Clustering of galaxies

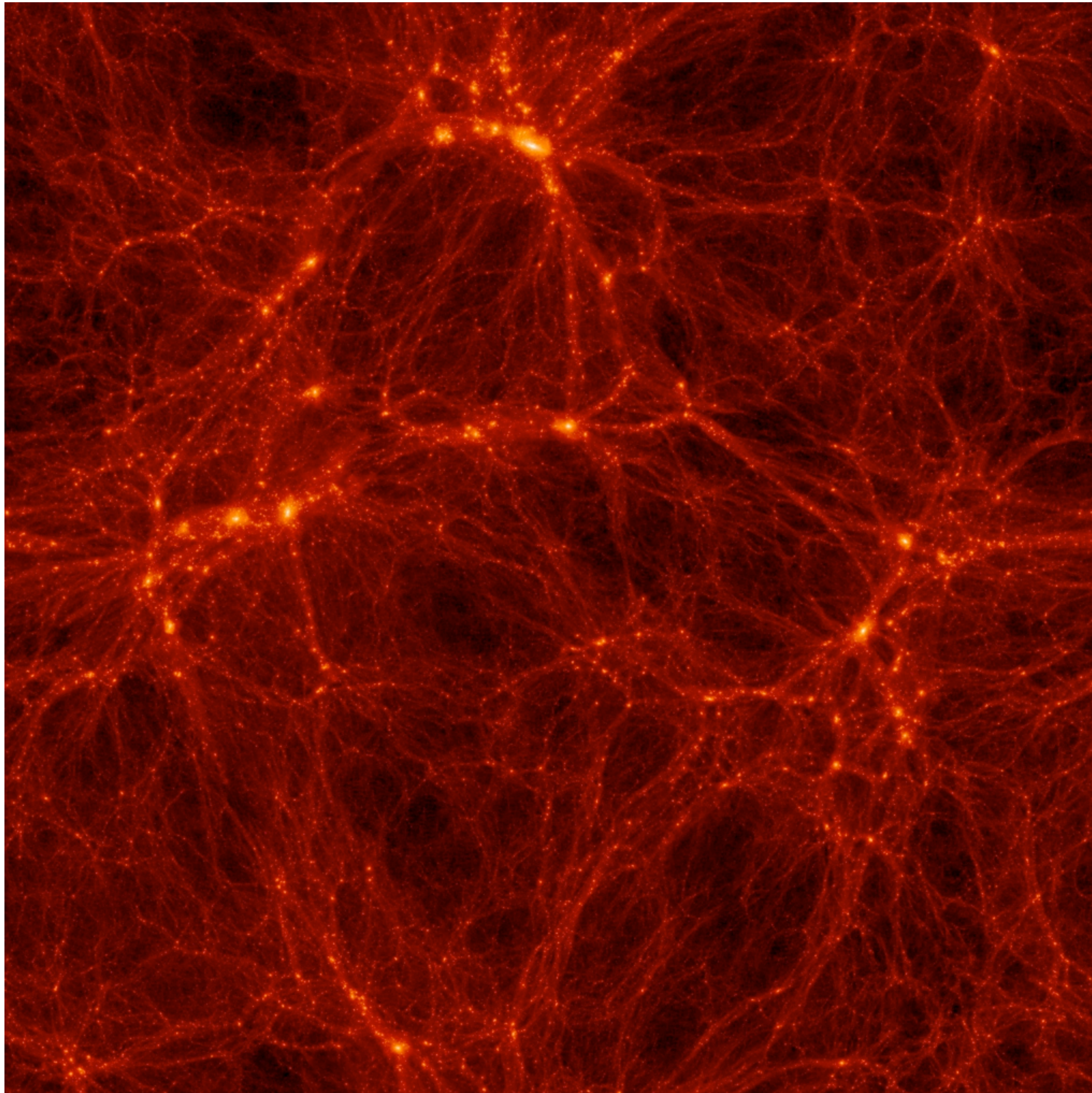
- Notions: Comoving coordinates, peculiar velocities, redshift space
- Correlation function: definitions and methods of estimation
- Power Spectrum
- Effects: redshift distortions
- Biases and biasing
- Observations

Distribution of mass
in supergalactic plane

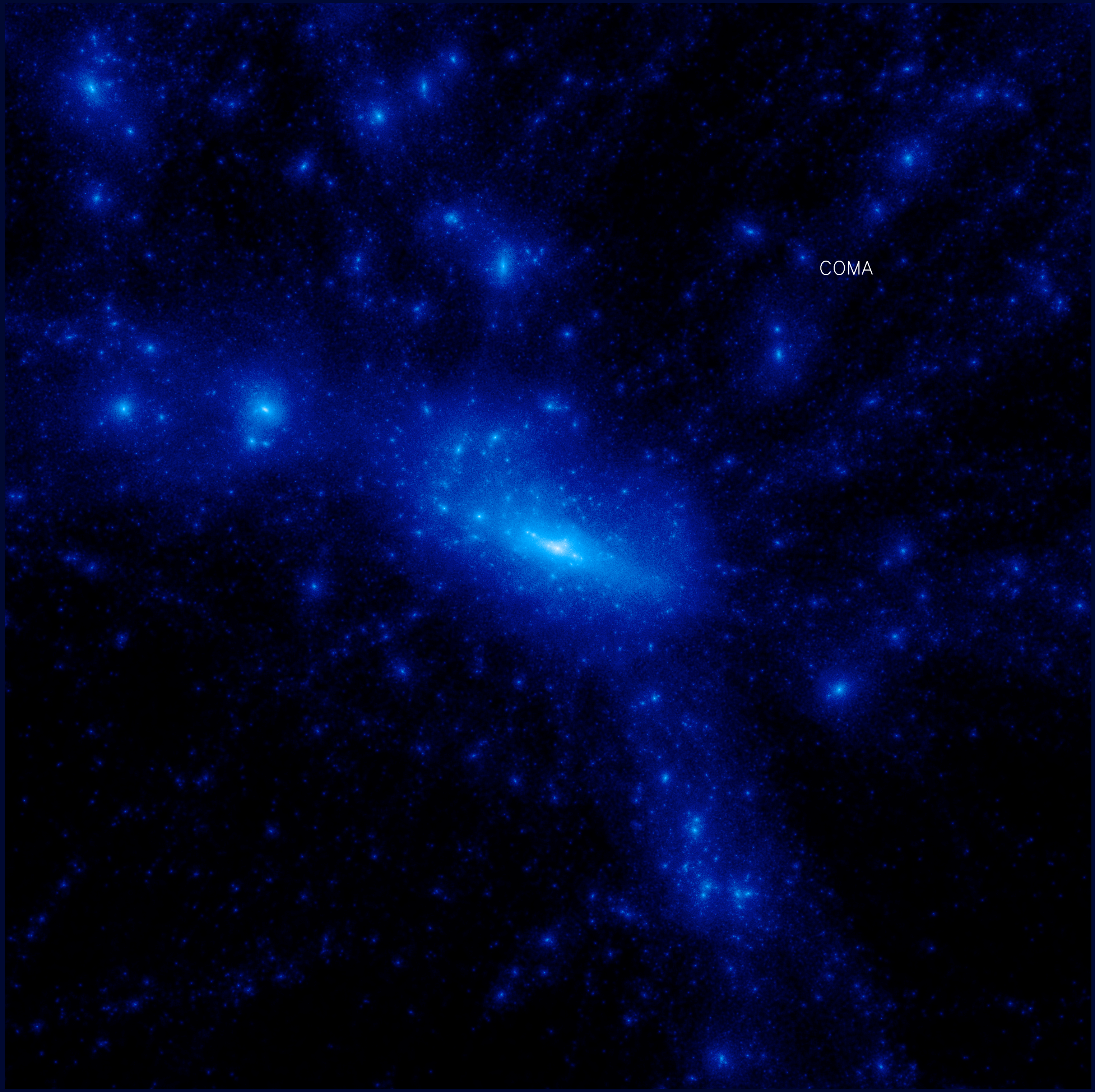
Long waves are taken from observations. Short waves are added according the standard LCDM model

