[redacted]

there are 5 questions. each can get maximum 5 points. read questions and try to understand what you are being ask to do. often you may not know the exact numbers, and that is OK, as long as your answers/numbers are reasonable (and not completely off). please keep your handwriting as clear as you can.

Q1) Single Stars (general).

a) draw H-R diagram and mark each evolutionary stage of star life (e.g., main sequence, giant branch....). also describe max 2 sentences per each stage: basically name the stage, say what is the source of energy and what is the basic stellar structure during the stage

b) delineate initial mass range (at Zero Age Main Sequence: ZAMS) for a star to form a white dwarf, neutron star, black hole

c) mass is the most important factor driving star evolution. give the time a star of 1, 10, 100 Msun is going to take to finish its evolution (time from ZAMS to remnant formation).

Q2) Binary Stars (general).

a) what is a binary star?

b) identify the processes that change binary component separation and note for what type of stars these processes are important. most processes decrease binary separation, but not all. while naming the process say which way it can change the separation.

c) what is a fraction of binaries in Galaxy? is it the same for low-mass and high-mass stars? do stars form in multiples?

Q3) Single Stars (calculus).

consider initial mass function (IMF) for stars to be a three component broken power law:

```
        a1 M^{-1.3}   for 0.08 <= M < 0.5 Msun
f(M)=   a2 M^{-2.2}   for 0.5 <= M < 1.0 Msun
        a3 M^{-2.7}   for 1.0 <= M <= 100.0 Msun
```

60/6

(*) note that this function is continuous, e.g., power-laws meet at M=0.5 and M=1.0 Msun.
(**) note that the integral of f(M) is required to be 1.0.
if you use (*) and (**) you should be able to get a1, a2, a3.

a) what is an average mass of a star in a stellar population (containing single stars only) described by the above IMF?

b) what is a fraction of massive stars (with mass over 8 Msun) in this population?

Q4) Binary Stars (calculus).

consider a circular (e=0) binary system described by
M1=10 Msun, giant
M2=8 Msun, main sequence star
A=500 Rsun, binary separation
star 1 (giant) has the core mass of M1,core=3 Msun and envelope mass M1,~~core~~ envelope=7 Msun.
say this system goes through mass transfer (MT) phase. more massive star is a donor, and it losses its entire envelope in the process.
to answer the following questions you will need to know 2 equations:
(*) Kepler law
(*) binary orbital angular momentum

Jorb= M1 M2 sqrt[AG(M1+M2)] / (M1+M2)

a) assume conservative mass transfer (both mass and angular momentum is conserved) and calculate post-MT separation

b) calculate post-MT binary orbital period, express answer in days, you may need to use:
   G=6.672 10^{-11} m^3 s^{-2} kg^{-1};
   1 Rsun = 6.9590 10^{10} cm
   1 Msun = 1.989 10^{33} g

Q5) General Knowledge/Writing Skills.

Come up with a short text (maximum 200 words; each word counts, e.g. articles "the", "a" count as a word too. you go over the specified maximum, and you get 0 points no matter how good your writing is) describing the recent important (in your opinion) breakthrough in astronomy. I want to see how well you can convey (abstract style) information. Consider using scratch paper first.

I suggest you include:

a) a short general introduction/opening statement
b) precise and to-the-point description of the discovery
c) note on what are the implications
d) some speculations/conclusion statement

[redacted]

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Q1) Single Stars (general)

a) draw H-R diagram and mark each evolutionary stage of star life (e.g., main sequence, giant branch....). also describe max 2 sentences per each stage: basically name the stage, say what is the source of energy and what is the basic stellar structure during the stage

Main Sequence -- central H burning, no clear core-envelope structure, only small change in radius (factor ~2), the longest phase

Hertzsprung Gap -- basically no nuclear burning, core-envelope structure emerges, contraction of the core, very rapid expansion of the star (factors ~ 10-100)

Red Giant Branch -- H-shell burning, clear core-envelope structure, continuous expansion of star

Horizontal Branch -- core He burning, star makes a loop on H-R diagram, contraction followed by expansion (second longest phase)

Asymptotic Giant Branch -- H-shell and He-shell burning, continuous expansion of star, eventually loss of an envelope and WD formation

b) delineate initial mass range (at Zero Age Main Sequence: ZAMS) for a star to form a white dwarf, neutron star, black hole

```
  < 8-10 Msun      -> white dwarf
8-10 < M < 20-25   -> neutron star
M > 20-25          -> black hole
```

c) mass is the most important factor driving star evolution. give the time a star of 1, 10, 100 Msun is going to take to finish its evolution (time from ZAMS to remnant formation).

```
1 Msun    -> 12-15 Gyr
10 Msun   -> 25-30 Myr
100 Msun  -> 3-5 Myr
```

Q2) Binary Stars (general)

a) what is a binary star?

