Recombination and Decoupling

In the early Universe radiation and matter (baryons) were coupled due to interactions between photons and electrons. Atoms were ionized because of high T and large n_γ

at $T \simeq 0.3\,eV$, $z = 1000 - 1500$

three events:
1) gas became neutral (recombination)
2) radiation gets free (decoupling of matter and radiation
3) freeze out of free electrons ($x_e \simeq 10^{-5}$)

recombination: reaction: $\boxed{p + e^- \leftrightarrow H + \gamma}$

we neglect ^4He (only $1/10$ of all atoms)

for p, e^-, and H in thermal equilibrium:

$$n_i = g_i \left(\frac{m_i T}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{m_i - \mu_i}{kT}\right)$$

g_i: stat. weight $= 2$ for p and e^-, $= 4$ for H

n_i: $\begin{pmatrix} n_H \\ n_e \\ n_p \end{pmatrix}$

conservation of chemical potential: $\mu_H = \mu_p + \mu_e$

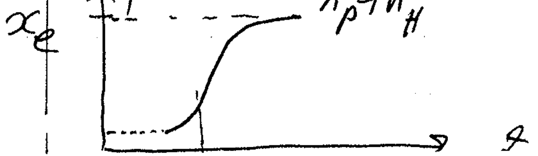
Binding energy of H: $13.6\,eV = m_p + m_e - m_H$

Expressing μ_p and μ_e through n_p and n_e, we get Saha equation:

$$\boxed{n_H = \frac{g_H}{g_e g_p} n_p n_e \left(\frac{m_e T \cdot k}{2\pi\hbar^2}\right)^{-3/2} \exp\left(\frac{B_H}{kT}\right)}$$

we can rewrite this for the ionization fraction

$$x_e \equiv \frac{n_p}{n_p + n_H} : \quad \boxed{\frac{1 - x_e}{x_e^2} = \frac{4\sqrt{2}\,\zeta(3)}{\sqrt{\pi}} \cdot \eta \left(\frac{kT}{m_e c^2}\right)^{3/2} \exp\left(\frac{B_H}{kT}\right)}$$



Two-photon emission: out-of-equilibrium recombination

transitions 2S-1S are important because they produce photons, which have $\lambda > \lambda_{Ly\alpha}$, which escape. Expansion of the universe also helps to change frequency of ~~ph~~ photons with $\lambda_{Ly\alpha}$ - stretch them so they can not ionize gas again.

fractional ionization near recombination $z \simeq 1000$ is approximated by:

$$x(z) = 2.4 \cdot 10^{-3} \frac{(\Omega h^2)^{1/2}}{\Omega_{baryon} h^2} \left(\frac{z}{1000}\right)^{12.75}$$

optical depth to Thomson scattering

$$\tau(z) = \int n_e x \sigma_T \, dr_{prop} \simeq$$

$$\simeq 0.37 \left(\frac{z}{1000}\right)^{14.25}$$

The surface of last scattering is well-defined at $z \sim 1065$, but it has some width, which is important for small-scale anisotropies of $\frac{\Delta T}{T}$

$$\boxed{T_{rec} = 3600\,K, \quad z_{rec} = 1300, \quad t_{rec} = 1.4 \cdot 10^{5} \left(\Omega_{tot} \cdot h^2\right)^{-1/2} yrs}$$

<u>Decoupling</u>: Condition for equilibrium:

$$\tau_{reaction} < \frac{1}{H}$$

Thomson scattering is ~~most~~ important at this late stages. Other processes - Compton scattering and free-free already have time-scales, which are longer than the age of the Universe at this stage.

$$\tau_{thompson} = \frac{1}{\sigma_T\, n_e\, c} < \frac{1}{H}$$

take n_e from recombination: $n_e = x_e \cdot \eta \cdot n_\gamma$

This gives $\boxed{T_{Decoupling} = 0.26\,eV, \quad z_{dec} = 1100}$

<u>Freeze out of ionization</u>: at T_{Dec}, $x_e \simeq 3 \cdot 10^{-5} \frac{\Omega_{tot}^{1/2}}{\Omega_b h}$

after decoupling temperature of neutral matter would go down as $T \propto 1/a^2$ (because $v \propto 1/a$)
But residual ionization keeps temperature high:
Until $z \simeq 20$ $\boxed{T_{baryons} \propto T_\gamma \propto 1/a}$

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Intergalactic Medium after recombination

$T_\gamma = 2{,}736(1+z)\ K$ | $T_{gas} \sim T_\gamma$ until $z \simeq 100$. Then it drops as $T_{gas} \sim a^{-2}$

The mean number density of hydrogen

$$n_H(z) = \frac{\rho_{IGM}}{1{,}3\, m_H} \sim 8.6 \cdot 10^{-6}\, \Omega_{b,0}\, h^2 (1+z)^3\ cm^{-3}$$

The mean optical depth of IGM

Suppose a source at high z_{em} emits energy $\mathcal{L}(\nu)d\nu$. If the light is not absorbed and no other sources are present,

$$F(\nu)d\nu = \frac{\mathcal{L}(\nu(1+z_e))}{4\pi d_L^2}(1+z_e)d\nu$$

If photons can be absorbed than

$$\frac{dn_\gamma}{dt}(\nu_e, t) = -\Lambda\left[\nu_e \frac{a(t_e)}{a(t)}, t\right] n_\gamma(\nu_e, t)$$

where $\Lambda(\nu, t)$ = absorption coefficient

$$\Rightarrow n_\gamma(\nu_e, t) = e^{-\tau} n_\gamma(\nu_e, t_e)$$

$$\text{where } \tau = \int_{t_e}^{t} \Lambda\left(\nu_e \frac{a(t_e)}{a(t)}, t'\right) dt'$$

