Name:______________________________
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# 8 Estimating the Earth's Density

## 8.1 Introduction

We know, based upon a variety of measurement methods, that the density of the Earth is 5.52 grams per cubic centimeter. [This value is equal to 5520 kilograms per cubic meter. Your initial density estimate in Table 8.3 should be a value similar to this.] This density value clearly indicates that Earth is composed of a combination of rocky materials and metallic materials.

With this lab exercise, we will obtain some measurements, and use them to calculate our own estimate of the Earth's density. Our observations will be relatively easy to obtain, but they will involve contacting someone in the Boulder, Colorado area (where the University of Colorado is located) to assist with our observations. We will then do some calculations to convert our measurements into a density estimate.

As we have discussed in class, and in previous labs this semester, we can calculate the density of an object (say, for instance, a planet, or more specifically, the Earth) by knowing that object's mass and volume. It is a challenge, using equipment readily available to us, to determine the Earth's mass and its volume directly. [There is no mass balance large enough upon which we can place the Earth, and if we could what would we have available to "balance" the Earth?] But we have through the course of this semester discussed physical processes which relate to mass. One such process is the gravitational attraction (force) one object exerts upon another.

The magnitude of the gravitational force between two objects depends upon both the masses of the two objects in question, as well as the distance separating the centers of the two objects. Thus, we can use some measure of the Earth's gravitational attraction for an object upon its surface to ultimately determine the Earth's mass. However, there is another piece of information that we require, and that is the distance from the Earth's surface to its center: the Earth's radius.

We will need to determine both the MASS of the Earth and the RADIUS of the Earth. Since we will use the magnitude of Earth's gravitational attraction to determine Earth's mass, and since this magnitude depends upon the Earth's radius, well first determine Earth's circumference (which will lead us to the Earth's radius and then to the Earth's volume) and then determine the Earth's mass.

## 8.2 Determining Earth's Radius

Earlier this semester you read (or should have read!) in your textbook the description of Eratosthenes' method, implemented two-thousand plus years ago, to determine Earth's circumference. Since the Earth's circumference is related to its radius as:

$$\text{Circumference} = 2 \times \pi \times \text{RADIUS (with } \pi = \text{“pi”} = 3.141592)$$

and the Earth's volume is a function of its radius:

$$\text{VOLUME} = (4/3) \times \pi \times \text{RADIUS}^3$$

We will implement Eratosthenes' circumference measurement method and end up with an estimate of the Earth's radius.

Now, what measurements did Eratosthenes use to estimate Earth's circumference? Eratosthenes, knowing that Earth is spherical in shape, realized that the length of an object's shadow would depend upon how far in latitude (north-or-south) the object was from being directly beneath the Sun. He measured the length of a shadow cast by a vertical post in Egypt at local noon on the day of the northern hemisphere summer solstice (June 20 or so). He made a measurement at the point directly beneath the Sun (23.5 degrees North, at the Egyptian city Syene), and at a second location further north (Alexandria, Egypt). The two shadow lengths were not identical, and it is that difference in shadow length plus the knowledge of how far apart the the two posts were from each other (a few hundred kilometers), that permitted Eratosthenes to calculate his estimate of Earth's circumference.

As we conduct this lab exercise we are not in Egypt, nor is today the seasonal date of the northern hemisphere summer solstice (which occurs in June), nor is it locally Noon (since our lab times do not overlap with Noon). But, nonetheless, we will forge ahead and estimate the Earth's circumference, and from this we will estimate the Earth's radius.

**TASKS:**

- Take a post outside, into the sunlight, and measure the length of the post with the tape measure.
- Place one end of the post on the ground, and hold the post as vertical as possible.
- Using the tape measure provided, measure to the nearest 1/2 centimeter the length of the shadow cast by the post; this shadow length should be measured three times, by three separate individuals; record these shadow lengths in Table 8.1.
- You will be provided with the length of a post and its shadow measured simultaneously today in Boulder, Colorado.
- Proceed through the calculations described after Table 8.1, and write your answers in the appropriate locations in Table 8.1. **(10 points)**

## 8.3 Angle Determination:

With a bit of trigonometry we can transform the height and shadow length you measured into an angle. As shown in Figure 8.1 there is a relationship between the length (of your shadow in this situation) and the height (of the shadow-casting pole in this situation), where:

Table 8.1: **Angle Data**

| Location | Post Height (cm) | Shadow Length (cm) | Angle (Degrees) |
|---|---|---|---|
| Las Cruces Shadow #1 | | | |
| Las Cruces Shadow #2 | | | |
| Las Cruces Shadow #3 | | | |
| Average Las Cruces Angle: | | | |
| Boulder, Colorado | | | |

