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Big Bang Nucleosynthesis

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1 THE THERMAL HISTORY OF THE UNIVERSE

At very early epochs of the Universe interactions between different particles we so frequent that particles can be considered being in nuclear equilibrium at each moment of time. In equilibrium the rate of creation of particles is equal to the rate of their destruction. For example, if m_e is the mass of electron, then for moments when $kT \gg m_e c^2$ the abundance of electrons and positrons is kept in equilibrium by reactions $e^- + e^+ \Longleftrightarrow \gamma + \gamma$.

For photos in equilibrium the mean number per unit volume per mode is given by the occupation numbers:

$$\mathcal{N} = \frac{1}{e^{\hbar\omega/kT} - 1}. \tag{1}$$

In order to find the number of photons with different frequencies in a cubic volume $V = L^3$ we count all photons (plane waves) with wavenumbers:

$$\vec{k} = \frac{2\pi}{L}\vec{n}, \quad \vec{n} = \{n_x, n_y, n_z\}. \tag{2}$$

Here $\vec{n} = \{n_x, n_y, n_z\}$ is a triplet of integer numbers. The number of photons in volume V with $\vec{k}$ in the range d^3k is a product (occupation number)x(number of modes)x(number of polarization states):

$$L^3 n(\omega) d\omega = 2 \cdot \mathcal{N} \cdot \frac{L^3}{(2\pi)^3} = \frac{L^3}{\pi^2 c^3} \frac{\omega^2 d\omega}{e^{\hbar\omega/kT} - 1}. \tag{3}$$

Here we used $k = \omega/c$ and $d^3k = 4\pi k^2 dk$. This is the Planck formula:

$$n(\omega) = \frac{\omega^2}{\pi^2 c^3 (e^{\hbar\omega/kT} - 1)}. \tag{4}$$

This can be integrated over all frequencies to give the total number of photonsn_γ per unit volume:

$$n_\gamma = \frac{2\zeta(3)}{\pi^2}\frac{(kT)^3}{(\hbar c)^3} = 414\left(\frac{T}{2.726K}\right)^3. \tag{5}$$

Here $\zeta(x)$ is the Riemann function: $\zeta(3) \approx 1.202$. The specific entropy of radiation is $s = (4/3)a_\gamma T^3$. Thus the entropy of radiation and the number of photons are proportional one to the other.

For $t > 1sec$ when $kT < 1\,\mathrm{GeV}$ the comoving number-number density of baryons (protons and neutrons in all atoms) is preserved. The comoving number-density of photons is preserved after electron-positron annihilation epoch $kT \approx 0.5\,\mathrm{MeV}$. The ratio of the number of baryons n_B to the number of photons $\eta n_B/n_\gamma$ is an important number characterizing our Universe:

$$n_B = \frac{3H_0^2}{8\pi G}\frac{\Omega_b}{m_p}\left(\frac{T}{T_0}\right)^3, \tag{6}$$

and

$$\eta = \frac{n_B}{n_\gamma} = \frac{3\pi(\hbar c)^3}{16\zeta(3)}\frac{\Omega_b H_0^2}{G m_p (kT_0)^3}. \tag{7}$$

Numerically $\eta \sim 10^{-10}$. So, it is a very small number.

These derivations were done for one type of particles – photons. The same type of analysis is valid for all relativistic bosons.

In more general case one can write the occupation number as:

$$\mathcal{N} = \frac{1}{e^{(E-\mu)/kT} \pm 1}. \tag{8}$$

Here the "+" sign is for fermions (e.g. electrons, protons, neutrons) that obey the Fermi-Dirac distribution. The "-" sign is for bosons (e.g., photons, neutrinos, gluons, W and Z bosons,Higgs boson, mesons, deuterium, helium-4) that obey the Bose-Einstein distribution. The energy E is related with momentum p and particle mass: $E^2 = (cp)^2 + (mc^2)^2$. Chemical potential μ defines the number of particles for non-relativistic case $kT \ll mc^2$. For relativistic particles $\mu = 0$ and their number is defined solely by the temperature.

