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November 10, 2012

CUME EXAM # 374 – WITH SUGGESTED SOLUTIONS

This exam is worth 65 points. It is based on the accompanying paper by Ehrenreich *et al.* (2012), "Transmission spectrum of Venus as a transiting exoplanet" *A&A* 537, L2. A grade of 70% or higher is expected to be a passing grade. **You may use a calculator, but only for algebraic and trigonometric types of calculations – you may NOT use a calculator to store formulae, constants, etc.**

Things You Might Need to Know

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$

$$\text{Venus' orbital semimajor axis} = 0.723 \text{ AU}$$

$$\text{Saturn's orbital semimajor axis} = 9.539 \text{ AU}$$

$$\text{Venus' orbital inclination} = 3.3944^\circ$$

$$\text{Venus' mass} = 4.87 \times 10^{24} \text{ kg}$$

$$\text{Venus' radius} = 6.051 \times 10^6 \text{ m}$$

$$\text{radius of the Sun} = 6.955 \times 10^8 \text{ m}$$

$$c_p \text{ for CO}_2 = 850 \text{ J K}^{-1} \text{ kg}^{-1}$$

$$G = 6.674 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

$$\text{Solar irradiance} = 1365 \text{ W m}^{-2}$$

$$\text{Venus' effective temperature} = 240 \text{ K}$$

$$\sigma = 5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

Additional Instructions

- Start each question on a new page and then staple your packet of pages together at the end. Put your name on every page.
- Write legibly! If I cannot read your writing, you likely will not receive as much credit as you deserve because I won't understand what you're trying to convey.

Transit Geometry

1. The paper states that the June 2012 Venus transit was the last one until 2117. If Venus' orbit were not at all inclined with respect to the ecliptic, how often would Venus transits occur? (5 points)

To answer this question we need to calculate the *synodic period* of Venus, which describes the time interval between successive inferior conjunctions, or Sun-Venus-Earth alignments. The answer is not simply the orbital period of Venus, because during the time that it takes Venus to orbit the Sun once, the Earth moves in its orbit as well. We can use the following relationship for inferior planets:

$$\frac{1}{P_{sid}} = \frac{1}{P_{\oplus}} + \frac{1}{P_{syn}} \quad (1)$$

where P_{sid} is Venus' sidereal period (measured with respect to the background stars), P_{syn} is its synodic period, and $P_{\oplus}$ is the Earth's orbital period. To get a rough estimate of Venus' sidereal period, we can use the simplified version of Kepler's Third Law: $p^2 = a^3$, with p in units of years and a in AU. $p^2 = (0.723)^3$, so Venus' sidereal period $P_{sid} = 0.614763$ yrs = 224.54 d. Now we can solve for Venus' synodic period:

$$P_{syn} = \left(\frac{1}{P_{sid}} - \frac{1}{P_{\oplus}} \right)^{-1} = \left[(224.54)^{-1} - (365.25)^{-1} \right]^{-1} = 583 \text{ d} = 1.60 \text{ yr} \quad (2)$$

Thus, Venus is in the correct orbital position (as far as the Sun-Venus-Earth alignment goes) every ~ 583 days for a transit to occur.

2. What is the maximum distance (in km) that Venus can be above or below the ecliptic such that a transit will occur? (5 points)

We use trigonometry to solve this problem, referring also to Fig. ?? below. The Sun's angular size is about 0.5° , so $\theta \approx 0.25^\circ$. The Earth-Venus distance is $\sim 1 - 0.723 \text{ AU} = 0.277 \text{ AU} = 4.14 \times 10^7 \text{ km}$, so $x = (4.14 \times 10^7) \tan(0.25^\circ) = 1.8 \times 10^5 \text{ km}$.