TANGENT of the ANGLE = far-side length/ near-side length

Since you know the length of the post (the near-side length, which you have measured) and the length of the shadow (the far-side length, which you have also measured, three separate times), you can determine the shadow angle from your measurements, using the ATAN, or TAN$^{-1}$ capability on your calculator (these functions will give you an angle if you provide the ratio of the height to length):

ANGLE = ATAN (shadow length / post length)
or
ANGLE = TAN$^{-1}$(shadow length / post length)

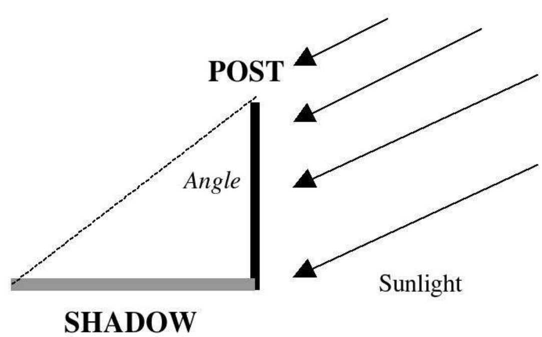


Figure 8.1: The geometry of a vertical post sitting in sunlight.

**Calculate the shadow angle for each of your three shadow-length measurements, and also for the Boulder, Colorado shadow-length measurement.** Write these angle values in the appropriate locations in Table 8.1. Then calculate the average of the three Las Cruces shadow angles, and write the value on the "Average Las Cruces Angle" line.

The angles you have determined are: 1) an estimate of the angle (latitude) difference between Las Cruces and the latitude at which the Sun appears to be directly overhead (which is currently ∼ 12 degrees south of the equator since we are experiencing early northern autumn), *and* 2) the angle (latitude) difference between Boulder, Colorado and the latitude at which the Sun appears to be directly overhead. The difference (Boulder angle minus Las Cruces angle) between these two angles is the angular (latitude) separation between Las Cruces and Boulder, Colorado.

We will now use this information and our knowledge of the actual distance (in kilometers) between Las Cruces' latitude and Boulder's latitude. This distance is:

**857 kilometers north-south distance between Las Cruces and Boulder, Colorado**

In the same way that Eratosthenes used his measurements (just like those you have made today), we can now determine an estimate of the Earth's circumference. Using your calculated Boulder Shadow Angle and your Average Las Cruces Shadow Angle values, calculate the corresponding EARTH CIRCUMFERENCE value, and write it below:

$$\textbf{Average Earth Circumference (kilometers)} =$$

$$\textbf{857 kilometers} \times \mathbf{(360^o)/(Boulder\ angle — Avg\ LC\ Angle)} =$$

$$\mathbf{857 \times [360^o/(\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_)] = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\ km\ (2\ points)}$$

The CIRCUMFERENCE value you have just calculated is related to the RADIUS via the equation:

$$\text{EARTH CIRCUMFERENCE} = 2 \times \pi \times \text{EARTH RADIUS}$$

which can be converted to RADIUS using:

$$\text{EARTH RADIUS} = \text{R}_\text{E} = \text{EARTH CIRCUMFERENCE} / (2 \times \pi)$$

For your calculated CIRCUMFERENCE, calculate that value of the Radius (in units of kilometers) in the appropriate location below:

**AVERAGE EARTH RADIUS VALUE = $\mathbf{R_E}$ = __________________ kilometers (3 points)**

**Convert this radius ($\mathbf{R_E}$) from kilometers to meters, and enter that value in Table 8.3.** *(Note we will use the radius in meters the rest of this lab.)*

You have now obtained one important piece of information (the radius of the Earth) needed for determining the density of Earth. We will, in a bit, use this radius value to calculate the Earth's volume. Next, we will determine Earth's mass, since we need to know both the Earth's volume and its mass in order to be able to calculate the Earth's density.

## 8.4 Determining the Earth's Mass

The gravitational acceleration (increase of speed with increase of time) that a dropped object experiences here at the Earth's surface has a magnitude defined by the Equation (thanks to Sir Isaac Newton for working out this relationship!) shown below:

$$\text{Acceleration (meters per second per second)} = \mathrm{G} \times \mathrm{M_E}/\mathrm{R_E}^2$$

Where $\mathbf{M}_\mathrm{E}$ is the mass of the Earth in *kilograms*, $\mathbf{R}_\mathrm{E}$ is the radius of the Earth in units of *meters*, and the Gravitational Constant, $\mathbf{G} = 6.67 \text{ x } 10^{-11}$ meters$^3$/(kg-seconds$^2$). You have obtained several estimates, and calculated an average value of $\mathrm{R_E}$, above. However, you currently have no estimate for $\mathrm{M_E}$. You can estimate the Earth's mass from the measured acceleration of an object dropped here at the surface of Earth; you will now conduct such an exercise.