Our Galaxies is at the center of this picture

Size of the box is 160Mpc/h

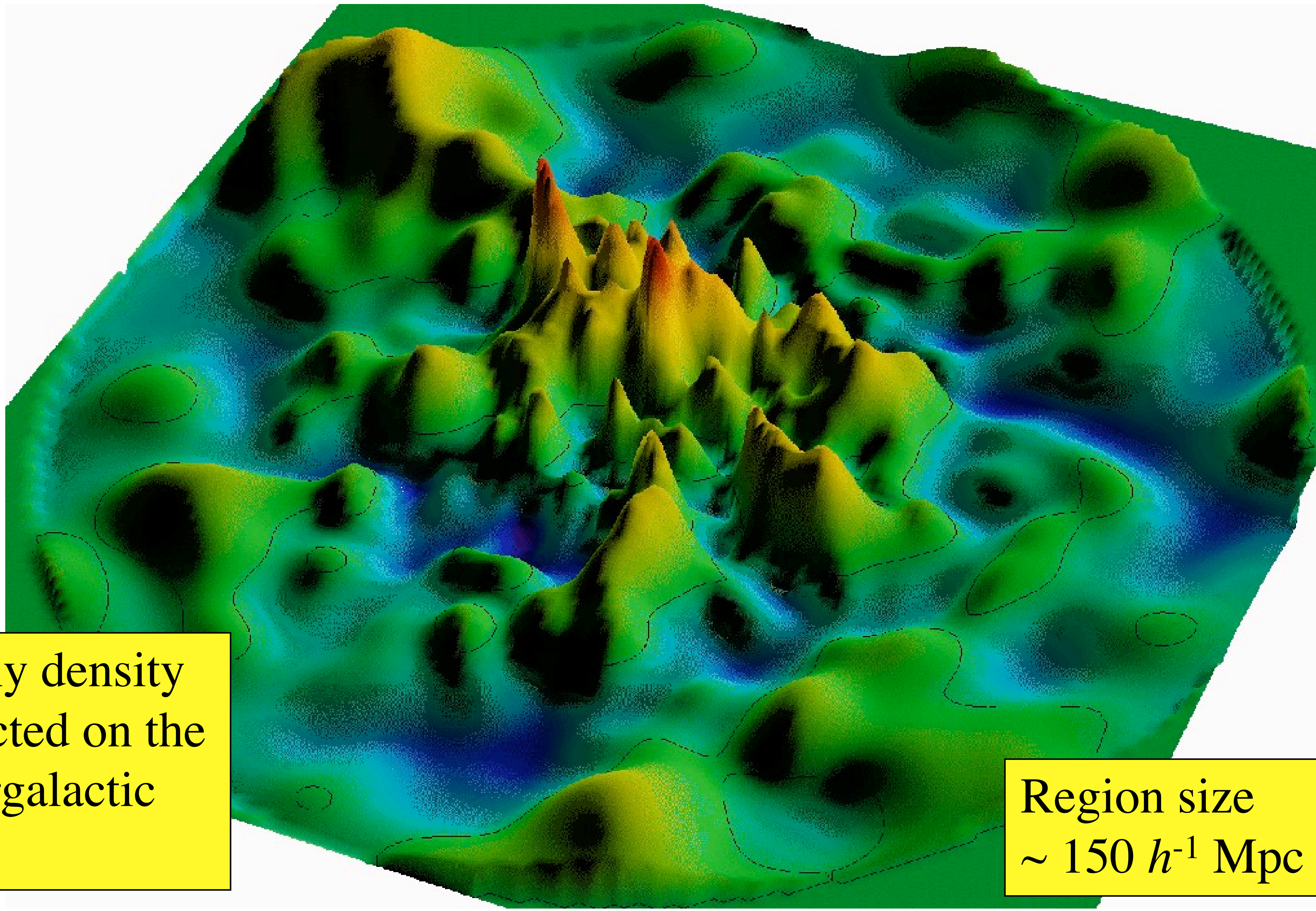


Possible Environment of the Coma cluster



30Mpc frame
Cosmological simulations

The PSCz Redshift Survey: One of several redshift surveys which used the IRAS satellite (IR) selected galaxy catalogs, in order to avoid the extinction effects



Galaxy density projected on the Supergalactic plane

Region size ~ 150 h^{-1} Mpc

$\vec{r} = a(t)\,\vec{x}$

r is radius-vector

x is comoving coordinates

a(t) is the expansion factor:

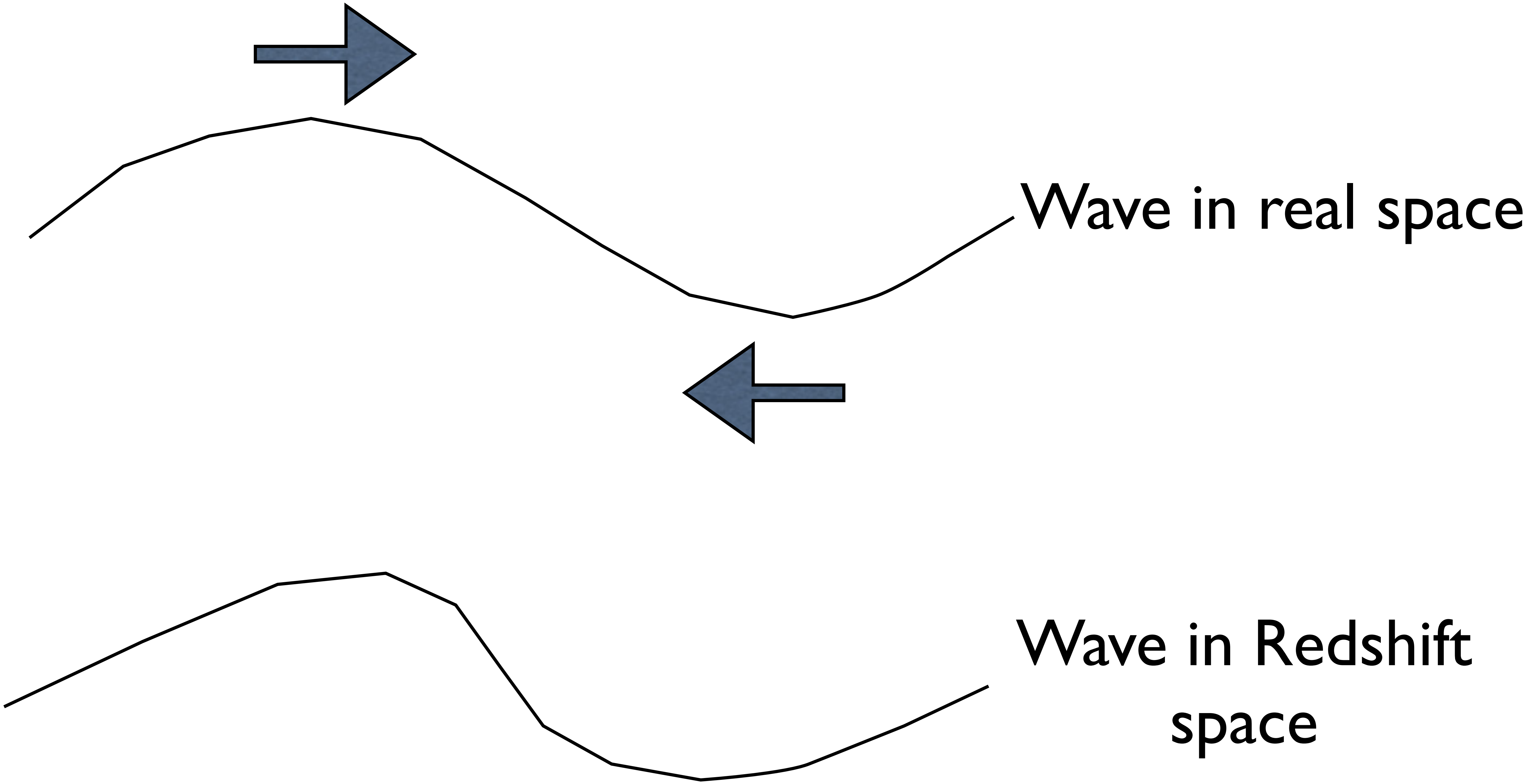
$$a(t) = \frac{1}{1+z}$$

$$\dot{r} = \dot{a}(t)\,x + a(t)\,\dot{x} = H(t)r + v_{pec}$$

$R = V_{obs}/H$ is the redshift-space coordinate

v_{pec} is the peculiar velocity

Redshift distortions: long waves



Correlation function: definition

Suppose there are N objects in a volume V. The number of objects dN expected in a small volume element dV is simply:

$$dN = ndV, \tag{1}$$

where n is the number-density of objects: $n = N/V$. Note that we can call dN the expected average number of objects for a randomly selected volume element in space. If dV is sufficiently small, than dN is less than unity and we can treat dN as a probability to find an object in a dV element.