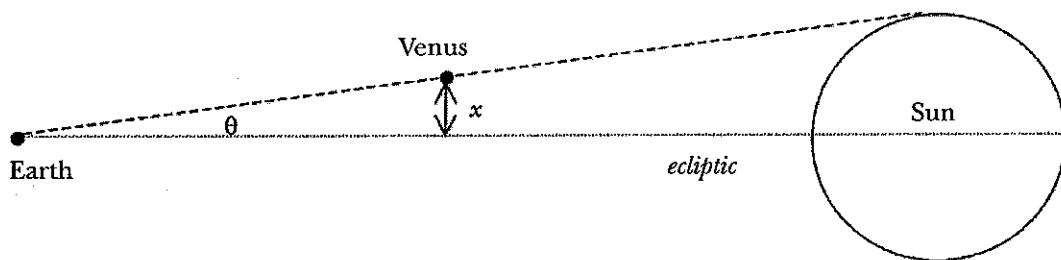


Figure 1: Observing geometry for a Venus transit as seen from an edge-on perspective.

3. Given Venus' orbital inclination with respect to the ecliptic, calculate how far it gets above/below the ecliptic (in km). Compare your answer to the one you arrived at in Problem 2, and comment on whether it makes sense given the *actual* frequency of Venus transits versus the frequency of transits that you computed in Problem 1. (5 points)

Again, we can use trigonometry to solve this problem, referring also to Fig. ?? below. $\sin\theta$, where now θ is the orbital inclination, is equal to x divided by the Venus-Sun distance. The Venus-Sun distance is $0.723 \text{ AU} =$

1.082×10^8 km, so $x = (1.082 \times 10^8) \sin(3.3944^\circ)$, or $x = 6.40 \times 10^6$ km. This number is more than an order of magnitude *larger* than the maximum allowable distance above or below the ecliptic in order for a transit to occur. Thus, it makes sense that even though the Sun, Venus, and the Earth are aligned favorably every ~ 1.6 yr, transits occur much less frequently than that. During those alignments, Venus is likely going to be too far above or below the ecliptic to transit across the disk of the Sun.

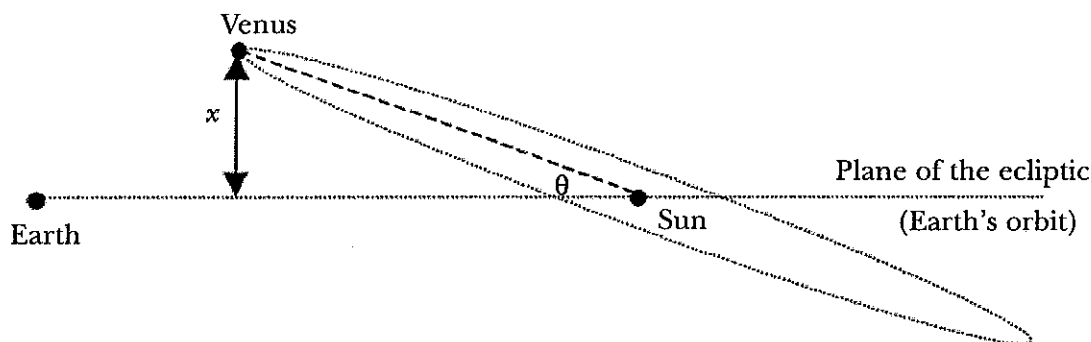


Figure 2: Height of Venus above the ecliptic.

Venus' Atmosphere

4. Using the information in the paper as your guide, sketch Venus' pressure-temperature profile, labeling the axes (using *altitude* on the y-axis instead of pressure) and the different layers in the atmosphere. Indicate on your P-T profile where Venus' cloud deck lies. (5 points)

Figure ?? below shows Fig. 1a from the paper, which was blocked out for your exams, illustrating Venus' P-T profile. The paper describes the altitude ranges for the different atmospheric layers: 0-60 km is the troposphere, 60-100 km is the mesosphere, and $z > 100$ km up to ~ 400 km is the "upper atmosphere" for the purposes of this paper. T at ($z=100$ km) is given as 200 K, the surface temperature is 740 K, and the temperature at the tropopause is 220 K. The cloud deck extends from 45-70 km, and an upper haze layer extends from 70-90 km. *These values were all given in the paper.*