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For resonant absorption: transition from state 1 to state 2, energies E_1 and E_2

$$h\nu_{12} = E_2 - E_1$$

absorption cross-section $\sigma_{12}(\nu) \Rightarrow \Lambda(\nu, t) = c\, n_1(t)\, \sigma_{12}(\nu)$

In other words, absorption is proportional to the number-density of atoms in the first state $n_1(t)$ and to the cross-section of the absorption $\sigma_{12}(\nu)$

For resonant transitions absorption $\sigma_{12}(\nu)$ is peaked sharply at ν_{12} $\Rightarrow$

$$q_{12} = \int \sigma_{12}(\nu)\, d\nu$$

The optical depth becomes

$$\tau = \int_{t_{em}}^{t} \underbrace{\left\{1 - \exp\left[-\frac{h\nu_e}{kT(t')} \cdot \frac{a(t_{em})}{a(t')}\right]\right\}}_{\text{stimulated transitions}} \Lambda\left[\nu_{em} \frac{a(t_e)}{a(t')}, t'\right] dt'$$

Now suppose a photon emitted at z_{em} (t_{em}, a_{em}) is absorbed at redshift z_a. Frequencies:

$$\underbrace{\nu_e}_{\text{emitted}} = \frac{1+z_e}{1+z_a}\, \nu_{12} = (1+z_e)\underbrace{\nu_o}_{\text{observed}}$$

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The optical depth at observed frequency ν_0 is approximately

$$\tau(\nu_0) \simeq \frac{c\, n_1(t_a)}{a(t_a) H_0 E(z_a)} I_{12}$$

where $I_{12} = \frac{1}{\nu_{12}} \int \sigma_{12}(\nu)\, d\nu = \frac{a_{12}}{\nu_{12}}$

Here we assumed that $kT(t_a) \ll h\nu_{12}$

Gunn-Peterson effect: optical depth in $Ly\alpha$

$\lambda_{Ly\alpha} = 1216$ $\quad$ $1s \to 2p$ transition

$$T_{Ly\alpha} = \frac{h\nu_{Ly\alpha}}{k} = 1.18\cdot 10^5 K$$

$$\tau(\nu_0) = \frac{c\, n_{HI}(t_a)}{a(t_a) H_0 E(t_a)} \cdot I_{Ly\alpha}$$

where $I_{Ly\alpha} = 4.5\cdot 10^{-18}\ cm^{-2}$

This can be used to measure the proper number-density of neutral hydrogen:

$$\boxed{n_{HI} = 2.4\cdot 10^{-11}\, h E(z_a)\, \tau \ \ cm^{-3}}$$

Flux decrement $D_A = 1 - \frac{F_{observed}(\lambda)}{F_{continuum}(\lambda)} \simeq \langle 1 - e^{-\tau} \rangle$

For $z \leq 5.5$

measured $\Rightarrow$ $$\tau(z) = (0.85 \pm 0.06)\left(\frac{1+z}{5}\right)^{4.3}$$

$$\Rightarrow \quad n_{HI}(z) \sim 2 \cdot 10^{-11}\, h\, E(z) \left(\frac{1+z}{5}\right)^{4.3}$$

$$\Downarrow \quad \Omega_{HI}(z) = \frac{n_{HI}\, m_p}{\rho_{crit,0}(1+z)^3} \sim 1.4 \cdot 10^{-8}\, h^{-1} E(z) \left(\frac{1+z}{5}\right)^{1.3}$$

$\rightarrow$ very high ionization of hydrogen

Optical depth from CMB

CMB photons can interact with free electrons throug Thomson scattering. Optical depth of Thomson scattering:

$$\tau(z) = \sigma_T \int_0^z n_e(z') \frac{dl}{dz'}\, dz'$$

$$n_e(z) = n_p(z)\, X_e(z)$$

$X_e(z)$ ← ionization fraction

$n_p(z)$ ← number-density of protons

$$n_p(z) = \frac{0.88}{m_p} \rho_{baryon}(z)$$ (assuming 24% of He by mass)

$$\tau(z) = 0.063\, h\, \Omega_{b,0} \int_0^z \frac{(1+z)^2}{E(z)} [1 + \delta_{bar}(z)]\, X_e(z)\, dz$$

$\delta_{bar}(z)$ ← density contrast

$$\delta_{bar} = \frac{\rho_{bar}}{\langle \rho_{bar} \rangle} - 1$$

For instant re-ionization at $z = z_{ri}$ the optical depth by re-ionization

$$\tau(z_{ri}) = 0.07 \left(\frac{h}{0.7}\right)\left(\frac{\Omega_{b,0}}{0.04}\right)\left(\frac{\Omega_{m,0}}{0.3}\right)^{-1/2}\left(\frac{1+z_{ri}}{10}\right)^{3/2}$$

WMAP $\Rightarrow \boxed{\tau = 0.1} \Rightarrow \underline{\underline{9 < z_{ri} < 14}}$