A stellar system consisting of two stars orbiting about a common center of mass and often appearing as a single visual or telescopic object.

b) identify the processes that change binary component separation and note for what type of stars these processes are important. most processes decrease binary separation, but not all. while naming the process say which way it can change the separation.

stellar winds: mostly massive stars and late evolutionary stages for all masses (separation increase)

magnetic braking: low mass stars (M<1.2-1.5 Msun) with convective envelopes, for example cataclysmic variables (separation decrease) gravitational radiation: close compact object binaries like double white dwarfs or double neutron stars (separation decrease)

mass transfer: all close binaries (separation decrease or increase)

tidal interactions: for all close binaries in which the stars have significant (relative to the orbit) size (separation decrease or increase)

supernova explosion: only for binaries with massive stars (M>8-10 Msun) that explode in supernova (separation decrease, increase or orbit disruption)

c) what is a fraction of binaries in Galaxy? is it the same for low-mass and high-mass stars? do stars form in multiples?

binary fraction is rather large ~ 50% for low- and intermediate-mass stars, and can be as high as ~100% for massive stars.

yes, stars (especially massive) are commonly found in triplets and quadruplets.

Q3) Single Stars (calculus).
consider initial mass function (IMF) for stars to be a three component broken power law:

```
        a1 M^{-1.3}   for 0.08 <= M < 0.5 Msun
f(M)=   a2 M^{-2.2}   for 0.5 <= M < 1.0 Msun
        a3 M^{-2.7}   for 1.0 <= M <= 100.0 Msun
```

(*) note that this function is continuous, e.g., power-laws meet at M=0.5 and M=1.0 Msun.
(**) note that the integral of f(M) is required to be 1.0.
if you use (*) and (**) you should be able to get a1, a2, a3.

a) what is an average mass of a star in a stellar population (containing single stars only) described by the above IMF?

answer: Mave=0.453 Msun (see last page for calculation)

b) what is a fraction of massive stars (with mass over 8 Msun) in this population?

answer: frac=0.002 (0.2%) (see last page for calculation)

Q4) Binary Stars (calculus)
consider a circular (e=0) binary system described by
M1=10 Msun, giant
M2=8 Msun, main sequence star
A=500 Rsun, binary separation
star 1 (giant) has the core mass of M1,core=3 Msun and envelope mass M1,core=7 Msun.
say this system goes through mass transfer (MT) phase. more massive star is a donor, and it losses its entire envelope in the process.
to answer the following questions you will need to know 2 equations:
(*) Kepler law
(*) binary orbital angular momentum
Jorb= M1 M2 sqrt[AG(M1+M2)] / (M1+M2)

a) assume conservative mass transfer (both mass and angular momentum is conserved) and calculate post-MT separation

orbital angular momentum (for circular binary) is:
Jorb= M1 M2 sqrt[AG(M1+M2)] / (M1+M2)

conservation of angular momentum requires
Jorb,i = Jorb,f

that gives a very easy derivation of final separation
Af = Ai [(M1,i M2,i)/(M1,f M2,f)]^2

conservation of mass requires
M1f=3 Msun, M2f=15 Msun

and that results in new orbit
Af= 500 * 3.160 = 1580 Rsun

b) calculate post-MT binary orbital period, express answer in days, you may need to use:
G=6.672 10^{-11} m^3 s^{-2} kg^{-1};
1 Rsun = 6.9590 10^{10} cm
1 Msun = 1.989 10^{33} g

from Kepler law we get
(2 Pi / P)^2 = G(M1+M2) / A^3

we get
P=2 Pi sqrt[A^3]/ sqrt[G(M1+M2)]= 1715.8 days

as simple as it is, i want to check that they know (or know how to derive) Kepler law and that they can deal with unit conversion