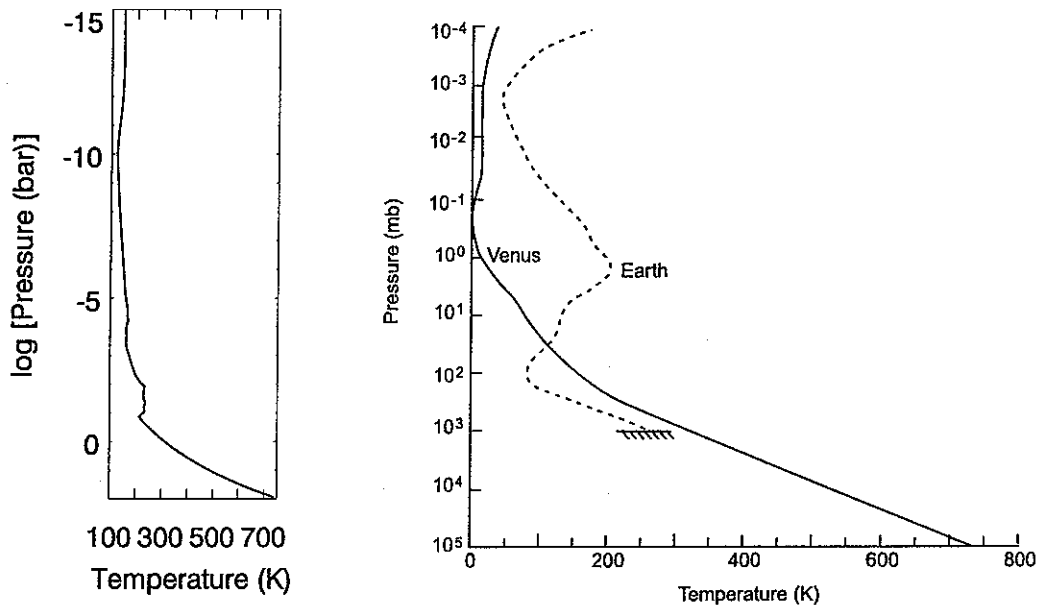


Figure 3: Left: Venus' P-T profile from the paper used in this exam. Right: Venus' P-T profile as compared with Earth's, from Taylor and Grinspoon (2009), Climate evolution of Venus, *J. Geophys. Res.* 114, E00B40, doi:10.1029/2008JE003316.

5. Explain what the *dry adiabatic lapse rate* is and calculate it for Venus. Then compare your result with the temperature lapse rate quoted in the second paragraph of §2.1 of the paper and comment on any differences. (10 points)

An *adiabatic* process is one in which no heat is added to or removed from a system. So an adiabatic lapse rate describes the change in temperature as a function of altitude for an air parcel that is moved adiabatically, i.e. without exchanging heat with its surroundings. The dry adiabatic lapse rate is given by

$$\Gamma_d = -\frac{dT}{dz} = -\frac{g}{C_p} \quad (3)$$

Venus' gravitational acceleration is given by $g = GM/R^2 = (6.674 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(4.87 \times 10^{24} \text{ kg})/(6.051 \times 10^6 \text{ m})^2 = 8.88 \text{ m/s}^2$. So $\Gamma_d = -(8.88 \text{ m/s}^2)/(850 \text{ J K}^{-1} \text{ kg}^{-1}) = -0.0104 \text{ K/m} = -10.4 \text{ K/km}$.

Now we can compare this number to that given in the paper, $+8.5 \text{ K/km}$. As for the sign of the two values, do not worry that they are different. The paper states that they used their lapse rate going from the tropopause down to the surface, whereas we implicitly assumed that our value starts from the surface and goes upward. So the fact that they have different

signs is consistent. However, the value we computed above is *larger* than the one quoted in the paper. This is presumably because the authors are not using a *dry* adiabatic lapse rate, but a *wet* one instead. The latent heat released due to condensation causes the lapse rate to decrease, i.e. the rate of temperature decrease with height also decreases.

