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Silk dumping

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1 SILK DAMPING: PHOTON DIFFUSION AT SMALL SCALES

Before the recombination photons provide the pressure support for waves in baryons. So far we treated photons as an ideal fluid, which is a good approximation for long waves. At very small scales photons can experience diffusion and that will reduce the pressure resulting in decline of the amplitude of acoustic waves.

The diffusion originates from random jumps of photons between collisions with electrons. The mean-free path for the photons is defined by the Thompson cross-section σ_T and electron density n_e:

$$\lambda_\gamma = \frac{1}{\sigma_T n_e}. \tag{1}$$

In turn, the electron density is defined by ionization state and gas density:

$$n_e = x_e n_H = x_e \rho_b \Omega_{bar}/m_H. \tag{2}$$

We need to find the typical scale $\lambda_{\rm Silk}(t)$ over which photons diffuse by the moment t. The amplitude of fluctuations is significantly reduced for all waves shorter than $\lambda_{\rm Silk}$. We estimate the effect of photon diffusion using the random walk approximation.

There is one complication in applying the standard random walk formulas: the mean-free-path changes (increases) with time. So, we need to integrate the diffusion equation over time. Per time interval Δt a photon experiences N collisions with electrons where $N = c\Delta t/\lambda_\gamma$. Note that λ_γ in eq. (1) is the proper (physical) distance. In order to estimate the effect in the expanding Universe, we need to sum effects of diffusion in comoving coordinates at different moments of time. At an expansion parameter a and during time interval dt the comoving mean-free-path is $\Delta x_1 = \lambda_\gamma(a)/a$. Over dt time interval the photons diffuse by

$$\Delta x^2(dt,t) = N\Delta x_1^2 = \frac{cdt}{\lambda_\gamma}\cdot\left(\frac{\lambda_\gamma(a)}{a}\right)^2. \tag{3}$$

Now we can integrate this over time to find the total photons diffusion distance and the comoving scale of Silk damping:

$$\lambda_{\rm Silk}^2(t) = \int_0^t \frac{cdt}{\lambda_\gamma(t)} \frac{\lambda_\gamma^2(t)}{a^2} \approx ct \cdot \frac{\lambda_\gamma(t)}{a^2}. \tag{4}$$

The last estimate is a good approximation because most of the diffusion happens close to the upper limit of the integral.

Numerically the damping scale at the moment of recombination is equal to

$$\lambda_{\rm Silk} \approx 5.7\left(\Omega_{m,0}h^2\right)^{-3/4}\left(\frac{\Omega_{\rm bar,0}}{\Omega_{m,0}}\right)^{-1/2}\left(\frac{X_e}{0.1}\right)^{-1/2}\left(\frac{1+z_{\rm rec}}{1100}\right)^{-5/4} {\rm Mpc}, \tag{5}$$

where X_e is the ionization fraction at $z = z_{\rm rec}$. This damping suppresses angular fluctuations in CMB on scales less than few arcminutes.

2 RANDOM WALK APPROXIMATION FOR BROWNIAN MOTION

If a particle (photon in our case) experiences random scattering every time it collides with another particle (electron in our case), then over time interval t its average distance from the initial position x is equal to zero: $\langle x \rangle = 0$. This simply means that there is no preferred direction for diffusion of the particle. For example, as many particles diffuse to the right ($x > 0$) as to the left ($x < 0$). However, the dispersion of this distance is not zero. It is equal to

$$\langle x^2 \rangle = 2Dt, \tag{6}$$

where D is the diffusivity coefficient. This coefficient appears in the equation of thermal diffusion:

$$\frac{\partial \rho(\vec{x},t)}{\partial t} = D\nabla^2 \rho(\vec{x},t) \tag{7}$$

Here is a simple example of solution of this equation. Imagine that at initial moment $t = 0$ we put N particles at rest at $r = 0$. They start to random walk and at any later moment their number-density profile is

$$n(r,t) = \frac{N}{(4\pi Dt)^{3/2}} e^{-\frac{r^2}{4Dt}} \tag{8}$$

In our case it is more convenient and more transparent to write the random approximation in a different form. If N is the average number of scattering experienced by a particle and Δ_1 is the average distance the particle travels between scatterings, then the dispersion of the distance from the initial position is

$$\langle \Delta x^2 \rangle = N\Delta_1^2. \tag{9}$$

Note that in this case the typical distance from the initial position scales $x \propto \sqrt{N}$.