final draft
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SUGGESTED RESPONSES FOR CUME EXAM # 333 March 8, 2008

1. **A companion paper to this one states that Enceladus is "difficult to observe from Earth because of the scattered light of the planet and its rings."**

a) Calculate the angular separation between Enceladus and Saturn as seen from Earth when Saturn is at opposition. (5 points)

Orbital radius of Enceladus is given as 3.94 R_S, or 3.94 × 60,268 km = 237,456 km. Using small angle approximation, $\sin\theta \approx \theta = \frac{R_{orb}}{D}$. D ~ 8.5 AU (from Earth at opposition), so $\theta = (2.375 \times 10^5\ km)/(8.5 \times 1.5 \times 10^8\ km) = 1.86 \times 10^{-4}$ rad = 38 arcsec.

b) Explain whether you agree with the statement above. Give at least one suggestion for how could you observe Enceladus from Earth and mitigate these possible effects. (5 points)

An angular separation of ~ 38 arcsec does not seem that horrible if we consider a site with average seeing of 1 arcsec. However, given that Saturn's rings extend beyond 2R_S, Enceladus is likely to be hampered by scattered light. One could observe in wavelengths where the planet is dark and the moons are bright, such as a methane absorption band in the near-IR (although the problem remains that the RINGS will still be bright). One could employ techniques such as adaptive optics that improve the spatial resolution, although this is not without problems (size of isoplanatic patch, moons may be too faint to serve as natural guide stars, etc.). One could use a mask or coronagraph to block out the signal from Saturn and its rings. Alternatively, one could observe during Saturn's ring plane crossing, when the signal from the rings is minimized. [*Any* of these suggestions would be acceptable answers to this question.]

2. **The paper states that the "visual geometric albedo" of Enceladus is 1.4.**

a) What is the definition of *geometric albedo?* (5 points)

The geometric albedo, A_g, is defined as the ratio of apparent brightness *when the object is at zero phase angle* to that of a perfectly diffusing (Lambertian) disk of the same angular size.

b) How is this quantity different from an object's *Bond albedo?* (3 points)

The Bond albedo, A_b, is defined as the ratio of the total radiation reflected or scattered by an object to the total incident light from the Sun. It is integrated over all wavelengths.

c) Is it physical to have an albedo value > 1? Explain. (3 points)

A geometric albedo value greater than one means that the total radiation reflected/scattered by an object exceeds the amount of incident solar radiation it receives. One could envision this happening if the surface is extremely reflective, and its surfacing scattering coefficients must be such that there is a significant amount of radiation scattered back into the line of sight between target and observer. Enceladus is thought to have just such a surface.

3. The authors state that the closest approach during Cassini's third flyby of Enceladus was 168 km, which was "well within Enceladus' Hill radius."

a) What is a Hill radius? Provide a clear description of the physical nature of this parameter. (5 points)

A Hill radius is the radius of an object's Hill sphere, which defines an object's gravitational sphere of influence in the presence of gravitational perturbations from a second object around which the first object orbits.

b) Calculate the size of Enceladus' Hill radius and list any assumptions that you make in this calculation. Compare your result with the value given in the paper; if your value is different, explain possible reasons for this discrepancy. (8 points)

$R_H = a\left(\frac{m_2}{3(m_1+m_2)}\right)^{\frac{1}{3}}$, where

m_2 is the mass of Enceladus, m_1 is the mass of Saturn, and a is the orbital radius of Enceladus. Using the fact that Enceladus' $GM = 7.2085\frac{km^3}{s^2}$ (from the paper, p. 1396), $m_2 = 1.08 \times 10^{20}$ kg. and we can solve for Enceladus' Hill radius $R_H \sim 947$ km, which is pretty close to the value given on p. 1393.

c) If the spacecraft passed well within Enceladus' Hill radius, why didn't it enter into orbit around Enceladus? (3 points)

The spacecraft did not alter its orbit to permanently reside inside of Enceladus' Hill sphere, and it passed through the Hill sphere with a velocity greater than Enceladus' escape velocity, thus it remained in orbit around Saturn.