If we know the temperature, we can find the number-density (n) and the mass density (ρ) for bosons and fermions. We also need to know the statistical weight g of particles. For example, for photons the statistical weight is $g = 2$ to account for two polarizations. For fermions the statistical weight is given by the spin (which is half-integer). For electrons the spin is $l = 1/2$ and statistical weight is $2l + 1 = 2$. Neutrino has spin $1/2$, but there are neutrinos and anti neutrinos and there are three types of neutrinos. So, statistical weight of all neutrinos is 6. For relativistic particles ($kT \gg mc^2$) the number-density and mass-density are:

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2}\left(\frac{k}{\hbar c}\right)^3 \cdot gT^3, & \text{for bosons,} \\ \frac{3}{4}\frac{\zeta(3)}{\pi^2}\left(\frac{k}{\hbar c}\right)^3 \cdot gT^3, & \text{for fermions.} \end{cases} \tag{9}$$

$$\rho c^2 = \begin{cases} \frac{\pi^2 k^4}{30\hbar^3 c^3} \cdot gT^4, & \text{for bosons,} \\ \frac{7}{8}\frac{\pi^2 k^4}{30\hbar^3 c^3} \cdot gT^4, & \text{for fermions.} \end{cases} \tag{10}$$

For nonrelativistic particles ($kT \ll mc^2$)

$$n = g\frac{(2\pi mkT)^{3/2}}{(2\pi\hbar)^3} \exp\left[\frac{(\mu - mc^2)}{kT}\right]. \tag{11}$$

At very early epochs ($t < 1\,\text{sec}$) the main contribution to the density is due to relativistic particles. If g_* is the effective (total) statistical weight of particles in equilibrium, than the relation between density and temperature is given by:

$$\rho c^2 = \frac{\pi^2 k^4}{30\hbar^3 c^3} \cdot g_* T^4, \tag{12}$$

where

$$g_* = \sum_{\text{bosons}} g_i + \frac{7}{8}\sum_{\text{fermions}} g_j. \tag{13}$$

Here is an example. For $1\,\text{MeV} < kT < 100\,\text{MeV}$ relativistic particles are photons, electrons, positrons, and neutrinos. Thus the statistical weight is $g_* = 2 + (7/8)(2 + 2 + 3 \cdot 2) = 10.75$. Here the first term is for photons (two polarizations). The next two are for electrons and positrons (each has spin 1/2) and the last term is for three types of neutrinos (electron, muon and tau) each with neutrino and anti-neutrino. Note that there are no anti-protons: they annihilated with most of protons at $kT \approx 1\,\text{GeV}$. Contribution of protons and neutrons to the mass-density is very small. First, because they are not in equilibrium (too low temperature). Second, because most of protons disappeared during proton-antiproton annihilation. Not all protons annihilated because there is an asymmetry in nuclear rates producing protons and antiprotons.

2 FREEZE-OUT OF PARTICLE INTERACTIONS

At some moment the rate of nuclear reactions becomes slow as compared to the rate of expansion leading to the "freeze-out" of particle concentrations. At this moment reactions effectively stop operating and the comoving number-density of particles becomes constant.

An example of this freeze-out is the concentration of neutrinos. At early times the neutrinos were kept in equilibrium with all other particles (including photons, electrons and positrons) thanks to reactions such as $e^- + e^+ \Longleftrightarrow \nu + \bar{\nu}$, $\nu + e^- \Longleftrightarrow \nu + e^-$. The cross-section of these reactions is $\sigma_\nu = G_F^2 T^2$, where $G_F = (292GeV)^{-2}$ is the Fermi constant. The number of reactions per neutrino per unit time is $\Gamma_\nu = n\sigma_\nu c$. For $n \propto T^3$ and $\sigma_\nu \propto T^2$ we have $\Gamma_\nu \propto T^5$, which is a very steep decline with increasing expansion parameter $a \propto T^{-1}$. Compare this with the rate of expansion: $H^2 \propto \rho \propto T^4$, which gives $H \propto T^2$ - a much slower decline. At $T = 1\,\mathrm{MeV}$ neutrinos decoupled from the rest of the particles. From that moment on their comoving number-density is preserved and temperature declined as $T_\nu \propto a^{-1}$.

