Cume #481

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What better way to commemorate the beginning of October with a cume?! Prior to receiving this exam, you will have received the corresponding paper "Comparison of models for the warm-hot circumgalactic medium around Milky Way-like galaxies" by Priyanka Singh et al. (2024). Herein, you will find questions on that paper and related topics.

There are 15 questions/subquestions with 49 points possible. A score of 34 points or more will warrant an automatic pass.

In reading the paper, I do not suggest that you read it linearly from front to back. For this particular paper, I might suggest reading the Abstract; read Section 1 (Introduction), examine Figure 1 and read the caption. Read Section 2.1, but do not get bogged down by the equations; this paper is all about making assumptions that greatly simplify these equations! Read Sections 2.2-2.5 and review the relevant panel of Figure 1 after each subsection. Now, read Section 5 (Summary). Finally, read Sections 3 and 4, reviewing Figures 2 and 3 and their captions as they are referenced.

As a reminder, references beyond this paper, notes, or communication (other than with me) are NOT permitted during this exam. You are permitted to use the basic functions of a calculator, i.e., not graphing or information stored prior to the exam. If you need a calculator that is not on your phone, please ask to borrow one; we have them available. Phones and all other electronic devices should be turned off and put away during the exam.

Please make sure your writing is legible. As a general rule, I try to assign partial credit for good efforts, but I *cannot if the writing is illegible.* Also, please show all the work and attempt each problem, showing your thought process even if you can't solve it completely. Note that most questions have multiple parts, so make sure you answer the entire problem.

If anything needs clarification, please just ask!

Possibly relevant information:

Redshift	Angular diameter distance [Mpc]	Distance modulus [mag]	Projected angular scale [kpc/arcsec]
0.005	22	31.7	0.01
0.010	44	33.2	0.20
0.050	208	36.8	0.94
0.100	393	38.4	1.7
0.500	1297	42.3	5.9

$k_B = 8.6 \times 10^{-5}$ eV/K $= 1.38 \times 10^{-16}$ erg / K
$m_p = 1.673 \times 10^{-24}$ g
$M_\odot = 1.988 \times 10^{30}$ kg
$G = 6.674 \times 10^{-11}$ m^3 / (kg s^2)
1 pc $= 3.08 \times 10^{16}$ m

1) This paper focuses on models of the 'warm-hot' circumgalactic medium (CGM). What approximate temperatures does this regime correspond to? (2 point)

   $10^{5\text{-}6}$ K (1 pt for something less than 10^7 K or greater than 10^4 K; 2pts for correct range)

2) What are two other temperature regimes or 'phases' that might be found in gaseous halos? What are their relevant temperatures? (3 points)

   'cool': 10^4 K; 'hot': $>10^6$ K (1.5 pts for each; approximations of the names are okay if the temperatures are right; accept 'cold' at 100 K even though it's far less prominent, we think)

3) In your own words and in 2-3 sentences, describe two advantages analytical models (like those studied here) have over hydrodynamical simulations? (3 points)

   Analytical models can be run very quickly relative to hydro sims. This enables one to try many choices of the input parameters and vary the physical assumptions. (1.5 pts for something about expediency; 1 point for varying parameters; 0.5 pt for attempting)

4) In your own words and in 2-3 sentences, describe two advantages hydrodynamical simulations have over analytical models? (3 points)

   Hydro sims allow one to simulate processes like gas flows, thermal instabilities, and feedback more directly rather than assuming their thermodynamic effects. One can also study interactions, such as galaxy mergers, that occur during structure formation. (1.5 pts for something about directness; 1 point for anything else reasonable; 0.5 pt for attempting)

5) As mentioned in Section 2.2, the cooling function $\Lambda(T, Z)$ depends on the temperature and metallicity.
   a. Does the cooling function increase, decrease, or neither with increasing metallicity? (1 point) increases

b. In 1-2 sentences, why is the cooling function so heavily dependent on the metallicity? Hint: what is the important cooling mechanism here? (3 points). Cooling can proceed radiatively by the line emission from the various species, particularly carbon, oxygen, and iron. (3 pts for line emission; 1.5 pts for just radiative cooling)