Now we make the situation more complicated. Let's select an object and ask question of finding another object at a distance R in a volume element dV. If positions of objects are not correlated (random), than the answer is the same: ndV. However, the objects may be correlated or anti-correlated. In this case we can write the probability as:

$$dN = n(1 + \xi(R))dV, \tag{2}$$

where $\xi(R)$ is called the correlation function. If $\xi(R) = 0$, we have uncorrelated distribution of objects in space. If it is positive, than there is excess probability to find an object at distance R.

Correlation function: definition

The correlation function is defined as the a quantity averaged over all objects. We take the first object and find $dN_1(R)$ for that object. Take the second object and do the same. Continue for all objects. Now average $dN_i(R)$ to find $\xi(R)$:

$$\xi(R) = \langle \frac{dN_i}{dV} \rangle - 1. \tag{3}$$

This gives us another definition of the correlation function: the correlation function is the average number-density profile of objects in excess of random around a randomly selected object.

Correlation function: definition

Another – equivalent – definition of the correlation function follows from counting pairs of objects with separations $(R, R+dR)$. We start with joint probability to find an object in a volume dV_1 and another in dV_2. It can be written as:

$$dN = n^2(1 + \xi(R))dV_1 dV_2, \tag{4}$$

This definition is easy to convert to algorithm to find the correlation function. Count the number of pairs of objects with given separation $(R, R + dR)$. Now estimate the same quantity for randomly placed objects. We call the first quantity $\langle(DD)\rangle$ (the term DD stands for data-data pairs). Now randomly place in space the same number of objects and count their pairs, which call RR for random-random pairs. The correlation function is:

$$\xi(R) = \frac{\langle DD\rangle}{\langle RR\rangle} - 1. \tag{5}$$

Correlation function: definition

This is usually quantified using the ***2-point correlation function, ξ (r),*** defined as an "excess probability" of finding another galaxy at a distance ***r*** from some galaxy, relative to a uniform random distribution; averaged over the entire set:

$$dN(r) = \rho_0 \left(1 + \xi(r)\right) dV_1 dV_2$$

Correlation function is often approximated with a power law:

$$\xi(r) = \left(r / r_0\right)^{-\gamma}$$

Parameter r_0 is called the correlation length

Estimators of the correlation function

- Simplest estimator: count the number of data-data pairs, $\langle DD \rangle$, and the equivalent number in a randomly generated (Poissonian) catalog, $\langle RR \rangle$:

$$\xi(r)_{est} = \frac{\langle DD \rangle}{\langle RR \rangle} - 1$$

- A better (Landy-Szalay) estimator is:

$$\xi(r)_{est} = \frac{\langle DD \rangle - 2\langle RD \rangle + \langle RR \rangle}{\langle RR \rangle}$$

where $\langle RD \rangle$ is the number of data-random pairs

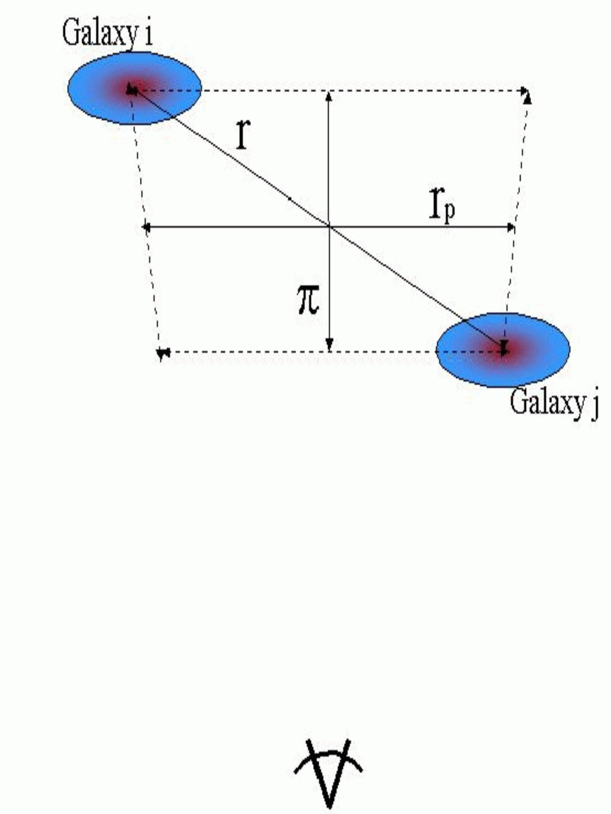
- This takes care of the edge effects, where one has to account for the missing data outside the region sampled, which can have fairly irregular boundaries

Angular and 3D correlation functions

$$w(r_p) = 2\int_{r_p}^{\infty} \xi(r)\left(r^2 - r_p^2\right)^{-1/2} dr$$

r_p: projected distance between pairs of galaxies,

π: distance parallel to the line of sight



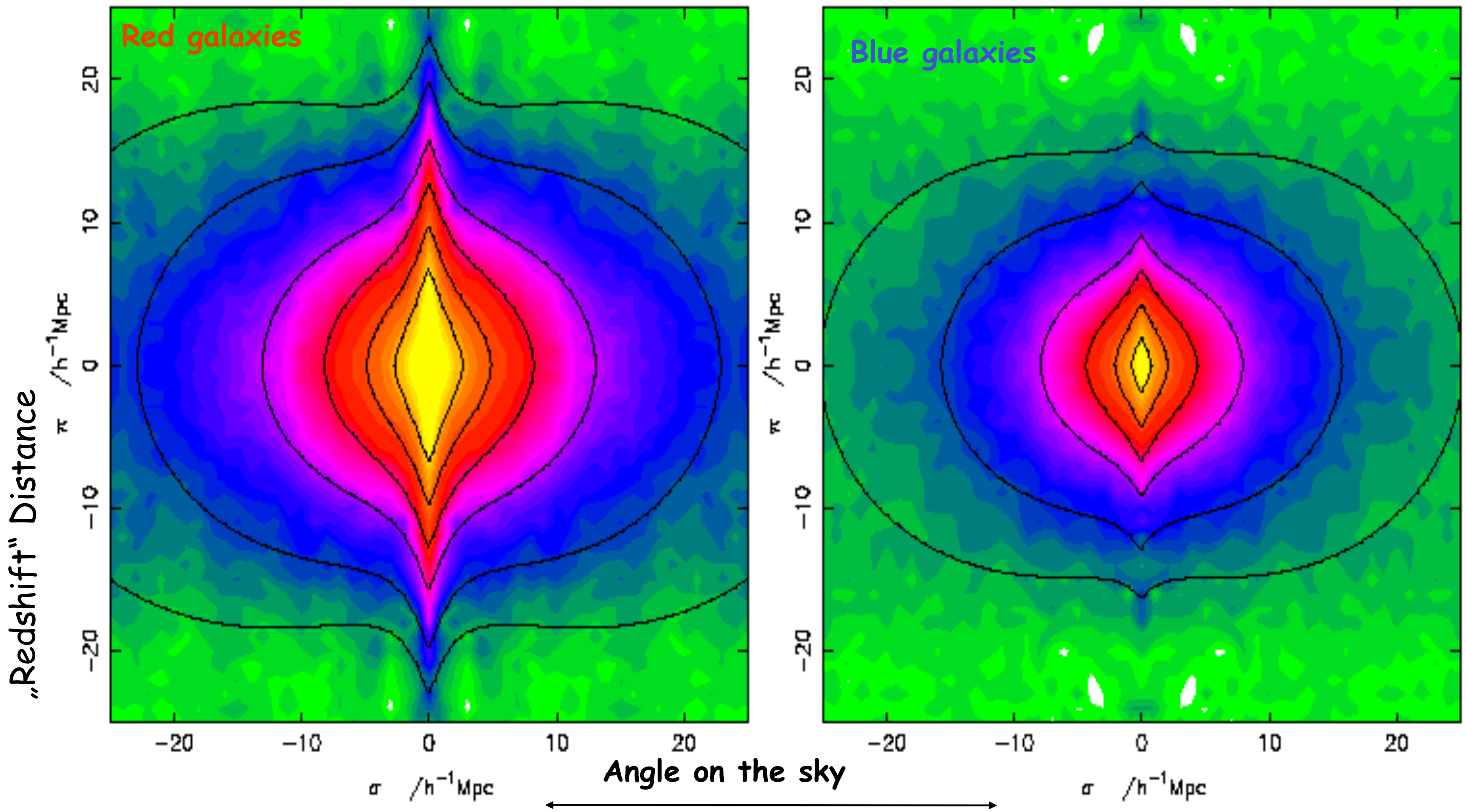
$$w(r_p) = \int_{-\delta\pi}^{+\delta\pi} \xi(r_p, \pi)\, d\pi$$