A bit later, at $T = 0.5\,\mathrm{MeV}$, the electrons and positrons annihilated leaving behind a small fraction of electrons to pair with protons. Electron-positron annihilation dumped their energy into photons, but not to neutrinos because neutrinos went out of equilibrium earlier. Simple estimates how much this affected the photon entropy show that the ratio of temperature of neutrinos to the temperature of photons should be

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma. \tag{14}$$

This gives $T_\nu = 1.95K$. We can find the contribution of the neutrinos to the density of the relativistic particles at present: $\rho_\nu c^2 = (7/8)3a_\gamma T_\nu^4 = 3(7/8)(4/11)^{4/3} a_\gamma T_\gamma^4$. Here the factor 7/8 comes from the fact that neutrinos are fermions and factor 3 is for three types of neutrinos. We can find the ratio of the densities of non-relativistic to relativistic particles and the moment of equality:

$$\frac{\Omega_{\rm NR}}{\rho_{\rm Rel}} = 1.88 \times 10^{-29} \frac{\Omega_m h^2}{a^3} \frac{a^4}{1.68 a_\gamma T_{\gamma,0}^4}, \tag{15}$$

$$1 + z_{\rm eq} = 2.3 \times 10^4 \Omega_m h^2. \tag{16}$$

3 FREEZE-OUT OF THE NEUTRON-PROTON RATIO

Another example of the "freeze-out" is the ratio of neutrons to protons at $t = 1 - 10\,\mathrm{sec}$. At early times ($T > 1\,\mathrm{MeV}$) protons and neutrons were in equilibrium thanks to reactions $e^+ + n \Leftrightarrow p + \bar{\nu}_e$ and $e^- + p \Leftrightarrow n + \nu_e$. Note that neutrons are slightly more massive than protons by $\Delta mc^2 = 1.28\,\mathrm{MeV}$. So, in equilibrium there should be more protons than neutrons:

$$\frac{n_n}{n_p} \approx e^{-\frac{\Delta mc^2}{kT}}. \tag{17}$$

When temperature was much bigger than $kT = 1.29\,\mathrm{MeV}$ the number of neutrons to the number of all baryons was about 1/2. By $t \approx 1\,\mathrm{sec}$ the equilibrium ratio was $n_n/(n_n + n_p) \approx 1/6$. However, the reactions that kept neutrons in equilibrium started to be not fast enough leading to much larger abundance of neutrons because the weak reactions that kept neutrons in equilibrium involve neutrinos that get out of equilibrium at the same time. The residual concentration of neutrons at $t \approx 2\,\mathrm{sec}$ was $n_n/(n_n + n_p) \approx 1/6$.

Free neutrons are unstable particles with exponential decay time $\tau_n \approx 15\,\mathrm{min}$. By $t = 100\,\mathrm{sec}$ when the Big Bang nucleosynthesis took place, the ratio of neutrons to baryons was

$$\frac{n_n}{n_p} \approx \frac{1}{7}. \tag{18}$$

4 BIG BANG NUCLEOSYNTHESIS

Epoch of Big Bang Nucleosynthesis (BBN): time $t \approx 100\,\mathrm{sec}$; temperature $kT \approx 0.1\,\mathrm{MeV}$. Involved particles:

particle	composition	binding energy, MeV
$^2H = D$	(p, n)	2.2
$^3H = T$	$(p, 2n)$	6.92
^3He	$(2p, n)$	7.72
^4He	$(2p, 2n)$	28.3

(19)

(20)