A falling object, as shown in Figure 8.2, increases its downward speed at the constant rate **"X"** (in units of meters per second per second). Thus, as you hold an object in your hand, its downward speed is zero meters per second. One second after you release the object, its downward speed has increased to **X** meters per second. After two seconds of falling, the dropped object has a speed of **2X** meter per second, after 3 seconds its downward speed is **3X** meters per second, and so on. So, if we could measure the speed of a falling object at some point in time after it is dropped, we could determine the object's acceleration rate, and from this determine the Earth's mass (since we know the Earth's radius). However, it is difficult to measure the instantaneous speed of a dropped object.

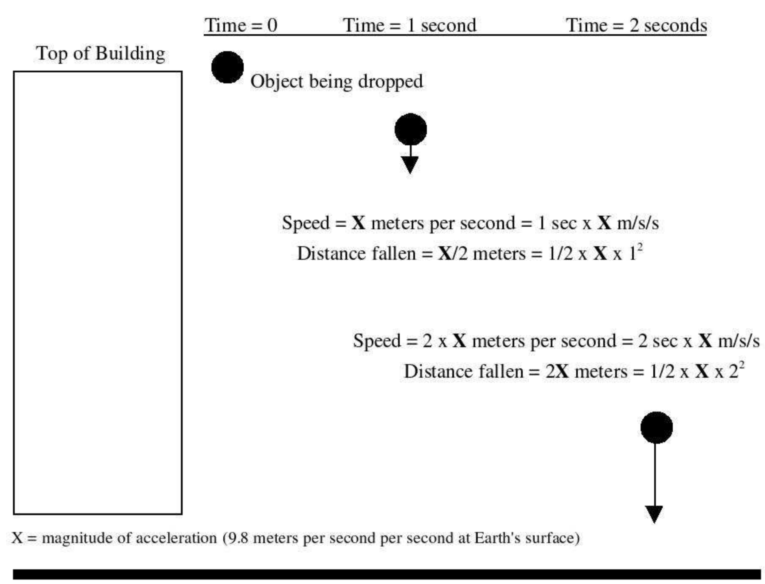


Figure 8.2: The distance a dropped object will fall during a time interval $t$ is proportional to $t^2$. A dropped object speeds up as it falls, so it travels faster and faster and falls a greater distance as $t$ increases.

We can, however, make a different measurement from which we can derive the dropped object's acceleration, which will then permit us to calculate the Earth's mass. As was pointed out above, before being dropped the object's downward speed is zero meters per second. One second after being dropped, the object's downward speed is X meters per second. During this one-second interval, what was the object's AVERAGE downward speed? Well, if it was zero to begin with, and X meters per second after falling for one second, **its average fall speed during the one-second interval is:**

**Average Fall speed during first second = (Zero + X) / 2 = X/2 meters per second**, which is just the average of the initial (zero) and final (X) speeds.

At an average speed of X/2 meters per second during the first second, the distance traveled during that one second will be:

$$\textbf{(X/2) (meters per second)} \times \textbf{1 second = (X/2) meters,}$$

since:

$$\textbf{DISTANCE = AVERAGE SPEED} \times \textbf{TIME = 1/2} \times \textbf{ACCELERATION x TIME}^2$$

So, if we measure the length of time required for a dropped object to fall a certain distance, we can calculate the object's acceleration.

**Tasks:**

- Using a stopwatch, measure the amount of time required for a dropped object (from the top of the Astronomy Building) to fall 9.0 meters (28.66 feet). Different members of your group should take turns making the fall-time measurements; write these fall time values for two "drops" in the appropriate location in Table 8.2. **(10 points for a completed table)**
- **Use the equation: Acceleration = [2.0 x Fall Distance] / [(Time to fall)$^2$]**

  and your measured Time to Fall values and the measured distance (9.0 meters) of Fall to determine the gravitational acceleration due to the Earth; write these acceleration values (in units of meters per second per second) in the proper locations in Table 8.2.
- Now, knowing the magnitude of the average acceleration that Earth's gravity imposes upon a dropped object, we will now use the "Gravity" equation to get $M_E$:

  Gravitational acceleration = $G \times M_E/R_E^2$ (where $R_E$ must be in meters!)

By rearranging the Gravity equation to solve for $M_E$, we can now make an estimate of the Earth's mass:

**$M_E$ = Average Acceleration × $(R_E)^2$ / G = ________________________ (5 points)**

Write the value of $M_E$ (in kilograms) in Table 8.3 below.

Table 8.2: **Time of Fall Data**

| | Time to Fall | Fall Distance | Acceleration |
|---|---|---|---|
| Object Drop #1 | | 9 meters | |
| Object Drop #2 | | 9 meters | |
| Average = | | | |

## 8.5 Determining the Earth's Density

Now that we have estimates for the mass ($M_E$) and radius ($R_E$) of the Earth, we can easily calculate the density: Density = Mass/Volume. You will do this below.

