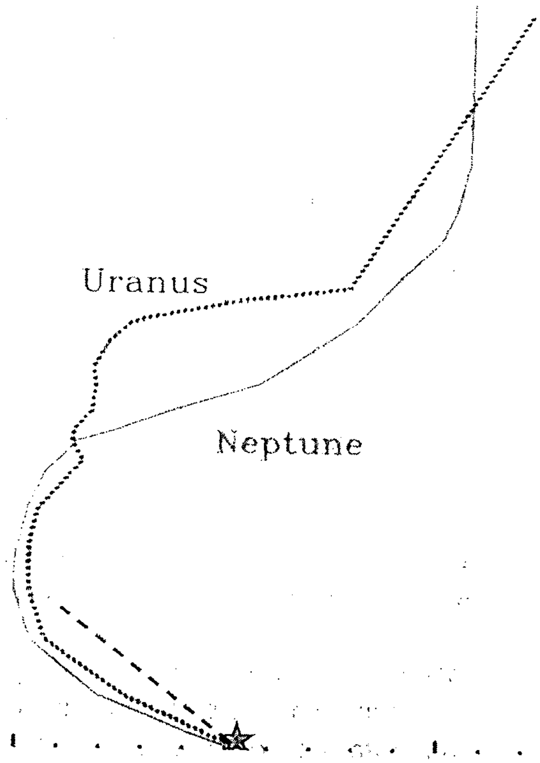

Figure 2: Uranus (blue dotted) and Neptune (red solid) pressure-temperature profiles (but with axes removed!).

expect you to remember this!): for a resulting transient crater greater than 100 m diameter, $D_t = 0.27\rho_p^{\frac{1}{6}}\rho_t^{-\frac{1}{2}}W^{0.28}(sin\theta)^{\frac{1}{3}}$, where D_t is the transient crater diameter, ρ_p is the density of the projectile, ρ_t is the density of the target, W is the kinetic energy of the impactor, and θ is the impact angle. Let's assume a vertical impact, so $sin\theta = 1$. The kinetic energy is 0.5 mv^2; plugging in the numbers for the projectile mass and velocity we get $W = 2.5 \times 10^9$ J. Plugging in numbers should give us something on the order of a density ~ 2 for a KBO that is a mostly icy body.

Putting it All Together

9. Given the science objectives of this ice giant + KBO mission, come up with a notional instrument suite that could be used to make measurements to support these objectives. Assume that you are limited to a payload of 4 instruments or fewer, and that you must address the objectives for Uranus given at the bottom of page 4 and the "first reconnaissance" objectives for KBOs given in the second paragraph of page 6. Consider only instruments that would be mounted on the main spacecraft bus, not an atmospheric entry probe or impactor. For each instrument that you list, discuss what type

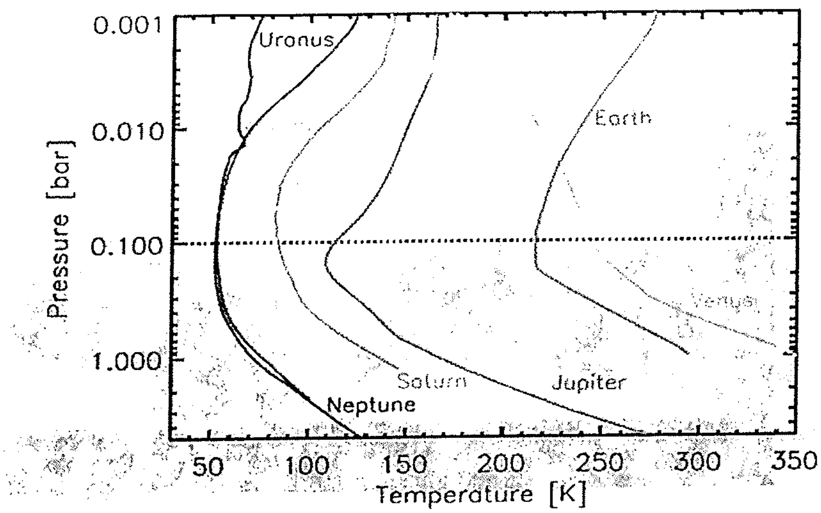

Figure 3: Pressure-temperature profiles for solar system targets (from http://arxiv.org/abs/1312.6859).