6) Equation 2 is the equation of momentum conservation for a fluid in a gravitational potential. Each of the four models considered here assume some form of hydrostatic equilibrium. Let us explore just how hydrostatic equilibrium simplifies matters.

a. Let's get Equation 2 into the more familiar form under hydrostatic equilibrium. Set velocities on the left-hand side equal to zero and assume there is only one contribution (thermal) to the pressure (remove the summation). Rewrite the equation with the pressure on one side and the gravitational potential on the other. (2 points)

$$\frac{1}{\rho}\frac{\partial P}{\partial R} = -\frac{\partial \Phi}{\partial r}$$

b. Assume an ideal gas to replace the pressure and assume a spherically symmetric gravitational potential (Φ = -GM/r where M is the total mass enclosed within radius r). Write down the resulting expression. (2 points)

$$\frac{1}{\rho}\frac{\partial nkT}{\partial R} = -\frac{GM(r)}{r^2}$$

c. Now, show that the total mass can be estimated (under the assumption of hydrostatic equilibrium) from the slopes of the density and temperature profiles (actually, the slopes of log ρ vs. log r and log T vs. log r). (3 points)
Hint: $\frac{d\ln n}{d\ln r} = \frac{dn}{dr}\frac{r}{n}$

$$\frac{kT}{\mu m_p}\left(\frac{dn}{n} + \frac{dT}{T}\right) = -\frac{GM(r)}{r^2}dr \quad \text{(1 pt)}$$

Use fact that $\frac{d\ln n}{d\ln r} = \frac{dn}{dr}\frac{r}{n}$

$$\frac{kT}{\mu m_p}\left(\frac{1}{r}\frac{d\ln n}{d\ln r} + \frac{1}{r}\frac{d\ln T}{d\ln r}\right) = -\frac{GM(r)}{r^2} \quad \text{(1 pt)}$$

$$M(r) = \frac{rkT(r)}{\mu m_p G}\left(\frac{d\ln \rho}{d\ln r} + \frac{d\ln T}{d\ln r}\right) \quad \text{(1 pt)}$$

d. Finally, let's assume further that the gas takes on a polytropic equation of state where P = Aρ^Γ (in addition to being an ideal gas), as in several of the cases explored in the paper. Show that the temperature profile of this halo would be proportional to the gravitational potential profile. Hints: Substitute the above expression for pressure into your expression from part (a) and eliminate the dr differential before integrating. Assume both the potential and the density go to zero at large radius. (3 points)

$$\frac{1}{\rho}\frac{\partial(A\rho^{\Gamma})}{\partial r} = -\frac{\partial\Phi}{\partial r}$$

$$\frac{1}{\rho}A\Gamma\rho^{\Gamma-1}\frac{\partial\rho}{\partial r} = -\frac{\partial\Phi}{\partial r}$$

$$A\Gamma\rho^{\Gamma-2}\frac{\partial\rho}{\partial r} + \frac{\partial\Phi}{\partial r} = 0$$

$$A\Gamma\rho^{\Gamma-2}\, d\rho + d\Phi = 0$$

$$A\frac{\Gamma}{\Gamma-1}\frac{\rho^{\Gamma}}{\rho} = -\Phi$$

$$\frac{P}{\rho} = \frac{1-\Gamma}{\Gamma}\Phi$$

$$\frac{P}{\mu m_p}\frac{kT}{P} = \frac{1-\Gamma}{\Gamma}\Phi$$

$$kT = \frac{1-\Gamma}{\Gamma}\mu m_p \Phi$$

1 pt for substitution and derivative; 1 pt for integration; 1 pt for solving for T (or kT)