Dominant nuclear reactions:

$$
\begin{aligned}
n+p &\Leftrightarrow D+\nu, \\
D+n &\Leftrightarrow T+\gamma, \\
D+D &\Leftrightarrow T+p, \\
D+D &\Leftrightarrow {}^3He+n, \\
{}^3He+n &\Leftrightarrow T+p, \\
T+D &\Leftrightarrow {}^4He+n.
\end{aligned}
\tag{21}
$$

Note that $D+D \Leftrightarrow {}^4He+\gamma$ does not play a role because of very small cross-section. The bottleneck of this chain of reactions is the deuterium because it has a low binding energy: once produced, it can easily be destroyed. Another peculiarity of our world is the absence of stable elements with atomic weights 5 or 8, which makes production of elements heavier than helium difficult. There are stable elements with atomic weights 6 and 7: ${}^6Li, {}^7Li, {}^7Be$. Two of those can be produced in small amounts in BBN: ${}^4He+T \Leftrightarrow {}^7Li+\gamma$ and ${}^4He+{}^3He \Leftrightarrow {}^7Be+\gamma$.

Precense of free neutrons is crucial for BBN. Without neutrons there would be no helium because normal $p+p \Rightarrow D+e^+ +\nu+\gamma$ is too slow because the reaction requires weak interaction at the moment of close collision of protons making the cross-section of the reaction very small.

Let's define the abundance of different elements as the fraction of mass in element A as compared with all baryons (neutrons and protons) in all nuclei:

$$
X_A = \frac{An_A}{\sum_{\text{all nuclei}}(n_n+n_p)}. \tag{22}
$$

For example, abundance of helium by mass Y for plasma with hydrogen, helium and carbon is $Y = 4n_{He}/(n_p+4n_{He}+12n_C)$.

We can estimate the abundance of helium predicted by BBN by simply assuming that all free neutrons are locked up in helium at the and of BBN. Thus, the abundance of helium is 1/2 of the abundance of neutrons before the onset of BBN. Because the initial ratio of neutrons is $n_n/n_p = 1/7$, the predicted helium abundance is

$$
Y = \frac{4(n_n/2)}{n_n+n_p} = 0.25. \tag{23}
$$

We start our treatment of BBN reactions assuming the nuclear equilibrium. The equilibrium abundance of a non-relativistic element with atomic weight A and charge Z is defined by (we use $k = 1$ and $c = 1$ units):

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} \exp\left[-\frac{m_a - \mu_a}{t}\right], \tag{24}$$

where g_A is the statistical weight of element A and μ_a is the chemical potential, which we do not know and find from condition that the sum of chemical potentials of all particles before the reaction is equal to the sum of potentials of all particles after the reaction. This reflects the fact the sum of all nuclei is preserved. Note that the chemical potential of relativistic particles (only photons in our case) is zero. For example, for the reaction of production/destruction of deuterium

$$n + p \Leftrightarrow D + \gamma \tag{25}$$

we have

$$\mu_D = \mu_p + \mu_n. \tag{26}$$

If Z protons and $A - Z$ neutrons are combined (through a chain of reactions) to produce an element with atomic weight A and charge Z, then its chemical potential is

$$\mu_A = Z\mu_p + (A - Z)\mu_n. \tag{27}$$

We can get rid of μ_p and μ_n by using abundance of neutrons and protons before the reactions started:

$$n_p = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_p - \mu_p)}{T}\right], \tag{28}$$

$$n_n = 2\left(\frac{m_p T}{2\pi}\right)^{3/2} \exp\left[-\frac{(m_n - \mu_n)}{T}\right]. \tag{29}$$

Using the definition of the abundance X_A we can write the comoving number-density as

$$n_A = \left(\frac{X_A}{A}\right)(n_n + n_p) = \left(\frac{X_A}{A}\right)\eta n_\gamma, \tag{30}$$

where $\eta = n_{\mathrm{baryons}}/n_\gamma$. Combining n_A, n_p and n_n expressions, we find the equilibrium abundance of element A:

$$X_A = F(A)\left(\frac{kT}{m_p c^2}\right)^{3(A-1)/2} \eta^{A-1} X_p^Z X_n^{A-Z} \exp\left[\frac{B_A}{kT}\right], \tag{31}$$