of measurements it would make and how those could address the science objectives. (12 points)

There is clearly no single correct answer to this question. The science objectives you are asked to address for Uranus are determining the planet's atmospheric heat balance, atmospheric composition, atmospheric dynamics, and ring and satellite surface composition and geology. For KBOs the preliminary science objectives are to determine their densities, assay their satellite and rings systems for content, map the surfaces and surface compositions of their satellites, and search for intrinsic magnetic fields. Here are some potential instruments that would satisfy these constraints, but this is by no means an exhaustive list:

(a) visible imaging camera: to acquire broad-, narrow-band, or hyperspectral images of surfaces, atmospheres, and rings, to reveal surface features

(b) infrared spectrometer: to infer chemical composition of atmosphere or surfaces

(c) magnetometer: to measure magnetic field strength

(d) UV spectrometer: to study composition of the upper atmosphere of Uranus and the satellites

(e) dust counter: to characterize the dust environment near the rings

(f) mass spectrometer: to directly sample the upper atmosphere to determine gas compositions

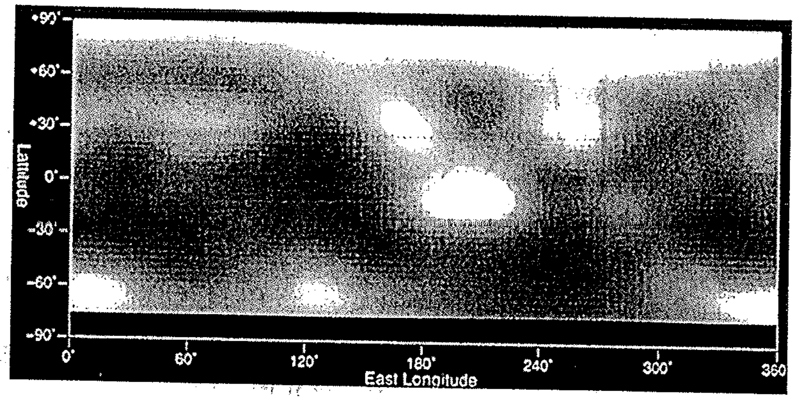

Figure 4: Surface map of target KBO (this is actually a surface map of Pluto made from HST observations!).

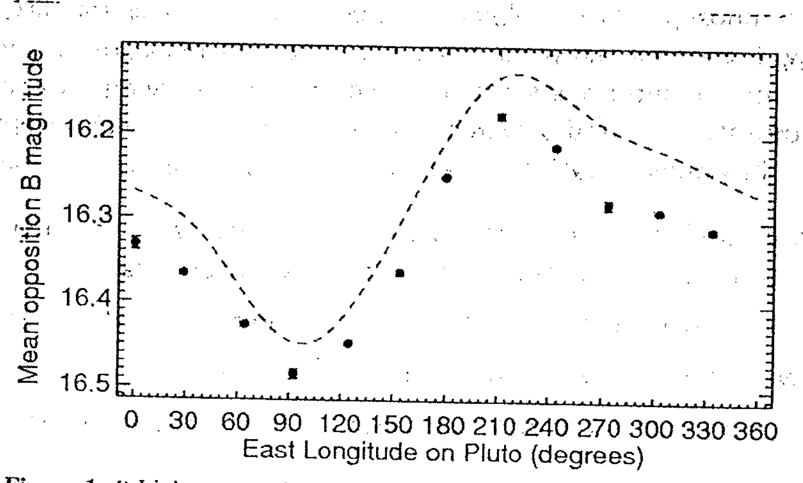

Figure 5: Light curve of Pluto, from Buie *et al.* (2010), DOI:10.1088/0004-6256/139/3/1117.