Inverting angular correlation function

$$w_p(r_p) = 2\int_0^\infty dy\,\xi\left[\left(r_p^2 + y^2\right)^{1/2}\right] = 2\int_{r_p}^\infty r\,dr\,\xi(r)\left(r^2 - r_p^2\right)^{-1/2} \tag{3}$$

$$\xi(r) = -\frac{1}{\pi}\int_r^\infty w_p'(r_p)(r_p^2 - r^2)^{-1/2} dr_p$$

Redshift distortions: 'finger-of-god' effect on small scales



Angular correlation function $w_p(r_p)$

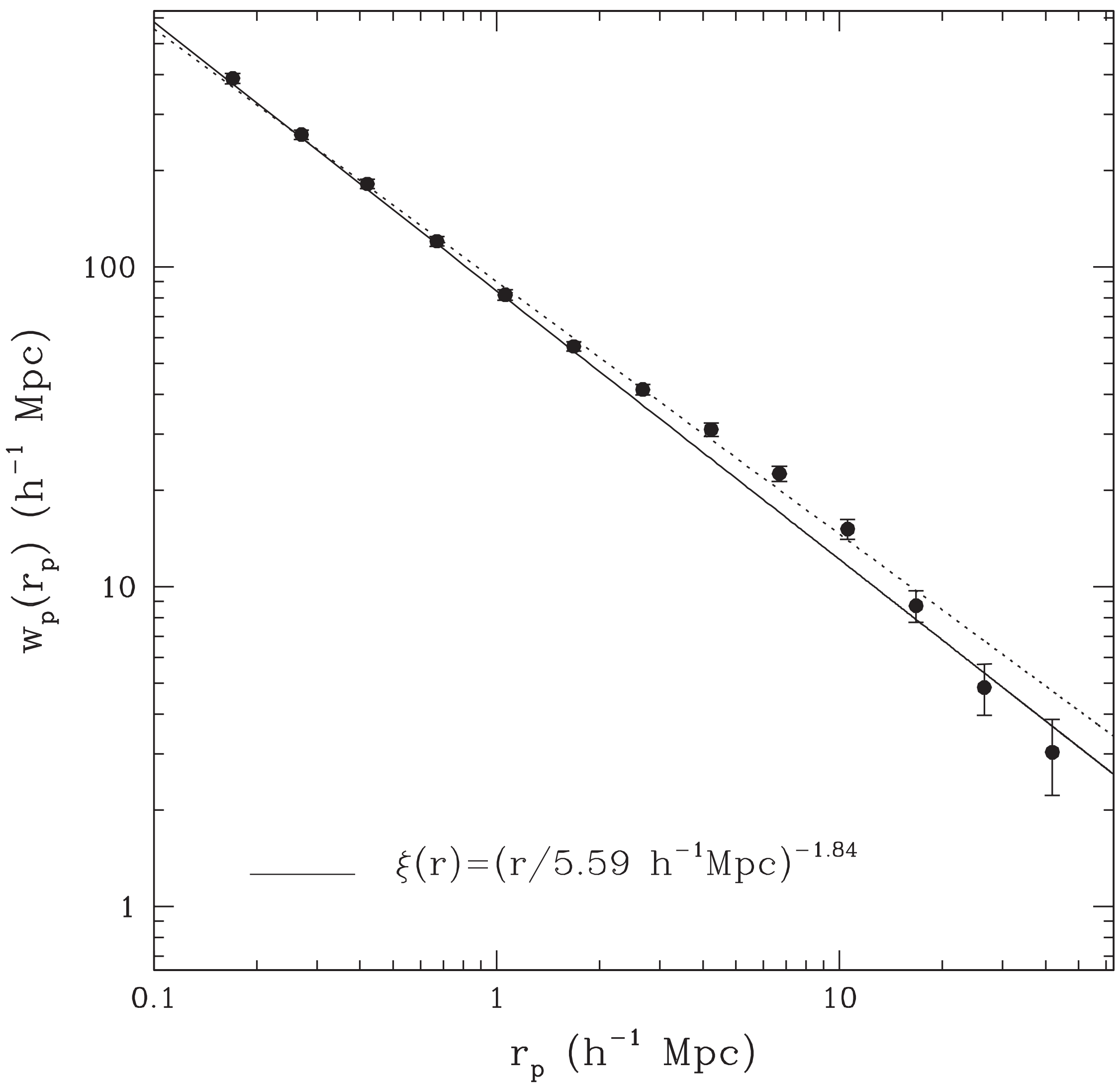


FIG. 6.—Projected galaxy correlation function $w_p(r_p)$ for the flux-limited galaxy sample. The solid line shows a power-law fit to the data points, using the full covariance matrix, which corresponds to a real-space correlation function $\xi(r) = (r/5.59\ h^{-1}\ \mathrm{Mpc})^{-1.84}$. The dotted line shows the fit when using only the diagonal error elements, corresponding to $\xi(r) = (r/5.94\ h^{-1}\ \mathrm{Mpc})^{-1.79}$. The fits are performed for $r_p < 20\ h^{-1}$ Mpc.

If only 2-D positions on the sky are known, then use angular separation $\boldsymbol{\theta}$ instead of distance $\boldsymbol{r}$:

$$\boldsymbol{w(\theta) = (\theta/\theta_0)^{-\beta}, \quad \beta = \gamma - 1}$$

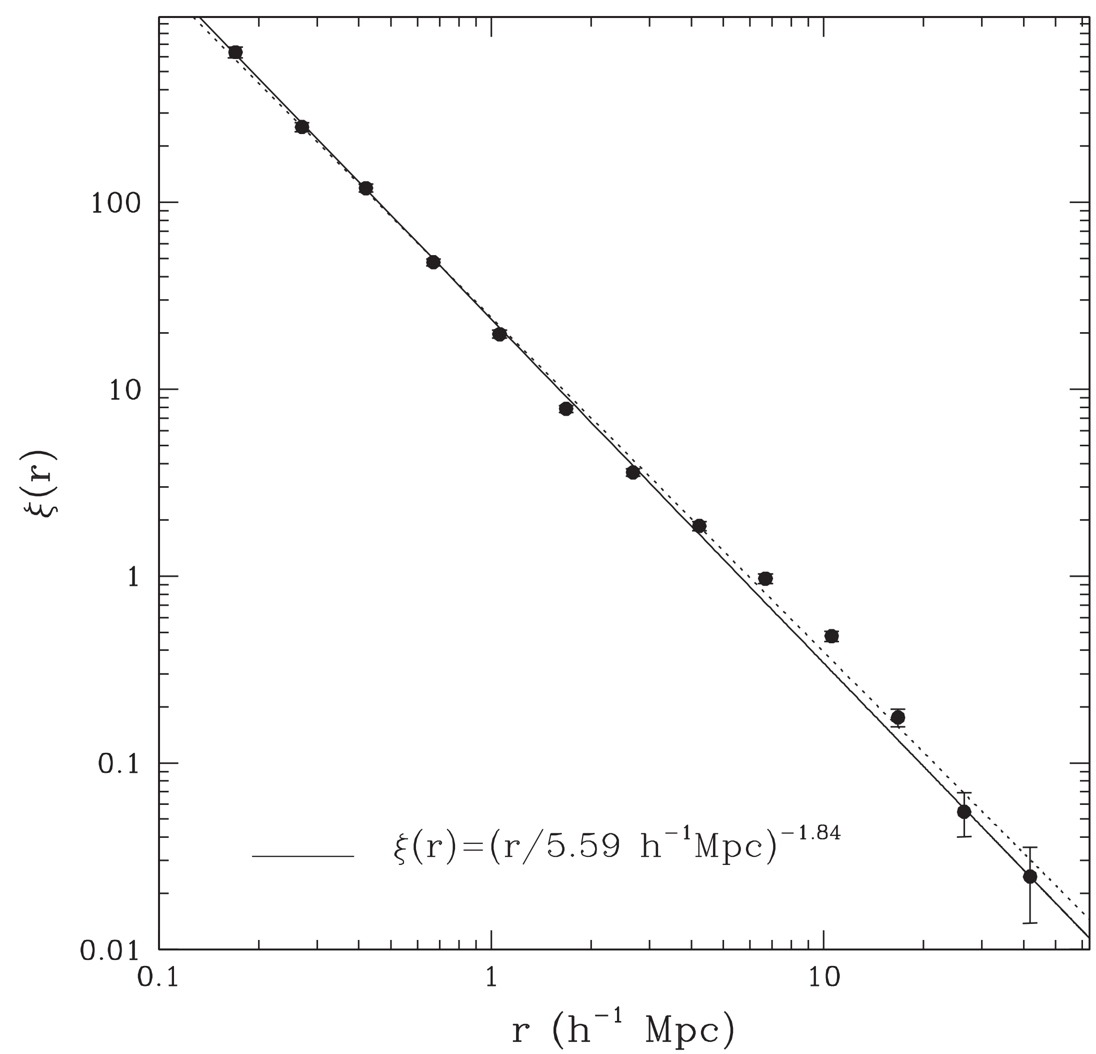


FIG. 7.—Real-space correlation function $\xi(r)$ for the flux-limited galaxy sample, obtained from $w_p(r_p)$ as discussed in the text. The solid and dotted lines show the corresponding power-law fits obtained by fitting $w_p(r_p)$ using the full covariance matrix or just the diagonal elements, respectively.

