Measuring Distances Using Parallax

Name:________________________________
Date:_________________________________

1 Introduction

How do astronomers know how far away a star or galaxy is? Determining the distances to the objects they study is one of the the most difficult tasks facing astronomers. Since astronomers cannot simply take out a ruler and measure the distance to any object, they have to use other methods. Inside the solar system, astronomers can simply bounce a radar signal off of a planet, asteroid or comet to directly measure the distance to that object (since radar is an electromagnetic wave, it travels at the speed of light, so you know how fast the signal travels–you just have to count how long it takes to return and you can measure the object's distance). But, as you will find out in your lecture sessions, some stars are hundreds, thousands or even tens of thousands of "light years" away. A light year is how far light travels in a single year (about 9.5 trillion kilometers). To bounce a radar signal of a star that is 100 light years away would require you to wait 200 years to get a signal back (remember the signal has to go out, bounce off the target, and come back). Obviously, radar is not a feasible method for determining how far away stars are.

In fact, there is one, and only one direct method to measure the distance to a star: "parallax". Parallax is the angle that something appears to move when the observer looking at that object changes their position. By observing the size of this angle and knowing how far the observer has moved, one can determine the distance to the object. Today you will experiment with parallax, and appreciate the small angles that astronomers must measure to determine the distances to stars.

To introduce you to parallax, perform the following simple experiment:

Hold your thumb out in front of you at arm's length and look at it with your left eye closed. Now look at it with your right eye closed. As you look at your thumb, alternate which eye you close several times. You should see your thumb move relative to things in the background. Your thumb is not moving but your <u>point of view</u> is moving, so your thumb *appears* to move.

- *Goals:* to discuss the theory and practice of using parallax to find the distances to nearby stars, and use it to measure the distance to objects in the classroom
- *Materials:* classroom "ruler", worksheets, ruler, protractor, calculator, small object

2 Parallax in the classroom

The "classroom parallax ruler" will be installed/projected on one side of the classroom. For the first part of this lab you will be measuring motions against this ruler.

Now work in groups: have one person stand at one wall and have the others stand somewhere in between along the line that goes straight to the other wall, and hold up a skewer or a pencil at the right height so the observer (person against the wall) can see it against the background ruler. The observer should blink his/her eyes and measure the number of lines on the background ruler against which the object appears to move. **Note that you can estimate the motion measurement to a fraction of tick mark, e.g., your measurement might be 2 1/2 tick marks).** Do this for three different distances, with the closest being within a few feet of the observer, and the furthest being at most half the classroom away. Leave the object (or a mark) where the object was located each of the three times, so you can go back later and measure the distances. Switch places and do it again. Each person should estimate the motion for each of the three distances.

How many tick marks did the object move at the closest distance? **(2 points)**:

How many tick marks did the object move at the middle distance? **(2 points)**:

How many tick marks did the object move at the farthest distance? **(2 points)**:

What is your estimate of the *uncertainty* in your measurement of the apparent motion? For example, do you think your recorded measurements could be off by half a tick mark? a quarter of a tick mark? You might wish to make the measurement several times to get some estimate of the reliability of your measurements. **(2 points)**

The apparent motion of the person against the background ruler is what we are calling parallax. It is caused by looking at an object from two different vantage points, in this case, the difference in the location of your two eyes. Qualitatively, what do you see? As the object gets farther away, is the apparent motion smaller or larger?

What if the vantage points are further apart? For example, imagine you had a huge head and your eyes were a foot apart rather than several inches apart. What would you predict for the apparent motion? Try the experiment again, this time using the object at one of the distances used above, but now measuring the apparent motion by using just one eye, but moving your whole head a few feet from side to side to get more widely separated vantage points.

How many tick marks did the person move from the more widely separated vantage points? __________________________ For an object at a fixed distance, how does the apparent motion change as you observe from more widely separated vantage points?

3 Measuring distances using parallax

We have seen that the apparent motion depends on both the distance to an object and also on the separation of the two vantage points. We can then turn this around: if we can measure the apparent motion and also the separation of the two vantage points, we should be able to infer the distance to an object. This is very handy: it provides a way of measuring a distance without actually having to go to an object. Since we can't travel to them, this provides the only direct measurement of the distances to stars.

We will now see how parallax can be used to **determine the distances to the objects you looked at** just based on your measurements of their apparent motions and a measurement of the separation of your two vantage points (your two eyes).