7) Consider the four CGM models described in Section 2.
   a. Which two models include pressure support from non-thermal sources, such as cosmic rays? (2 point) isentropic and baryon pasting
   b. Shown in Figure 2, how do the temperature profiles of these models compare to the other two? (2 points). They have the lowest temperature beyond 20 kpc. They are steeper too. (2 pts for saying T is lowest and something about the profile shape; 1pt for missing part (a) but saying something reasonable about what they picked)
8) Consider the observational predictions presented in Figure 3.
   a. What do the dotted lines and arrows signify in each panel? (1 point) These are the observational sensitivity limits for each diagnostic.

b. Pick one observational diagnostic capable of discriminating between two or more models and describe how that observation could be used, for example by taking a measurement at certain radii, to rule out one or more models. (3 points)
Could choose a few things here. For example, the soft X-ray surface brightness could distinguish between isentropic and the others if you could isolate the inner 10 kpc. (1pt for picking a diagnostic; 2pts for explanation)

c. What is a key instrument requirement beyond sensitivity (e.g., spatial or spectral resolution) for using this observation. (2 points) For the one I chose, it is spatial resolution, however energy resolution would boost sensitivity if one could isolate the O VIII line complex.

9) As discussed in the paper, O VI column densities appear to be highly promising in constraining these models, as seen in Figures 3 and A1. What is a potential major drawback to using O VI column densities alone? Why might the data shown in Figure A1 not rule out the cooling flow and precipitation models as they seem to do? Please answer with 1-2 sentences. (3 points)
One can also produce O VI through photoionization, which is not included in their modeling, so the predicted N(O VI) might be underestimated. (1pt for an attempt; 2pts for underestimated column densities)

10) The paper argues that the Sunyaev Zel'dovich effect offers a promising path forward to distinguish between models such as those they present. However, the CMB-S4 experiment mentioned in the paper, and whose specifications are shown in Figure 3, was recently delayed indefinitely by the National Science Foundation. Pretend you have received an anonymous donation for 3 billion dollars with the with the charge of saving galaxy formation science by distinguishing among the cooling flow, precipitation, and isentropic CGM models. You may build telescopes, operate them, design and execute surveys, etc. If you succeed, you can use the remaining funds for your own research, so the motivation is high.

a. Create a plan you would pitch to the foundation set up by the donor to outline your telescope and experiment needs. What kind of telescope would you build (or pre-existing one would you use)? Briefly describe the observations you would execute. Note: I am looking for an informed, well-reasoned 3-5 sentences here and am not looking for some specific approach or observation strategy. (4 points)
I would build a mm-wave array in the northern hemisphere with the requisite sensitivity and angular resolution to measure the kSZ to CMB-S4 limits. The advantage of doing this in the north is that one could leverage the infrastructure already in place for DESI and could simply execute a galaxy redshift survey that would yield enough galaxies at the appropriate redshift where the CMB survey's angular resolution matches the <20 kpc scales where the various models separate from isentropic at small and large radius. tSZ using the same mm-wave telescope can then distinguish the cooling flow/isentropic model from the others.

b. Suppose the board decides after consulting the authors of this paper that the kinematic Sunyaev Zel'dovich (kSZ) measurements are the way to go. Your CMB expert colleague on the project says they can measure T_{CMB} fluctuations on scales of 1.5 arcmin. Around what redshifts would recommend your team conduct their spectroscopic galaxy survey for the stacking experiment and why?

Hint: You may consult the table provided above. (3 points) 1.5 arcmin is 90 arcsec, so resolving <20 kpc scales would require 300,000 galaxies at z=0.1 at the very highest (full credit if they choose a larger size scale but use the correct scale from the table; 1pt for a reasonable attempt motivated by spatial resolution or magnitude limits)

c. Assume the absolute magnitude of the Milky Way is -20.6 in the V band. What would be the apparent magnitude of the Galaxy at the redshift you chose in part (b)? Would you say that this is difficult to observe with a 4-meter class telescope? Why or why not? (4 points) At z=0.1, it would be m_V = 12.6, which is cake for a 4m class telescope like the Mayall (where DESI is). DESI routinely reaches m~18 for the Bright Galaxy Survey. (1.5 pts for successfully adding an appropriate distance modulus; 0.5 pt for an answer yes or no; 2 pt for a coherent justification)