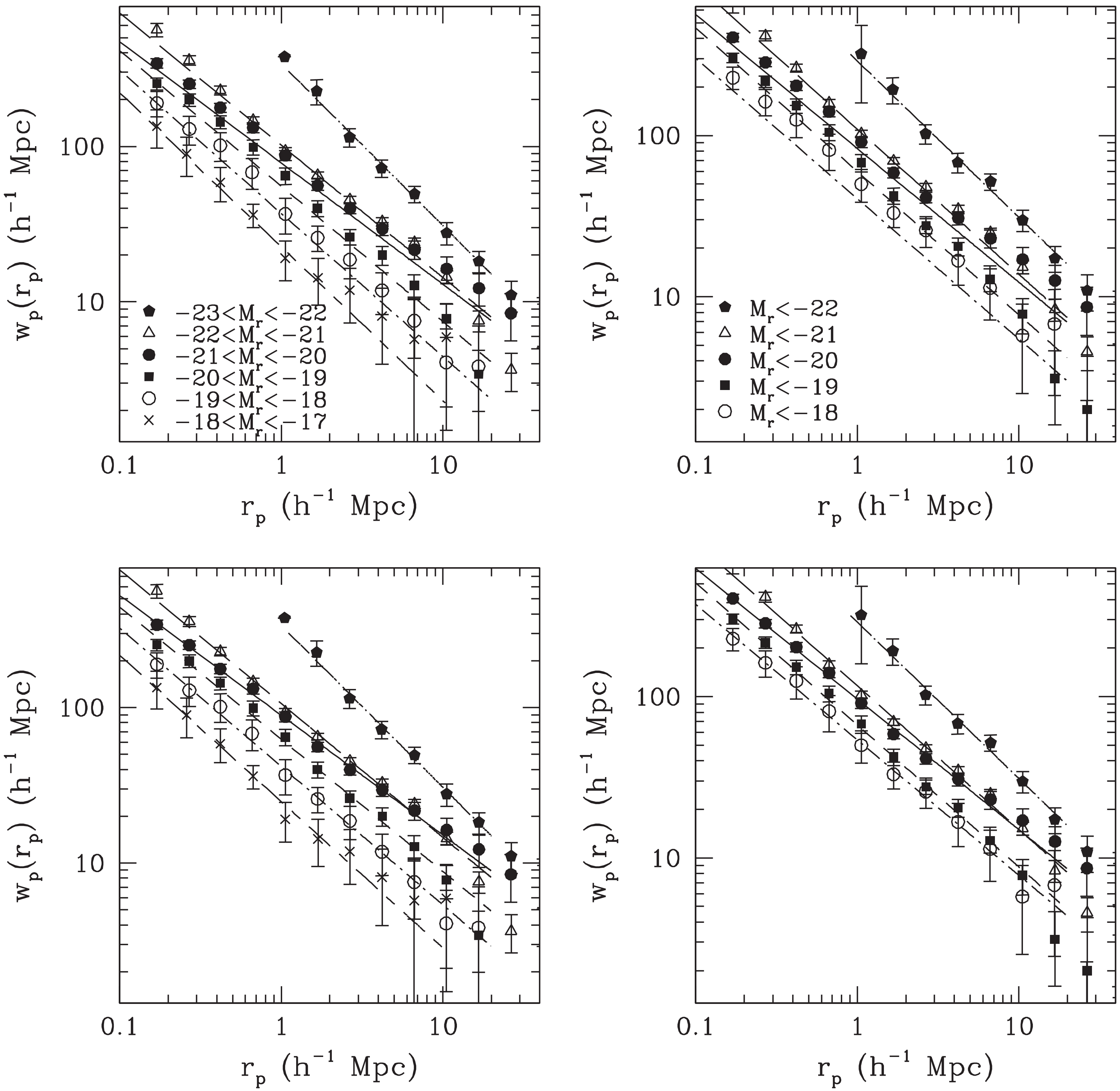
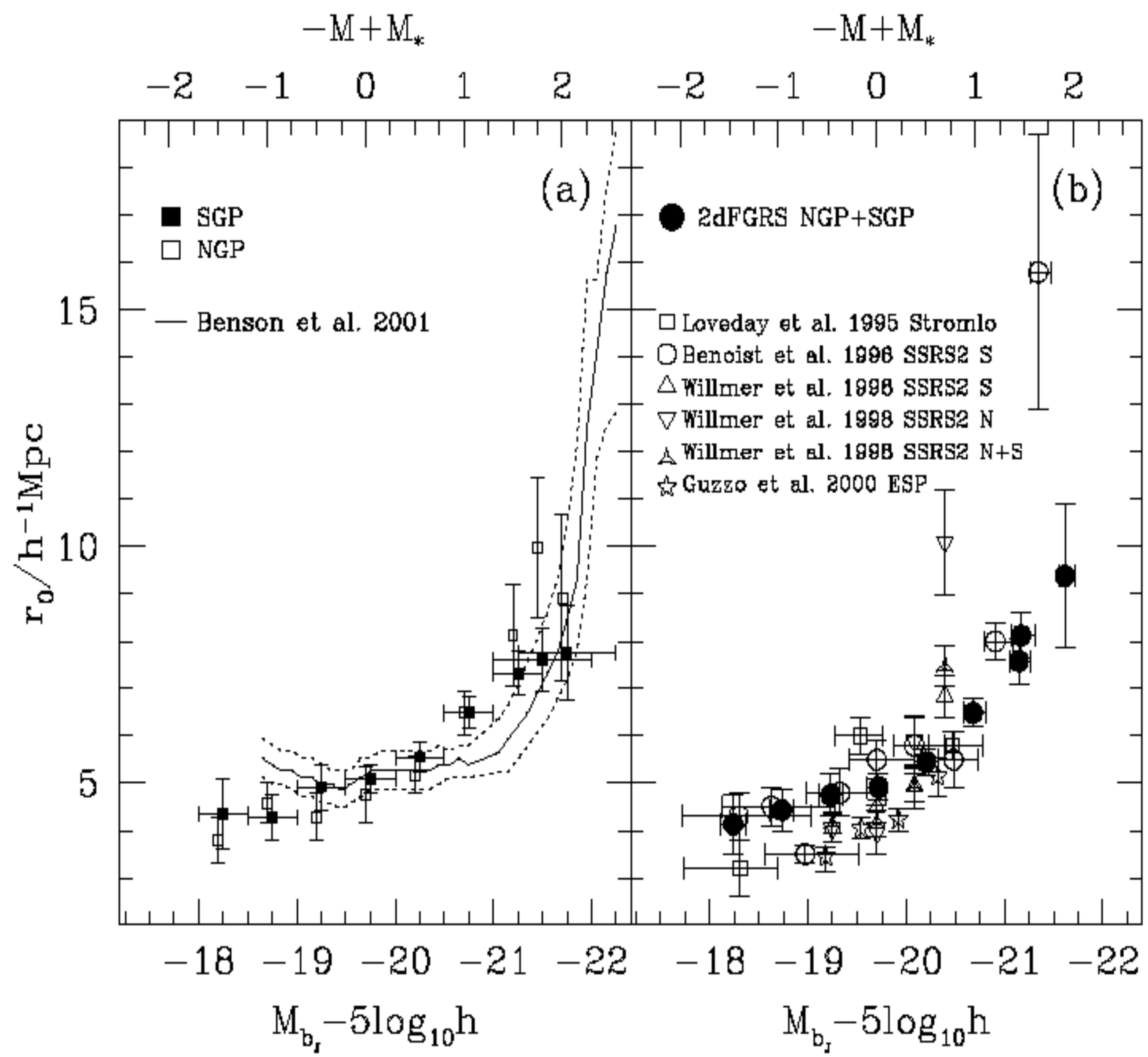


