Mon. Not. R. Astron. Soc. **000**, 000–000 (0000) Printed 12 March 2018 (MN LaTeX style file v2.2)

Halo Mass Function.

12 March 2018

1 MASS FUNCTION

To develop an approximation for the halo mass function we use the linear theory of growth of perturbations and gaussian statistics for the density field. Density field $\delta(\vec{x})$ is smoothed with a filter $W(R)$, where R defines the radius and mass of the filter:

$$M = \frac{4\pi}{3}\rho_b R^3, \qquad \text{for top} - \text{hat filter}, \tag{1}$$

$$= (2\pi)^{3/2}\rho_b R^3, \quad \text{for gaussian filter}. \tag{2}$$

Here ρ_b is the background density. It is customary to use the top-hat filter. For some reason, it works much better than the gaussian filter for the estimates of the halo statistics. According to the spherical infall model all regions with $\delta(\vec{x}) > \delta_c = 1.68$ have collapsed and by the time t are parts of halos with mass *larger* than M.

We can estimate the total fraction of the volume of the universe in all halos with mass larger than M, if we find the fraction of volume in regions where density contract is larger than δ_c. The distribution function of density – the probability that at the position $\vec{x}$ the density contrast is δ – is the gaussian. In other words, the probability to find δ in the interval $\delta, \delta + d\delta$ is

$$dP = P(\delta)d\delta = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\delta^2}{2\sigma^2}\right)d\delta. \tag{3}$$

The r.m.s of the density contrast $\sigma(M,t)$ is estimated at moment t using the density field smoothed with filter that has mass M. Thus, it is a function of mass M and time t.

The value of σ is found using the power spectrum $P(k)$ as estimated by the linear theory of growth of fluctuations:

$$\sigma^2(M,t) = \langle\left(\frac{\delta M}{M}\right)^2\rangle = \frac{1}{2\pi^2}\int_0^\infty b(t)P(k)W^2(k)k^2dk, \tag{4}$$

where $b(t)$ is the linear growth factor.

We can find the fraction of volume $F(>M)$ in all regions with density contrast above δ_c by integrating the distribution function:

$$F(>M) \equiv \int_{\delta_c}^{\infty} P(\delta) d\delta = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) d\delta \tag{5}$$

We can find the fraction of mass in halos with mass M by differentiating the cumulative fraction of mass: $f(M) = \partial F / \partial M$. Because each halo in the differential mass fraction $f(M)$ has mass M, we estimate the number-density of halos as

$$n(M) = \frac{\rho_b}{M} f(M). \tag{6}$$

We can write the cumulative mass fraction as:

$$F(>M) = \frac{1}{\sqrt{\pi}} \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) d\left(\frac{\delta}{\sqrt{2}\sigma}\right) = \tag{7}$$

$$= \frac{1}{\sqrt{\pi}} \int_{\frac{\delta_c}{\sqrt{2}\sigma}}^{\infty} e^{-x^2} dx \tag{8}$$

$$= \frac{1}{2} erfc\left(\frac{\delta}{\sqrt{2}\sigma}\right) \tag{9}$$

The fraction of mass should be twice of that estimate to include regions between peaks that formally have density below the threshold δ_c. The final expression is called the Press-Schechet approximation is:

$$n(M) = -\frac{\rho_b}{M} \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{\delta_c}{\sigma(M)}\right) \frac{1}{\sigma} \frac{d\sigma}{dM} \exp\left(\frac{\delta_c^2}{2\sigma^2}\right) \tag{10}$$

For the simplest case of the power spectrum $P(k) \propto k^n$ we have:

$$n(M) = -\frac{\rho_b}{\sqrt{2\pi}} (1 + \frac{n}{3}) \left(\frac{M}{M_*}\right)^{(3+n)/6} M^{-2} \tag{11}$$

For $n = -3$ the number-density declines as $n(M) \propto M^{-2}$. If $n = -2.5$,

$$n(M) \propto M^{-2+\frac{1}{12}} \exp\left(-\frac{1}{2}[M/M_*]^{1/6}\right). \tag{12}$$