3.1 Angular motion of an object

How can we measure the apparent motion of an object? As with our background ruler, we can measure the motion as it appears against a background object. But what are the appropriate units to use for such a measurement? Although we can measure how far apart the lines are on our background ruler, the apparent motion is not really properly measured in a unit of length; if we had put our parallax ruler further away, the apparent motion would have been the same, but the number of tick marks it moved would have been larger.

The apparent motion is really an *angular* motion. As such, it can be measured in *degrees*, with 360 degrees in a circle.

Figure out the angular separation of the tick marks on the ruler as seen from the opposite side of the classroom. Do this by putting one eye at the origin of one of the tripod-mounted protractors and measuring the angle from one end of the background ruler to the other end of the ruler. You might lay a pencil from your eye at the origin of the protractor toward each end and use this to measure the the total angle. Divide this angle by the total number of tick marks to figure out the angle for each tick mark.

Number of degrees for the entire background ruler: ________________________

Number of tick marks in the whole ruler: ________________________

Number of degrees in each tick mark: ________________________

Convert your measurements of apparent motion in tick marks above to angular measurements by multiplying the number of tick marks by the number of degrees per tick mark:

How many degrees did the object appear to move at the closest distance? **(2 points)**:

How many degrees did the object appear to move at the middle distance? **(2 points)**:

How many degrees did the object appear to move at the farthest distance? **(2 points)**:

Based on your estimate of the uncertainty in the number of tick marks each object moved, what is your estimate of the uncertainty in the number of degrees that each object moved?

(2 points)

3.2 Distance between the vantage points

Now you need to measure the distance between the two different vantage points, in this case, the distance between your two eyes. Have your partner measure this with a ruler. Since you see out of the pupil part of your eyes, you want to measure the distance between the centers of your two pupils.

What is the distance between your eyes? **(2 points)**________________________

3.3 Using parallax measurements to determine the distance to an object

To determine the distance to an object for which you have a parallax measurement, you can construct an imaginary triangle between the two different vantage points and the object, as shown in Figure 1.

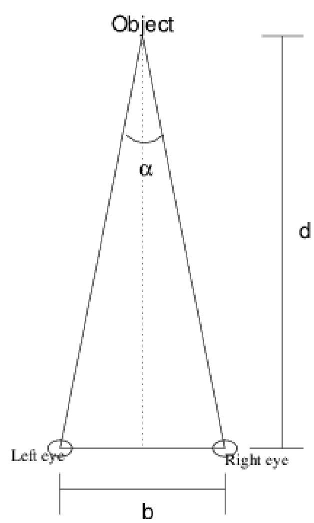

Figure 1: Parallax triangle

The angles you have measured correspond to the angle α on the diagram, and the distance between the vantage points (your pupils) corresponds to the distance b on the diagram.

The distance to the object, which is what you want to figure out, is d.

The three quantities b, d, and α are related by a trigonometric function called the *tangent*. Now, you may have never heard of a *tangent*, if so don't worry–we will show you how to do this using another easy (but less accurate) way! But for those of you who are familiar with a little basic trigonometry, here is how you find the distance to an object using parallax: If you split your triangle in half (dotted line), then the tangent of $(\alpha/2)$ is equal to the quantity $(b/2)/d$:

$$\tan\left(\frac{\alpha}{2}\right) = \frac{(b/2)}{d}$$

Rearranging the equation gives:

$$d = \frac{(b/2)}{\tan(\alpha/2)}$$

You can determine the tangent of an angle using your calculator by entering the angle and then hitting the button marked *tan*. There are several other units for measuring angles besides degrees (for example, radians), so you have to **make sure that your calculator is set up to use degrees** for angles before you use the tangent function.

The "Non-Tangent" way to figure out distances from angles

Because the angles in astronomical parallax measurement are very small, astronomers do not have to use the *tangent* function to determine distances from angles–they use something called the "small angle approximation formula":

$$\frac{\theta}{57.3} = \frac{(b/2)}{d}$$

In this equation, we have defined $\theta = \alpha/2$, where α is the same angle as in the earlier equations (and in Fig. 1). Rearranging the equation gives:

$$d = \frac{57.3 \times (b/2)}{\theta}$$

To use this equation your parallax angle "θ" has to be in degrees. (If you are interested in where this equation comes from, talk to your TA, or look up the definition of "radian" and "small angle formula" on the web.) Now you can proceed to the next step!

Combine your measurements of angular distances and the distance between the vantage points to determine the three different distances to your partner. The units of the distances which you determine will be the same as the units you used to measure the distance between your eyes; if you measured that in inches, then the derived distances will be in inches.