5) General Knowledge/Writing Skills
Come up with a short text (maximum 200 words; each word counts, e.g. articles "the", "a" count as a word too. you go over the specified maximum, and you get 0 points no matter how good your writing is) describing the recent important (in your opinion) breakthrough in astronomy. I want to see how well you can convey (abstract style) information. Consider using scratch paper first.
I suggest you include:
a) a short general introduction/opening statement
b) precise and to-the-point description of the discovery
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I would expect that students will describe one of the following:
-- WMAP results, expansion of Universe
-- discoveries of extrasolar planets
-- recent evidence for supermassive BHs (e.g., direct stellar orbits around Galactic BH)

However, I will be fine with any subject if it is presented as important and a reasonable motivation is given.

ad Q1) a) ANSWER

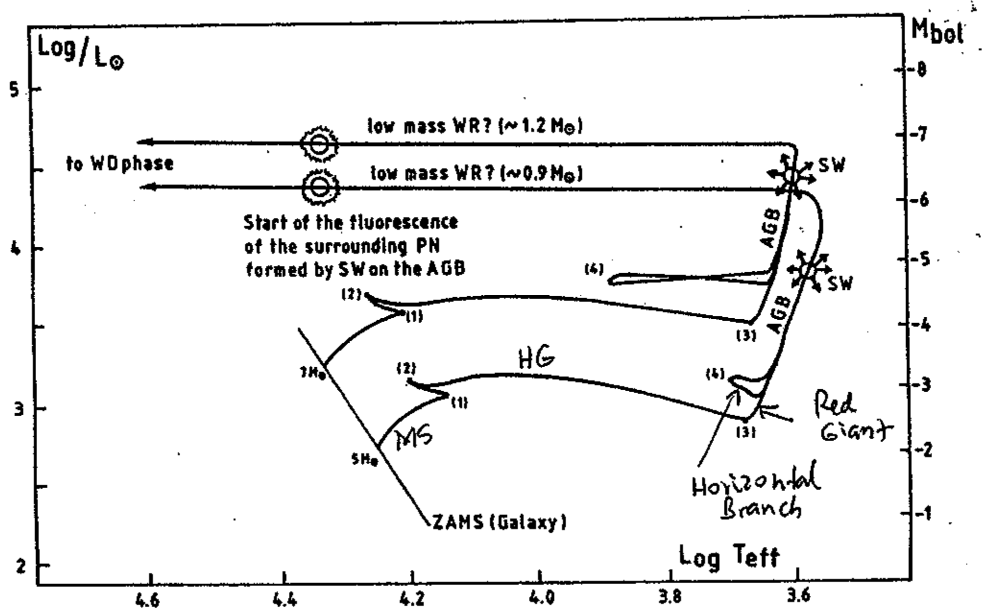

Figure 4.2: The effect of stellar wind mass loss at 'super rates' during the AGB phase on the evolution of a galactic star with initial mass = 5 M_o and 7 M_o.

Q3 a)

ANSWER

$$f(M) = \begin{cases} a_1 M^{-1.3} & 0.08 \leq M < 0.5 \\ a_2 M^{-2.2} & 0.5 \leq M < 1.0 \\ a_3 M^{-2.7} & 1.0 \leq M \leq 100 \end{cases}$$

I.) we estimate a_1, a_2, a_3

- first, our function $f(M)$ must be continious at meeting points $M = 0.5$ and $M = 1.0$ => that gives

$$\begin{cases} a_1 (0.5)^{-1.3} = a_2 (0.5)^{-2.2} & \Rightarrow a_2 = 0.536\, a_1 \\ a_2 (1.0)^{-2.2} = a_3 (1.0)^{-2.7} & \Rightarrow a_3 = 0.536\, a_1 \end{cases} \quad (*)$$

- second we normalize $f(M)$ so its total area is 1.0

$$\int f(M)dM = 1.0 \Rightarrow \int_{0.08}^{0.5} a_1 M^{-1.3} dM + \int_{0.5}^{1.0} a_2 M^{-2.2} dM + \int_{1.0}^{100} a_3 M^{-2.7} = 1.0 \quad (**)$$

now, we can input (*) to (**) to obtain a_1

$a_1 = 0.256$

that results [through (*)] in $a_2 = 0.137$

$a_3 = 0.137$

II.) once we have normalized $f(M)$ we can easily calculate average mass:

$$\overline{M} = \int_{0.08}^{100} f(M)\, M\, dM = 0.453\, M_\odot$$

Q3 b)

ANSWER

we can also obtain very easily the fraction of stars above $8 M_\odot$:

$$\text{frac}\,(M > 8M_\odot) = \int_{8.0}^{100} f(M)dM = 0.002 \quad (0.2\%)$$