FIG. 8.—*Top left*: Projected galaxy correlation functions $w_p(r_p)$ for volume-limited samples with the indicated absolute magnitude and redshift ranges. Lines show power-law fits to each set of data points, using the full covariance matrix. *Top right*: Same as top left, but now the samples contain all galaxies brighter than the indicated absolute magnitude; i.e., they are defined by luminosity thresholds rather than luminosity ranges. *Bottom panels*: Same as the top panels, but now with power-law fits that use only the diagonal elements of the covariance matrix. [*See the electronic edition of the Journal for a color version of this figure.*]

Clustering of different galaxies

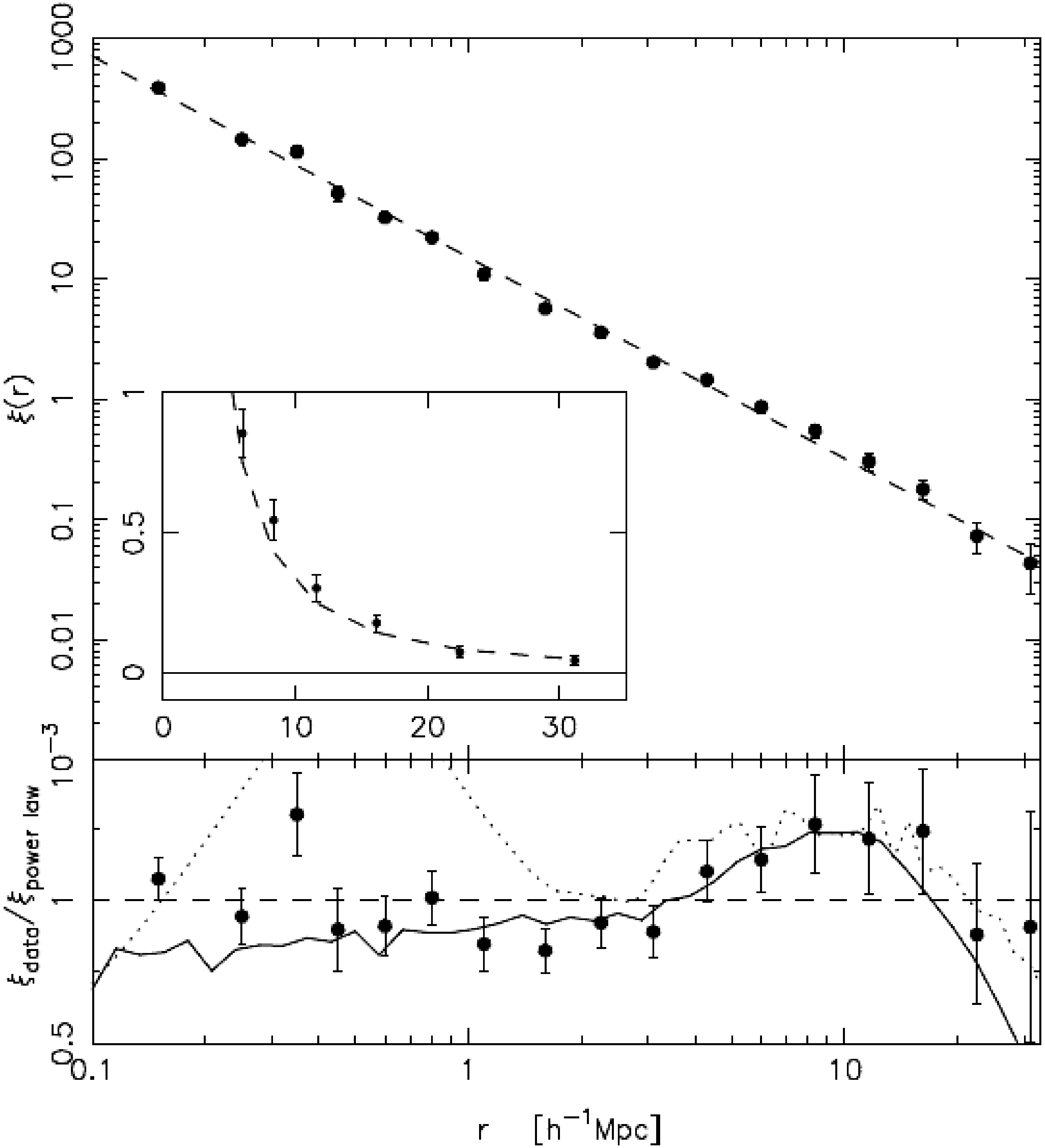


More luminous/massive galaxies are more strongly clustered

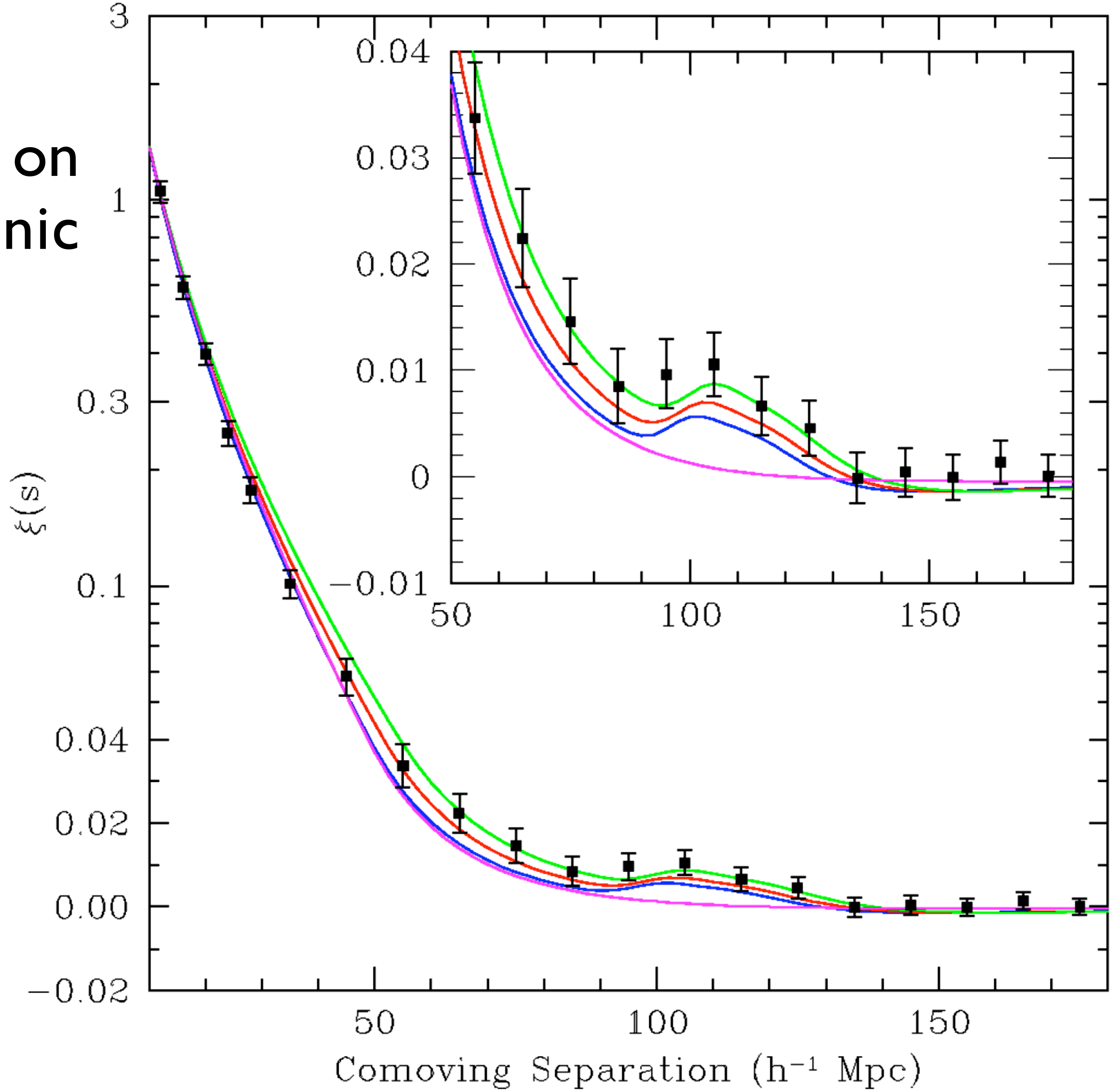
Galaxy Correlation Function

As measured by the 2dF redshift survey

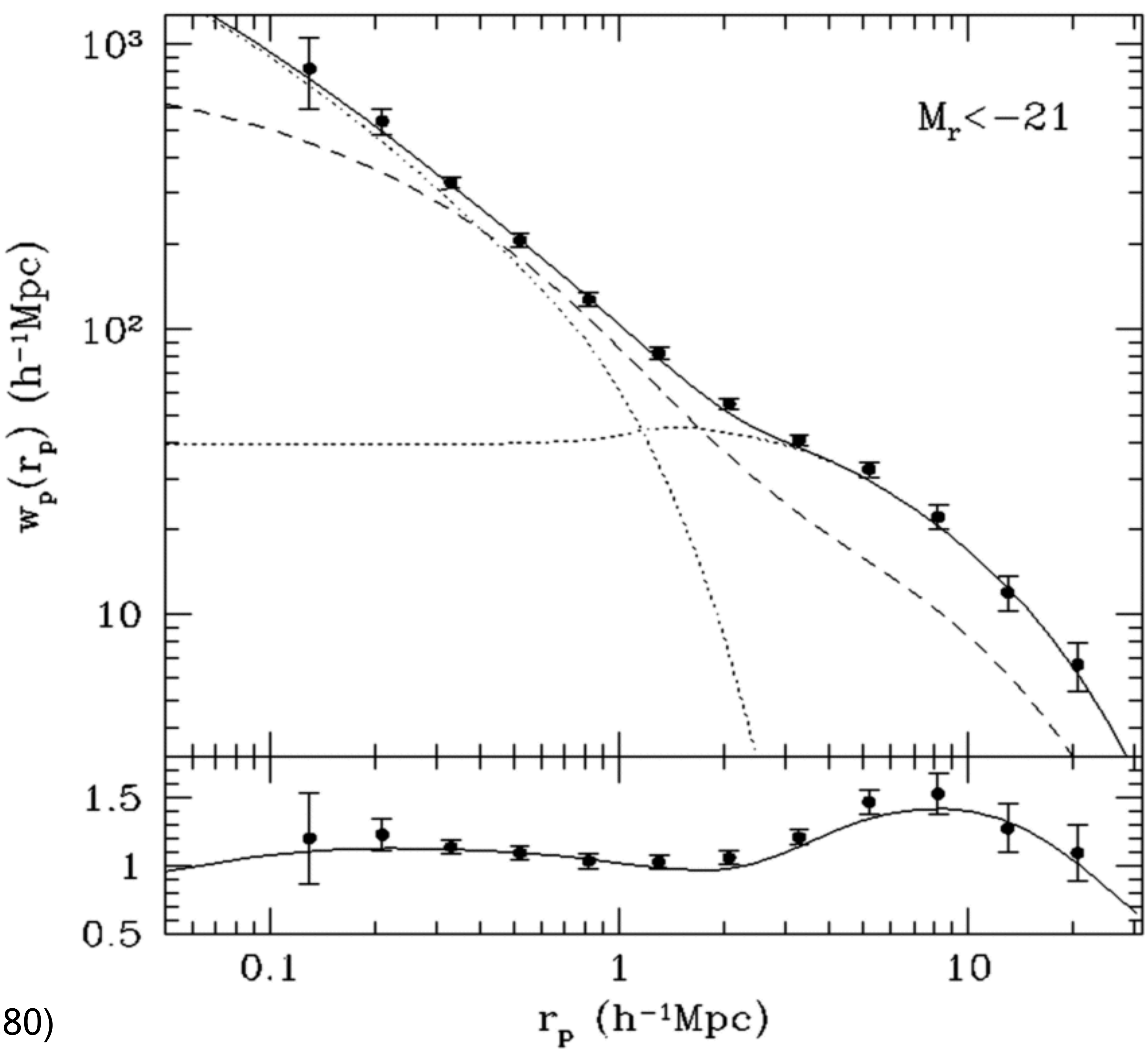
Deviations from the power law:



Correlation function on large scales: baryonic oscillations

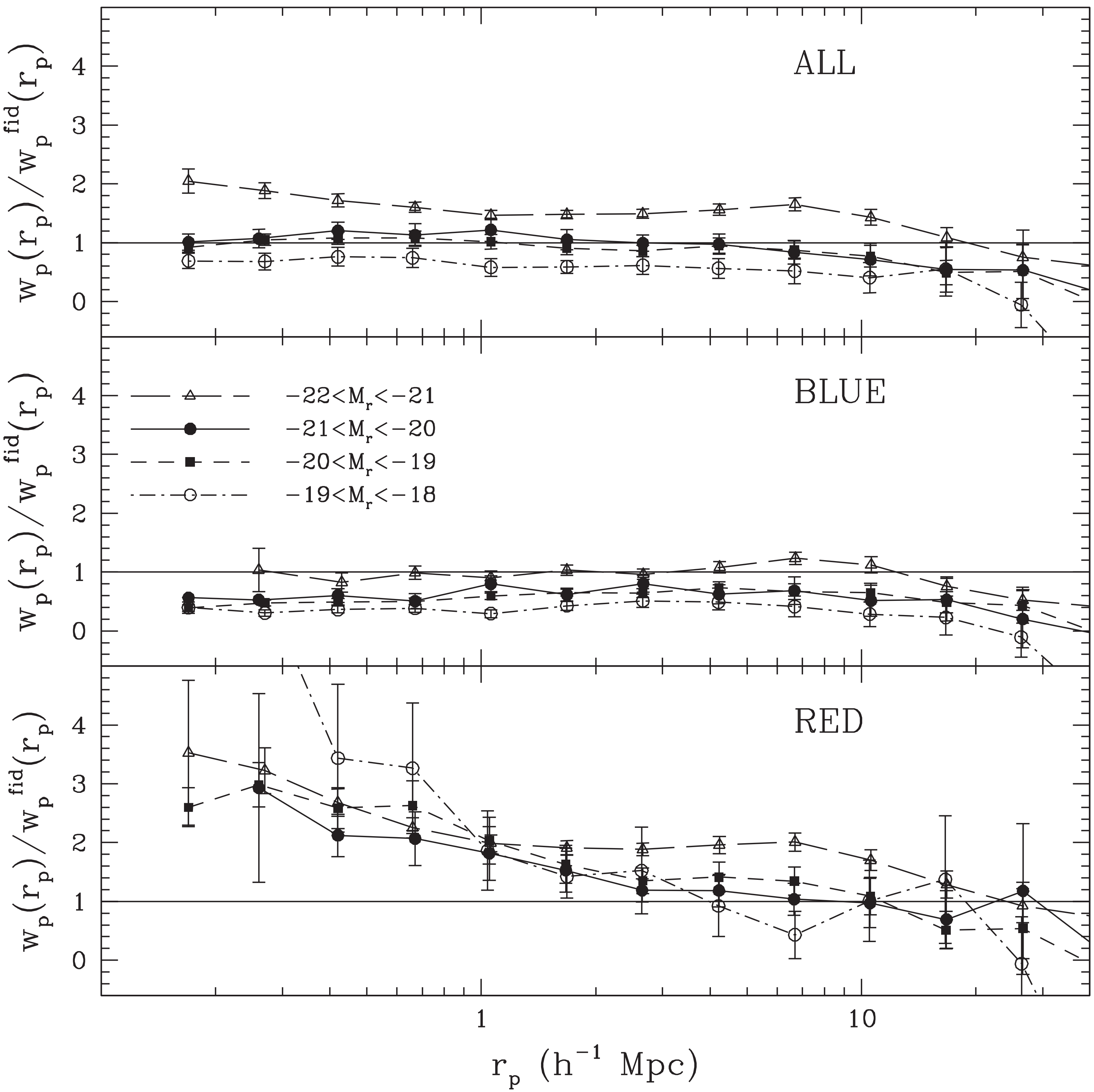


SDSS (Eisenstein et al.)