Distance when object was at closest distance: **(2 points)**________________________

Distance when object was at middle distance: **(2 points)**________________________

Distance when object was at farthest distance: **(2 points)**________________________

Based on your estimate of the uncertainty in the angular measurements and also on the uncertainty of your measurement of the separation of your eyes, estimate the uncertainty in your measurements of the distances to the objects. To do so, you might wish to redo the calculation allowing each of your measurements to change by your estimated errors. **(2 points)**

Now go and measure the actual distances to the locations of the objects using a yardstick, meterstick, or tape measure. How well did the parallax distances work? Are the differences between the actual measurements and your parallax measurements within your estimated errors? If not, can you think of any reasons why your measurements might have some additional error in them? **(5 points)**

4 Using Parallax to measure distances on Earth, and within the Solar System

We just demonstrated how parallax works in the classroom, now lets move to a larger scale then the classroom. *Using the small angle formula*, and your eyes, what would be the parallax angle (in degrees) for Organ Summit, the highest peak in the Organ mountains, if the Organ Summit is located 12 miles (or 20 km) from this classroom? [Hint: there are 5280 feet in a mile, and 12 inches in a foot. There are 1,000 meters in a km.]: **(2 points)**

You should have gotten a tiny angle! The smallest angle that the best human eyes can resolve is about 0.02 degrees. Obviously, our eyes provide an inadequate baseline for measuring this large of a distance. How can we get a bigger baseline? Well surveyors use a "transit" to carefully measure angles to a distant object. A transit is basically a small telescope mounted on a (fancy!) protractor. By locating the transit at two different spots separated by 100 yards (and carefully measuring this baseline!), they can get a much larger parallax angle, and thus it is fairly easy to measure the distances to faraway trees, mountains, buildings or other large objects.

How about an object in the Solar System? We will use Mars, the planet that comes closest to Earth. At favorable oppositions, Mars gets to within about 0.4 AU of the Earth. Remember, 1 AU is the average distance between the Earth and Sun: 149,600,000 km. Calculate the parallax angle for Mars (using the small angle approximation) using a baseline of 1000 km. **(2 points)**

Ouch! Also a very small angle.

5 Distances to stars using parallax, and the "Parsec"

Because stars are very far away, the parallax motion will be very small. For example, the nearest star is about 1.9×10^{13} miles or 1.2×10^{18} inches away! At such a tremendous distance, the apparent angular motion is very small. Considering the two vantage points of your two eyes, the angular motion of the nearest star corresponds to the apparent diameter of a human hair seen at the distance of the Sun! This is a truly tiny angle and totally unmeasurable by your eye.

Like a surveyor, we can improve our situation by using two more widely separated vantage points. The two points farthest apart we can use from Earth is to use two opposite points in the Earth's orbit about the Sun. In other words, we need to observe a star at two different times separated by six months. The distance between our two vantage points, b, will then be twice the distance between the Earth and the Sun: "2 AU". Figure 2 shows the idea.

Using 299.2 million km as the distance b, we find that the apparent angular motion (α) of even the nearest star is only about 0.0004 degrees. This is also unobservable using your naked eye, which is why we cannot directly observe parallax by looking at stars with our naked eye. However, this angle is relatively easy to measure using modern telescopes and instruments.

Time to talk about a new distance unit, the "Parsec". Before we do so, we have to review the idea of smaller angles than degrees. Your TA or professor might already have mentioned

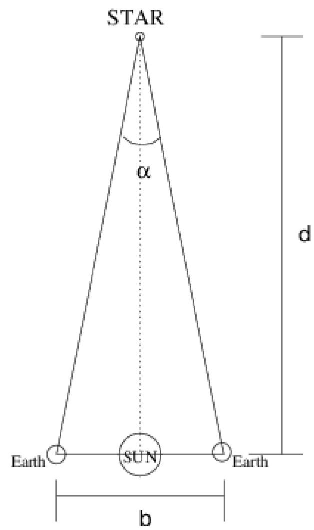

Figure 2: Parallax Method for Distance to a Star

that a degree can be broken into 60 arcminutes. Thus, instead of saying the parallax angle is 0.02 degrees, we can say it is 1.2 arcminutes. But note that the nearest star only has a parallax angle of 0.024 arcminutes. We need to switch to a smaller unit to keep from having to use scientific notation: the arcsecond. There are 60 arcseconds in an arcminute, thus the parallax angle (α) for the nearest star is 1.44 arcseconds. To denote arcseconds astronomers append a single quotation mark (") at the end of the parallax angle, thus $\alpha = 1.44$" for the nearest star. But remember, in converting an angle into a distance (using the tangent or small angle approximation) we used the angle $\alpha/2$. So when astronomers talk about the parallax of a star they use this angle, $\alpha/2$, which we called "θ" in the small angle approximation equation.