6. Assuming that the atmosphere is just a single slab of gas, calculate how much infrared radiation emitted from the surface will actually make it out of the atmosphere and leave the planet. Put another way, calculate the opacity of Venus' atmosphere. Approach this problem in two parts such that you are ultimately examining the energy balance *at the top of Venus' atmosphere*:

a) Calculate F_{in} , the flux of visible sunlight at the top of Venus' atmosphere. (6 points)

We must examine Venus' energy balance to answer this question. Fundamentally, $F_{out} = F_{in}$. F_{in} is the solar flux at Venus. $F_{in} = 1365/(4a^2)$ W m⁻², where a is the distance from the Sun in AU. [The factor of 4 comes from the ratio of the area of the Earth's disk (πR^2) to the Earth's surface area ($4\pi R^2$).] A fraction of the Sun's radiation is reflected, corresponding to A , the albedo. So $(1-A)F_{in}$ goes into the planet.

Note: I think I misspoke when one of you asked me about the definition of the solar irradiance - the value given in the list of constants is the "solar constant," i.e. the value measured at the top of the Earth's atmosphere at 1 AU. DO NOT WORRY ABOUT THIS. You were not penalized if you assumed that it corresponded to the top of Venus' atmosphere. As you can see below you can do this problem without using it.

b) Calculate F_{out} , the flux of energy leaving the top of Venus' atmosphere. (9 points)

F_{out} is the sum of $F_{reflected}$ and F_{IR} . The surface is emitting a flux of σT_g^4 W m⁻², where T_g is the ground temperature. In order to determine how much of this surface flux is absorbed by the atmosphere, we realize that the net amount of energy radiated to space is the same as the net amount absorbed:

$$F_{out} = (1 - A) \left(\frac{1365}{4a^2} \right) = \sigma T_{eff}^4 \quad (4)$$

where T_{eff} is the effective temperature, which you were given in the list of constants. If we define opacity as ϵ (not to be confused with the optical depth, τ), then the flux that is absorbed between the surface and the top of the atmosphere is $\epsilon \sigma T_g^4$, and the amount that is radiated to space is $(1-\epsilon)\sigma T_g^4 = \sigma T_{eff}^4$. Thus, $\epsilon \equiv 1 - (T_{eff}/T_g)^4$. For Venus, $\epsilon = [1 - (240/740)^4] = 0.99$. This

means that 99% of the energy is absorbed on its way from the ground to space!

Venus as a Transiting Extrasolar Planet

7. Briefly summarize the point of this paper. This should be no more than 1-2 paragraphs in length, and in your *own words*. In your summary, describe a) the observables, and their connection to physical parameters, b) the model developed by the authors, and c) the application of this work to future extrasolar planet detections. (10 points)

There is no single correct answer to this question. I was looking for the following points:

- **Observables:** transmission spectra, absorptions corresponding to molecules present in Venus' atmosphere, scattering signatures of aerosols
 - **Model:** radiative transfer model including appropriate atmospheric constituents, Mie scattering, hazes; used inputs from Venus Express observations
 - **Application:** using this model as a predictor of what will be seen in June 2012 Venus transit, could be extended to transmission spectra of transiting exoplanets, seeing Mie scattering signature would be indicative of an upper atmosphere haze
8. On December 21, 2012, the Cassini spacecraft will observe a transit of Venus. Furthermore, on July 20, 2020, the Earth will transit the Sun as seen from Saturn. Assume that the Cassini spacecraft will still be operational at that time, with its entire instrument suite still intact, so that both events will be observed by Cassini. Draw light curves for the two transits as seen by Cassini, assuming that the imaging camera was used as the primary instrument for these observations. Your plots should have **quantitative values** on both axes! (10 points)

We can calculate the approximate depth of the transits by computing the relative sizes of the Earth, Venus, and the Sun as seen from Saturn. Saturn's distance from the Sun ~ 9.539 AU. Earth is thus 8.539 AU from Saturn during a transit, while Venus is $(9.539 - 0.72) = 8.819$ AU. The angular size of the Sun as seen from Saturn is $2\theta = 2 \times \tan^{-1}\left(\frac{R_{\odot}}{d_{\odot S}}\right)$, where $d_{\odot S}$ is the Saturn-Sun distance. Plugging in numbers, I get $201''$.