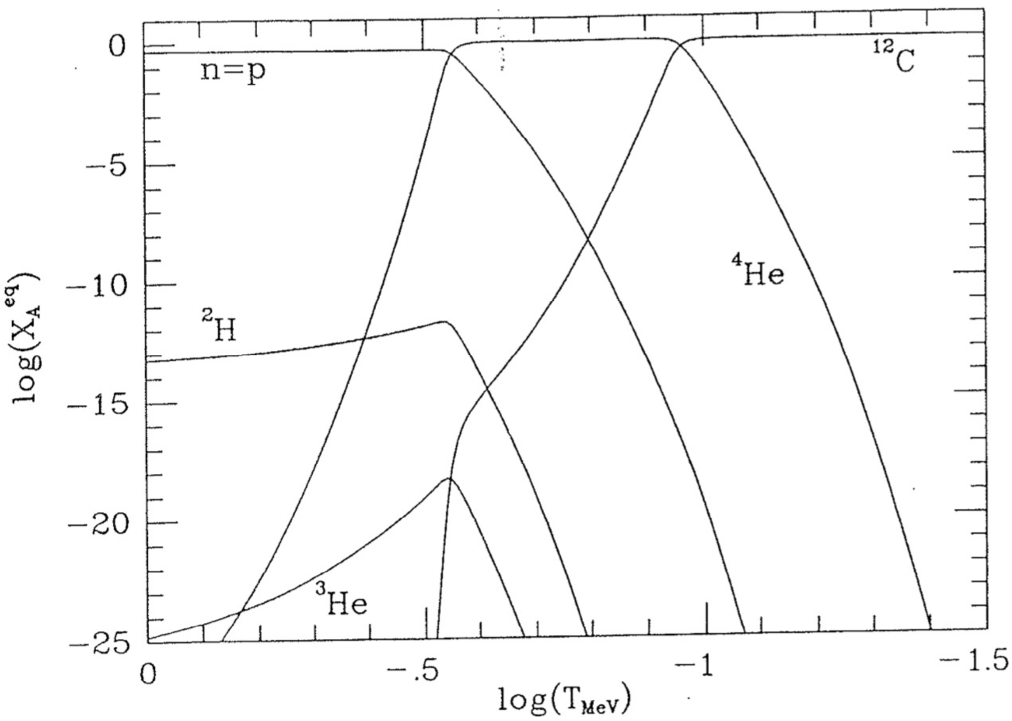

Fig. 4.2: The NSE mass fractions for the system of n, p, D, ^{3}He, ^{4}He, and ^{12}C as a function of temperature. For simplicity we have taken $X_n = X_p$.

where $F(A)$ is a numerical factor ($F \sim 1$) and B_A is the binding energy of element A: $B_A = (Zm_p + (A-Z)m_n - m_Z)c^2$. For deuterium and helium we find:

$$X_D = 16.3 \left(\frac{kT}{m_p c^2}\right)^{3/2} \eta X_p X_n \exp\left[\frac{B_D}{kT}\right], \quad (32)$$

$$X_{He} = 113 \left(\frac{kT}{m_p c^2}\right)^{9/2} \eta^3 X_p^2 X_n^2 \exp\left[\frac{B_{He}}{kT}\right]. \quad (33)$$

Note that $\eta^{(A-1)}$ is a very small number.

At $kT = 0.3\,\mathrm{MeV}$ the equilibrium mass fraction of helium is ≈ 0.15. soon after that nuclear reactions go out of equilibrium because of the Coulomb barrier and because of very low abundances of $D, T, ^3He$. At $kT \approx 0.1\,\mathrm{MeV}$ the abundance of "freeze-out" deuterium is $D/p \approx 10^{-5}$ and almost all neutrons ended up in helium.