How far away is a star that has a parallax angle of $\theta = 1$"? The answer is 3.26 light years, and this distance is defined to be "1 Parsec". The word Parsec comes from **Par**allax **Sec**ond. An object at 1 Parsec has a parallax of 1". An object at 10 Parsecs has a parallax angle of 0.1". Remember, the further away an object is, the smaller the parallax angle. The nearest star (Alpha Centauri) has a parallax of $\theta = 0.78$", and is thus at a distance of $1/\theta = 1/0.78$ = 1.3 Parsecs. Depending on your professor, you might hear the words Parsec, kiloparsec, Megaparsec and even Gigaparsec in your lecture classes. These are just shorthand methods of talking about distances in astronomy. A kiloparsec is 1,000 Parsecs, or 3,260 light years. A Megaparsec is one million parsecs, and a Gigaparsec is one billion parsecs. To convert to light years, you simply have to multiply by 3.26. The Parsec is a strange unit, but you have already encountered other strange units this semester!

Let's work some examples:

- If a star has a parallax angle of $\theta = 0.25$", what is its distance in Parsecs? **(1 point)**

- If a star is at a distance of 5 Parsecs, what is its parallax angle? **(1 point)**

- If a star is at a distance of 5 Parsecs, how many light years away is it? **(1 point)**

6 Questions

1. How does the parallax angle change as an object is moved further away? Given that you can usually only measure an angular motion to some accuracy, would it be easier to measure the distance to a nearby star or a more distant star? Why? **(4 points)**

2. Relate the experiment you did in lab to the way parallax is used to measure the distances to nearby stars in astronomy. Describe the process an astronomer has to go through in order to determine the distance to a star using the parallax method. What do your two eyes represent in that experiment? **(5 points)**

3. Imagine that you observe a star field twice one year, separated by six months and observe the configurations of stars shown in Figure 3:

   The star marked P appears to move between your two observations because of parallax. So you can consider the two pictures to be like our lab experiment where the left picture is what is seen by one eye and the right picture what is seen by the other eye. All the stars except star P do not appear to change position; they correspond to the background ruler in our lab experiment. If the angular distance between stars A and

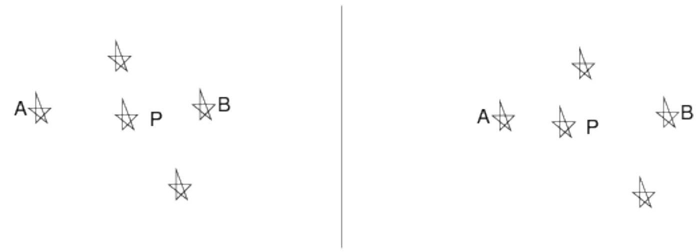

Figure 3: Star field seen at two times of year six months apart.

B is 0.5 arcminutes (remember, 60 arcminutes = 1 degree), then how far away would you estimate that star P is? Proceed by estimating the amount that star P moves between the two pictures relative to the distance between stars A and B. This gives you the apparent angular motion. You also know the distance between the two vantage points (which is the Earth at two opposite sides of its orbit) from the number given above). You can then use the parallax equation to estimate the distance to star P. **(11 points)**

4. Imagine that you did the classroom experiment by putting your partner all the way against the far wall. How big would the apparent motion be relative to the tick marks? What would you infer about the distance to your partner? Why do you think this estimate is incorrect? What can you infer about where the background objects in a parallax experiment need to be located? **(7 points)**

7 Summary (35 points)

Please summarize the important concepts discussed in this lab. Your summary should include:

- A brief description on the basic principles of parallax and how astronomers can use parallax to determine the distance to nearby stars

Also think about and answer the following questions:

- Does the parallax method work for all stars we can see in our Galaxy and why?
- Why do you think it is important for astronomers to determine the distances to the stars which they study?

Use complete sentences, and proofread your summary before handing in the lab.

8 Possible Quiz Questions

1) How do astronomers measure distances to stars?
2) How can astronomers measure distances inside the Solar System?
3) What is an Astronomical Unit?
4) What is an arcminute?
5) What is a Parsec?

9 Extra Credit (ask your TA for permission before attempting, 5 points)

Use the web to find out about the planned GAIA Mission. What are the goals of GAIA? How accurately can it measure a parallax? Discuss the units of milliarcseconds ("mas") and microarcseconds. How much better is GAIA than the best ground-based parallax measurement programs?