4. The paper describes high phase angle observations of the plume emanat-

ing from the south pole at the end of page 1393 and the beginning of page 1394.

a) What is the scientific value of this kind of observation? (3 points)

High phase angle observations of the plume can reveal the scattering nature of the particles, which tells you something about the particle size (see p. 1397, right column).

b) How can we parameterize the scattering properties of the plume particles, i.e. what would its *scattering phase function* look like, graphically and functionally? (4 points)

Because we see the particles most clearly when they are illuminated from behind (see diagram below), they are strongly forward scattering. An example of a forward scattering phase function would be a Henyey-Greenstein (HG) function, which has the form

$$P(cos\Theta) = \frac{(1-g^2)}{[1+g^2-2gcos\Theta]^{3/2}}, \tag{1}$$

where $g = 0$ corresponds to isotropic scattering, $g \rightarrow 1$ is for strongly forward scattering phase functions, and $g \rightarrow -1$ is for strongly backward scattering. A plot of the HG, taken from Petty (2006), *A First Course in Atmospheric Radiation, 2nd Ed.*, is shown below.

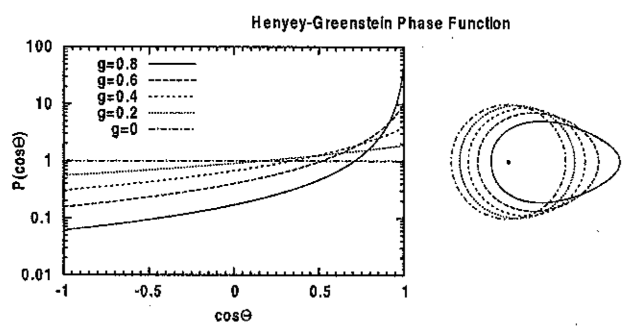

Fig. 11.2: The Henyey-Greenstein phase function plotted versus cos(Θ) (left) and as a log-scaled polar plot (right).

c) What scattering regime do these particles fall under (geometric, Rayleigh, Mie), and why? (4 points)

These particles fall under Mie scattering because they are small, $r \sim \lambda$. Geometric scattering works for large particles (e.g. water droplets in the Earth's atmosphere that form a rainbow), and Rayleigh scattering is most appropriate for gases where the particle size is much *smaller* than λ.

5. **Suppose that at the end of the Cassini mission, the JPL mission planners decide to crash the spacecraft into Enceladus.**

a) Would the resultant crater the impact would produce on Enceladus be larger than or smaller than the crater Cassini would form on the Moon if Cassini were orbiting Earth? You can assume that the spacecraft's orbital velocity is 10 km/sec, which is approximately matched to Saturn's orbital velocity. State any other assumptions you make! (9 points)

A crater diameter scales as approximately $\rho_P^{1/6}\rho_T^{1/2}K^{0.3}g^{-0.165}$, where ρ_P is the density of the projectile, ρ_T is the density of the target, K is the kinetic energy of the projectile, and g is the target's gravitational acceleration. If we compare the crater sizes on Enceladus vs. the Moon, we can take a ratio of the above relation, and the density of the projectile (the Cassini spacecraft) cancels out. You can find the density of Enceladus in the paper (p. 1396) and estimate the density of the Moon ~ 3 g cm^{-3}. The main difference will be the relative kinetic energies, and this is dependent on the impact velocity, which is the difference between the projectile's orbital velocity and the escape velocity of the target. [Assume that the impacts occurred with the same entry angle, so that doesn't come into play.]

For Enceladus, escape velocity $v_e = \left(\frac{2GM}{r}\right)^{1/2} = \left(\frac{2(7.2085\ km^3s^{-2})}{250\ km}\right)^{1/2} = 0.24$ km/s. This is much smaller than the orbital velocity of 10 km/s, so take Cassini's impact velocity on Enceladus ~ 10 km/s.

For the Moon, escape velocity $v_e = \left(\frac{2GM}{r}\right)^{1/2} = \left(\frac{2(6.673\times10^{-20}\ km^3kg^{-1}s^{-2})(7.35\times10^{22}\ kg)}{1737.53\ km}\right)^{1/2}$ $= 2.4$ km/s.

For the Moon, $g = 1.67$ m/s^2. For Enceladus, $g \sim 1.2$ m/s^2.