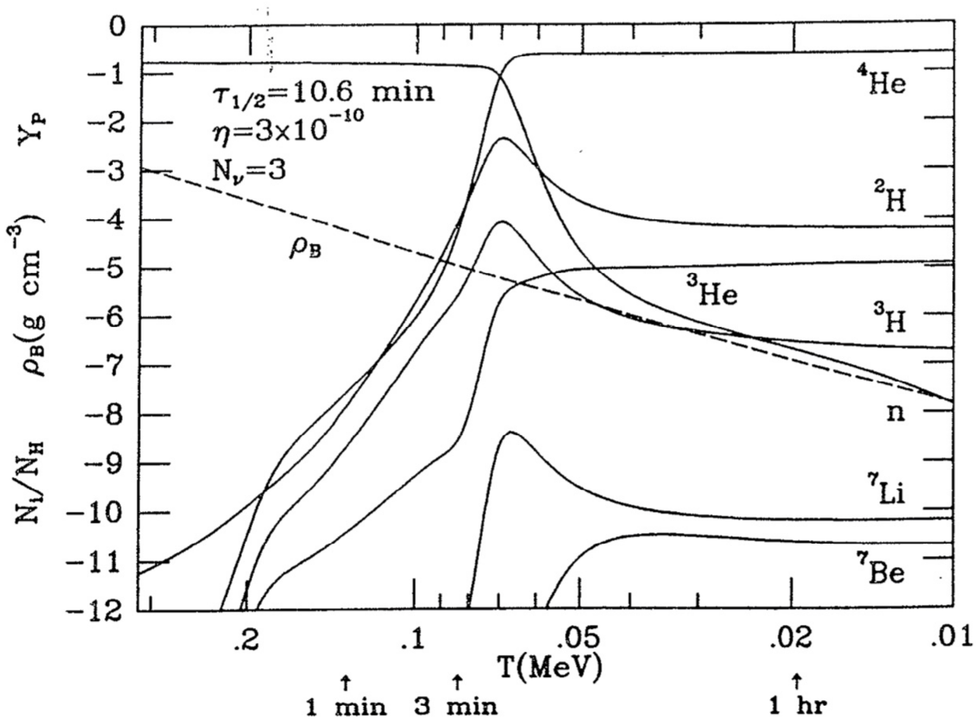

Fig. 4.3: The development of primordial nucleosynthesis. The dashed line is the baryon density, and the solid lines are the mass fraction of ^{4}He, and the number abundance (relative to H) for the other light elements.