Using a similar relation for Earth and instead using $d_{\oplus S}$ (the Saturn-Earth distance, 8.539 AU), I get $2.06''$. [Note: you had to either know what Earth's radius is or make an educated guess since it was not given.] Using a

similar relation for Venus and instead using d_{VS} (the Saturn-Venus distance, 8.819 AU), I get 1.89".

Now we can compute the fraction of the Sun's disk that is covered by Earth and Venus, respectively. The flux decrement, ΔF , is given by $(F_{no\ transit} - F_{transit})/F_{no\ transit}$. This is equal to $(R_p/R_\odot)^2$, where R_p is the radius of the planet. Note that my values are given in angular radii rather than linear units, but it doesn't matter. So for the Earth transit, $(\Delta F)_\oplus = (2.06/201)^2 = 1.05 \times 10^{-4}$. For the Venus transit, $(\Delta F)_V = (1.89/201)^2 = 8.84 \times 10^{-5}$. So the Venus transit light curve should not be quite as deep as the Earth transit light curve since the Earth blocks a slightly greater percentage of the Sun's disk.

Now we need to compute the angular velocities of Venus and Earth to determine how long it takes them to travel across the Sun (angular size of 201" as seen from Saturn). $\omega = 360^\circ/T$ where T is the object's orbital period. $\omega_\oplus = 360/(365.25\text{ d})(24\text{ hr/d})(3600\text{ s/hr}) = 1.141 \times 10^{-5}\text{ deg/s} = 0.041\text{ arcsec/s}$. So to travel 201" across the disk of the Sun, Earth will take $4894\text{ s} = 1.35\text{ hr}$. $\omega_V = 360/(224.54\text{ d})(24\text{ hr/d})(3600\text{ s/hr}) = 1.856 \times 10^{-5}\text{ deg/s} = 0.067\text{ arcsec/s}$. So to travel 201" across the disk of the Sun, Venus will take $3009\text{ s} = 0.84\text{ hr}$. The Venus transit will thus take less time to complete than the Earth transit, because Venus' orbital velocity is greater.

GRADING SCHEME FOR CUME EXAM # 374

November 10, 2012

#	Pts.	Point Distribution
1	5	1 pt for providing correct formula for Venus' synodic period 2 pts for computing Venus' sidereal period from Kepler's Third Law 2 pts for correctly computing Venus' synodic period
2	5	1 pt for angular size of Sun 2 pts for setting up correct geometry 2 pts for correct calculation
3	5	2 pts for setting up correct geometry 2 pts for correct calculation 1 pt for comments on relative heights and rarity of transits
4	5	2 pts for correct identification, placement of atmospheric layers 2 pts for locations of clouds 1 pt for correct labeling, values of axes
5	10	4 pts for explanation of what dry adiabatic lapse rate is (2 pts for knowing what <i>adiabatic</i> means), 3 pts for calculating it for Venus 3 pts for comparison with value given in paper
6(a)	6	2 pts for recognizing that it depends on distance from Venus to Sun 2 pts for correct use of energy balance equation (including albedo) 2 pts for calculation
6(b)	9	3 pts for recognizing need to do sum of reflected + thermal radiation 3 pts for concept of opacity 3 pts for calculation
7	10	3 pts for discussing observables and their connection to physical parameters 3 pts for discussing model 4 pts for discussing application to exoplanets
8	10	5 pts for relative depths of transit 5 pts for relative durations of transit