**Tasks:**

- Calculate the volume ($V_E$) of the Earth given your determination of its radius *in meters!*:

$$\mathbf{V_E = (4/3) \times \pi \times R_E^3}$$

  and write this value in the appropriate location in Table 8.3 below.

- *Divide your value of $M_E$ (that you entered in Table 8.3) by your estimate of $V_E$ that you just calculated (also written in Table 8.3):* the result will be your estimate of the Average Earth Density in units of kilograms per cubic meter. Write this value in the appropriate location in Table 8.3.

- *Divide your AVERAGE ESTIMATE OF EARTH'S DENSITY value that you just calculated by the number 1000.0*; the result will be your estimated Earth density value in units of grams per cubic centimeter (the unit in which most densities are tabulated). Write this value in the appropriate location in Table 8.3.

Table 8.3: **Data for the Earth**

| | |
|---|---|
| Estimate of Earth's Radius: | ______________________ **m** (4 points) |
| Estimate of Earth's Mass: | ______________________ **kg** (4 points) |
| Estimate of Earth's Volume: | ______________________$\mathbf{m^3}$ (4 points) |
| Estimate of Earth's Density: | ____________________ $\mathbf{kg/m^3}$ (4 points) |
| Converted Density of the Earth: | ___________________ $\mathbf{gm/cm^3}$ (4 points) |

## 8.6 In-Lab Questions:

1. Is your calculated value of the (Converted) Earth's density GREATER THAN, or LESS THAN, or EQUAL TO the actual value (see the Introduction) of the Earth's density? If your calculated density value is not identical to the known Earth density value, calculate the "percent error" of your calculated density value compared to the actual density value **(2 points)**:

   PERCENT ERROR =

   $$\frac{100\% \times (\text{CALCULATED DENSITY} - \text{ ACTUAL DENSITY})}{\text{ACTUAL DENSITY}} = \rule{3cm}{0.4pt}$$

2. You used the AVERAGE Las Cruces shadow angle in calculating your estimate of the Earth's density (which you wrote down in Table 8.3). If you had used the LARGEST of the three measured Las Cruces shadow angles shown in Table 8.1, would the Earth density value that you would calculate with the LARGEST Las Cruces shadow angle be larger than or smaller than the Earth density value you wrote in Table 8.3? Think before writing your answer! Explain your answer. **(5 points)**

3. If the Las Cruces to Boulder, Colorado distance was actually 200 km in length, but your measured fall times did not change from what you measured, would you have calculated a larger or smaller Earth density value? Explain the reasoning for your answer. **(3 points)**

4. If we had conducted this experiment on the Moon rather than here on the Earth, would your measured values (fall time, angles and angle difference between two locations separated north-south by 857 kilometers) be the same as here on Earth, or different? Clearly explain your reasoning. [It might help if you draw a circle representing Earth and then draw a circle with $1/4^{\text{th}}$ of the radius of the Earth's circle to represent the Moon.] **(5 points)**

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## 8.7 Take Home Exercise (35 points total)

1. Type a 1.5-2 page Lab Report in which you will address the following topics:

   a) The estimated density value you arrived at was likely different from the actual Earth density value of 5.52 grams per cubic centimeter; describe 2 or 3 potential errors in your measurements that could possibly play a role in generating your incorrect estimated density value.

   b) Describe 2-3 ways in which you could improve the measurement techniques used in lab; keep in mind that NMSU is a state-supported school and thus we do not have infinite resources to purchase expensive sophisticated equipment, so your suggestions should not be too expensive.

   c) Describe what you have learned from this lab, what aspects of the lab surprised you, what aspects of the lab worked just as you thought they would, etc.

## 8.8 Possible Quiz Questions

1. What is meant by the "radius" of a circle? (Drawing ok)
2. What does the term "circumference of a circle" mean?
3. How do you calculate the circumference of a circle if given the radius?
4. What is "pi" (or $\pi$)? What is the value of pi?
5. What is the volume of a sphere?
6. What does the term "density" mean?

## 8.9 Extra Credit (ask your TA for permission before attempting, 5 points)

Astronomers use density to segregate the planets into categories, such as "Terrestrial" and "Jovian". Using your book, or another reference, look up the density of the Sun and Jupiter (or, if you have completed the previous lab, use the data table you constructed for Take-Home portion of that lab). Compare the densities of the Sun and Jupiter. Do you think they are composed of same elements? Why/why not? What are the two main elements in the periodic table that dominate the composition of the Sun? If the material that formed the Sun (and the Sun *has* 99.8% of the mass of the solar system) was the original "stuff" from which all of the planets were formed, how did planets like Earth end up with such high densities? What do you think might have happened in the distant past to the lighter elements? (Hint: think of a helium balloon, or a glass of water thrown out onto a Las Cruces parking lot in the summer!).