Zehavi et al. (astro-ph/0301280)

Clustering: galaxy morphology



Bias $b^2 = W(r, sample1)/W(r,sample2)$

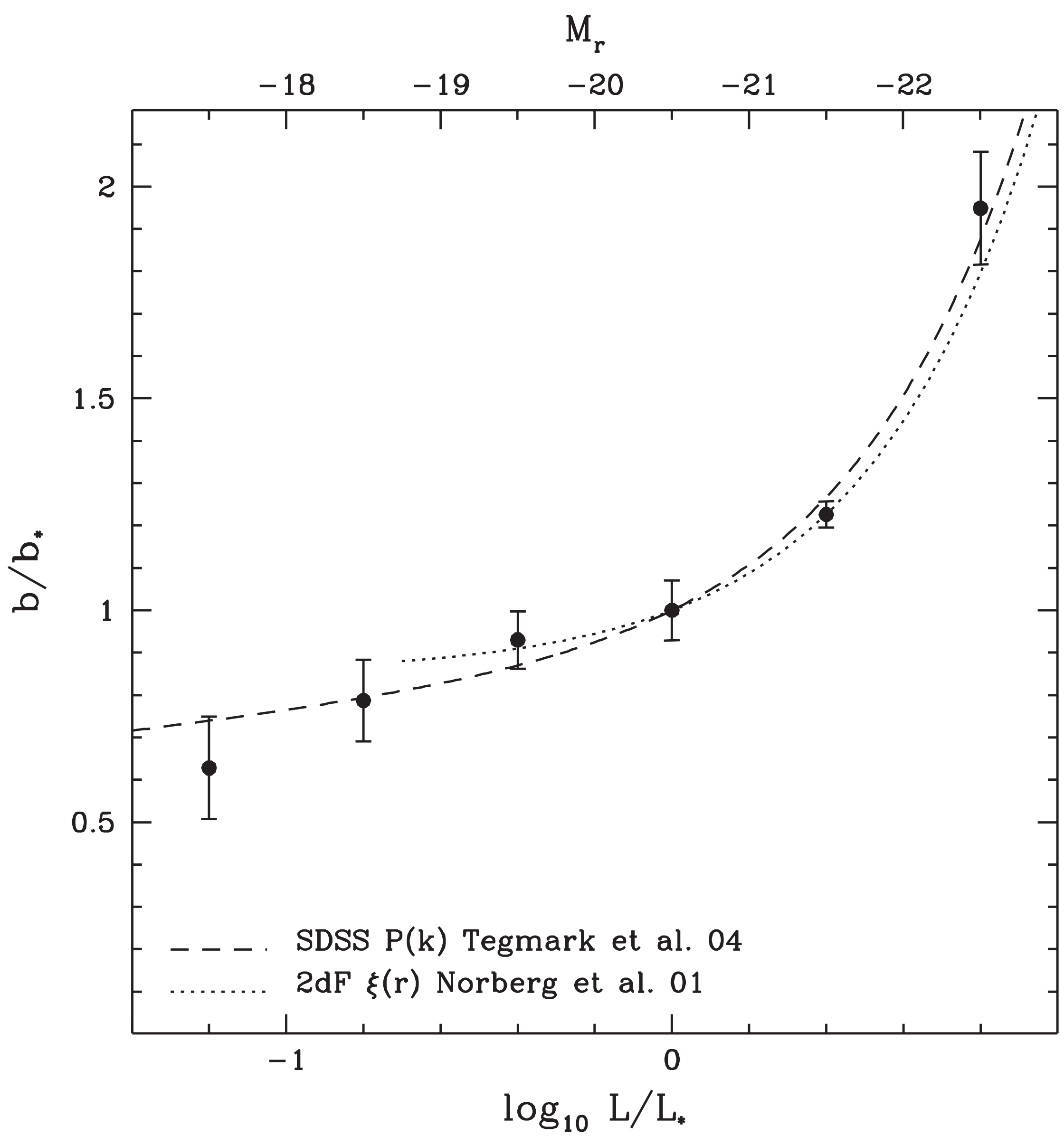


FIG. 11.—Relative bias factors for samples defined by luminosity ranges. Bias factors are defined by the relative amplitude of the $w_p(r_p)$ estimates at a fixed separation of $r_p = 2.7\ h^{-1}$ Mpc and are normalized by the $-21 < M_r < -20$ sample ($L \approx L_*$). The dashed curve is a fit obtained from measurements of the SDSS power spectrum, $b/b_* = 0.85 + 0.15L/L_* - 0.04(M - M_*)$ (Tegmark et al. 2004a), and the dotted curve is a fit to similar $w_p(r_p)$ measurements in the 2dF survey, $b/b_* = 0.85 + 0.15L/L_*$ (Norberg et al. 2001).

Power Spectrum

δ(k) is the Fourier amplitude

$$P(\mathbf{k}) = \left|\delta(\mathbf{k})\right|^2$$

- Naïve estimator for a discrete density field is

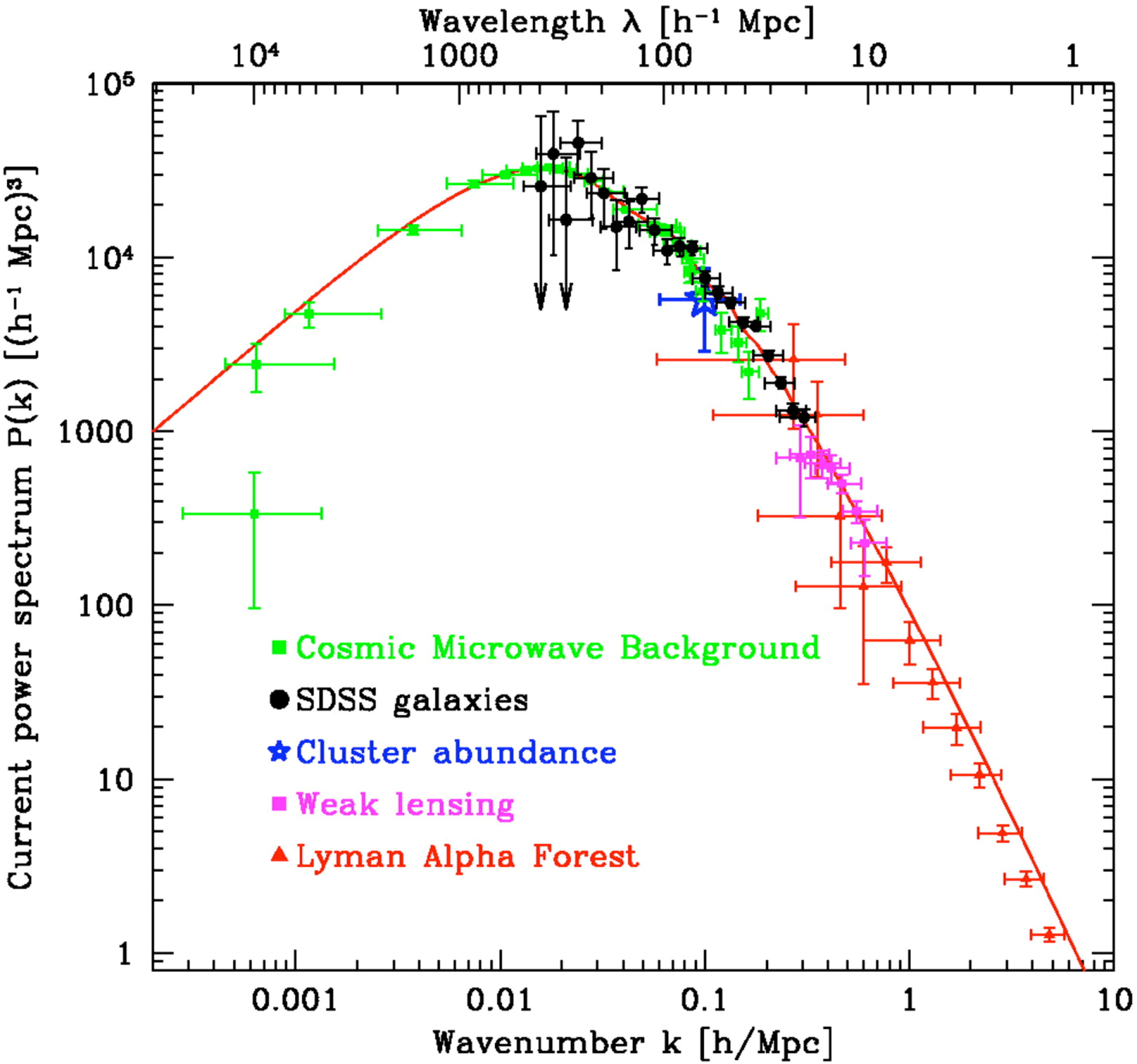
$$\hat{f}(\mathbf{k}) = \frac{1}{N}\sum_n e^{i\mathbf{k}\mathbf{r}_n}$$

- We need to take into account (1) selection function ϕ(r) and shot noise w(k)

$$\hat{f}(\mathbf{k}) = \sum_n \phi(\mathbf{r}_n) e^{i\mathbf{k}\mathbf{r}_n} - w(\mathbf{k})$$

$$\phi(\mathbf{r}) = \frac{\overline{n}(r)}{1 + \overline{n}(r) P(k)}$$

The Observed Power Spectrum



(Tegmark et al.)