If Cassini is orbiting Earth at Moon's distance, $v_{orb} = \left(\frac{GM}{r}\right)^{1/2} =$ $\left(\frac{(6.673\times10^{-20}\ km^3kg^{-1}s^{-2})(5.9736\times10^{24}\ kg)}{3.844\times10^{5}\ km}\right)^{1/2} \sim 1$ km/s.
We can thus take Cassini's impact velocity on the Moon to be the difference between this value the Moon's escape velocity, or ~ 3.5 km/s. Putting it all together,

$$\frac{D_E}{D_M} = \frac{(1.6)^{1/2}(10\ km/s)^{\cancel{2.3}\ .6}(1.67)^{-0.165}}{(3.34)^{1/2}(3.5\ km/s)^{\cancel{2.3}\ .6}(1.2)^{-0.165}} = \cancel{\frac{232}{32}}\quad \frac{4.63}{3.76} \tag{2}$$

So the impact crater on Enceladus would be larger than on the Moon.

b) Describe the cratering trend seen on Enceladus, both as a function of

latitude and as a function of crater diameter (hint: see paper, Figure 4 and related discussion). Has the source of impactors evolved over time, or is something else going on? (3 points)

As a function of latitude, the more heavily cratered regions are at mid-northern latitudes down to mid-southern southern latitudes, and the least heavily cratered regions are in the South Polar Terrain at southern latitudes $> 55°$. In terms of crater diameter, there are more smaller craters and fewer larger ones. The largest craters are found in the heavily cratered plains. The source of impactors could have evolved over time if it was a source similar to the impactors that cratered the moon during the early bombardment period of solar system formation. Alternatively, the impactor source could be constant (comets from the outer solar system), and the variations in cratered terrains could be due to geologic activity. The authors seem to favor the latter explanation for the cratering variations seen as a function of latitude.

6. **Write a paragraph describing how we can learn about an object's atmosphere by observing a stellar occultation. Explain the observing geometry, drawing an accompanying diagram, as well as specifically how one goes from star brightness measurements to parameters that are relevant to atmospheric structure. (10 points)**

A stellar occultation provides us with a remote probe of the structure of a planetary atmosphere. After geometrically reconstructing the path of the observer through the shadow plane, one can extract information about an atmosphere's *scale height* by analyzing the slope of the "fall-off" and "ramp-up" of the stellar signal as the star appears to go behind and then reappear from the planetary atmosphere. From this information, if one assumes something about the composition of the atmosphere, information about the atmosphere's temperature, pressure, and number density profiles can be inferred through numerical inversion of the scale height data. [If you would like to read more about this technique, I recommend the article by Elliot and Olkin (1996), Probing planetary atmospheres with stellar occultations, *Ann. Rev. Earth Planet. Sci.* **24**, 89–123.]

7. **According to a companion paper that discusses the occultation measurements in more detail, the occultation data showed that the best fit *scale length* for the water vapor in Enceladus' atmosphere is $L = 80$ km.**

   a) Is this *scale length* the same thing as an atmospheric *scale height*? (2 points)

No, it is not the same thing. A scale height is something we use to describe the e-folding distance for pressure or density within a gravitationally bound atmosphere. Enceladus has an escaping atmosphere, so it is not appropriate to talk about a pressure or density scale height in this context.

b) The Cassini CIRS (infrared) spectrometer mapped the temperature of Enceladus' surface, and found that the ice temperature at Enceladus' south pole is 145 K. Use this information, along with the aforementioned scale length and other information in the paper, to calculate the rate of water loss from Enceladus. (8 points)

The rate of water loss from Enceladus, S, is given by the product of the molecular abundance $N(= n/h)$, the plume area h^2, and the velocity v, as follows: $S = N \times h^2 \times v = n \times h \times v$. The linear dimension of the plume, h, is the measured scale length, $L = 80$ km (from part a). From the paper (p. 1398), the near-surface column abundance of water vapor is 1.5×10^{20} molecules m^{-2}, so $n = 1.5 \times 10^{16} cm^{-2}$. To determine v, assume kinetic energy = thermal energy, or

$$\frac{1}{2}mv^2 = \frac{3}{2}kT, \ or \ v = \left(\frac{3kT}{m}\right)^{1/2} \rightarrow \tag{3}$$

$$v = \left(\frac{(3(1.38 \times 10^{-16} erg/K)(145\ K)}{(1.66 \times 10^{-24} g)(18)}\right)^{1/2} = 44,822\ cm/s \tag{4}$$

Thus, $S = (1.5 \times 10^{16}\ cm^{-2})(80 \times 10^5\ cm)(44,822\ cm/s) = 5.38 \times 10^{27}$ molecules/sec. The mass of a water molecule $\sim 18 \times m_{amu} = 1.660 \times 10^{-24}$ g $= 1.660 \times 10^{-27}$ kg, so the rate of water loss from Enceladus is $\sim (5.38 \times 10^{27}$ molecules/sec$) \times (1.660 \times 10^{-27}$ kg/molecule$) \sim 150$ kg/sec.