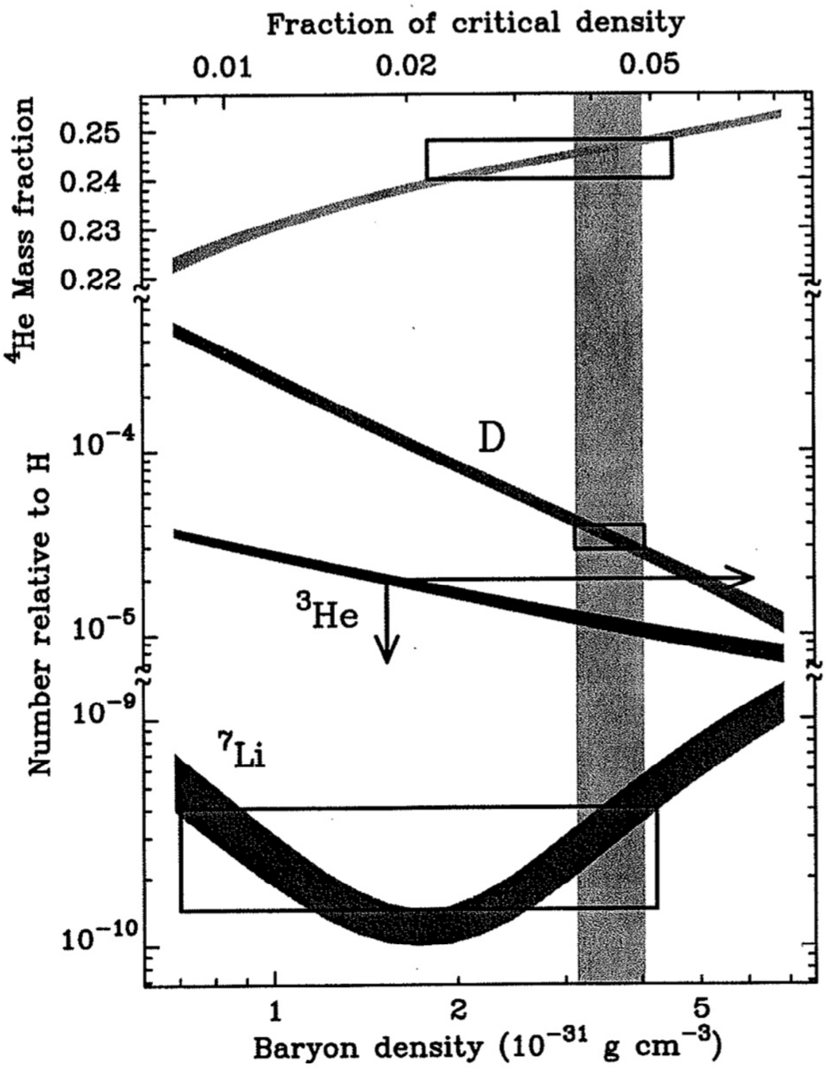

Figure 2: Abundances expected for the light nuclei ^{4}He, D, ^{3}He and ^{7}Li (top to bottom) calculated in standard BBN. New estimates of the nuclear cross-section errors from Burles et al. (1999a) and Nollet & Burles (1999) were used to estimate the 95% confidence intervals which are shown by the vertical widths of the abundance predictions. The horizontal scale, η, is the one free parameter in the calculations. It is expressed in units of the baryon density or critical density for a Hubble constant of 65 kms^{-1}Mpc^{-1}. The 95% confidence intervals for data, shown by the rectangles, are from Izotov and Thuan 1998a (^{4}He); Burles & Tytler 1998a (D); Gloeckler & Geiss 1996 (^{3}He); Bonifacio and Molaro 1997 (^{7}Li extended upwards by a factor of two to allow for possible depletion).

Detirium

$D/H = 2.8 \pm 0.4 \cdot 10^{-5}$ from observations of absorption lines of QSO's at $z \simeq 2.5$
In the same cloud the metallicity was low: $[O/H] = -2.8$, which means that abundance of D is likely primordial
This D/H ratio corresponds to $\eta = 5.9 \pm 0.5\, 10^{-10}$

$\Omega_b h^2 = 0.021 \pm 0.002$

^4He is observed in the ionized gas surrounding young luminous stars (HII regions). ^4He mass fraction Y in different galaxies is plotted as a function of O or N abundance. Regression gives the predicted Y for zero metallicity.

Recent estimates give

$Y = 0.244 \pm 0.002$ from O/H

and $Y = 0.245 \pm 0.001$ from N/H

^3He The primordial abundance of ^3He has not been measured. ^3He is more difficult to measure than D because the difference in wavelength of ^3He and ^4He lines is smaller than for D and H.
For galactic HII regions $^3He/H = 1.6 \pm 0.5\, 10^{-5}$ for a sample of seven HII regions. No variation is seen with abundance of O, there is little scatter. The results are consistent with cosmological origin of ^3He, but it is not clear that the abundance of ^3He is primordial

Li is observed in the solar system, the atmospheres of a wide variety of stars, and in the ISM.
Old halo stars with low metallicity show approximately constant $^7Li/H = 1.6\ 10^{-10}$ and little variation with Fe abundance of surface temperature from 5600 – 6300°K. This is the argument that ^7Li is primordial.
Observational estimates give
$^7Li/H \sim (1-1.7)\ 10^{-10}$, which is below BBN estimates of $(3.5 \pm 1) 10^{-10}$ for current D/H estimates