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Jeans Instability in the expanding Universe.

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1 GAS PROPERTIES

Here we consider fluctuations in baryons and waves shorter than the distance to the horizon $d_H(t)$.

We deal with ideal fluid with pressure P. For non-expanding medium the Jeans length can be estimated by equating the time needed for a wave to travel across an object $t_{\rm cross} = l_J/v_s$ to the object's free fall time $t_{\rm dynamic} = 1/\sqrt{4\pi G\rho}$. The critical wavelength is defined by the sound speed v_s and density of matter ρ:

$$l_J \approx \frac{v_s}{\sqrt{4\pi G\rho}}. \tag{1}$$

A better way of solving the problem is to find the critical wavelength using the dispersion relation for waves propogating in a homogeneous gas with given density and temperature. The answer gives the critical wave-length l_J that separates two regimes. Waves shorter than than l_J oscillate and their amplitude does not grow. Waves longer than l_J experience growth.

Sound speed in gas is

$$v_s^2 = \frac{dP}{d\rho} = \gamma\frac{P}{\rho}, \tag{2}$$

For normal ideal gas the pressure is defined by the random motions of atoms, ions and electrons:

$$v_s^2 = \frac{\gamma kT}{\mu m_H}, \tag{3}$$

where

$$P = nkT, \qquad \rho = \mu n m_H, \tag{4}$$

and γ is the ratio of heat capacities $\gamma = c_p/c_v$. It is equal to $\gamma = 5/3$ for mono-atomic gas. μ is the molecular weight (depends on the ionization state). m_H mass of a proton. k is the Boltzmann constant.

Before the recombination ($z \approx 1100$) the gas is ionized and its pressure is provided mostly by the photons through Thompson scattering, not by random motions of atoms and electrons:

$$P = \frac{\rho_{gas} kT}{\mu m_H} + \frac{\rho_\gamma c^2}{3}, \qquad \rho_\gamma c^2 = \sigma T^4. \tag{5}$$

If the second term dominates, then $v_s^2 = c^2/3$. This means that the sound speed is close to the velocity of light c. In turn, this implies that the Jeans length is only slightly smaller than the distance to the horizon. If this is the case, then fluctuations in the baryons do not grow and the waves in gas are acoustic waves ("sound") that travel through the universe without much of a change.

The amplitude of the density fluctuations is nearly the same as that in the radiation field. We can estimate the relation between perturbations in the gas $\Delta\rho_{gas}/\rho_{gas}$ and perturbations in temperature $\Delta T/T$ by observing that when a patch of gas is compressed, the photons are compressed too, but their number is preserved:

$$\frac{\Delta\rho_{gas}}{\rho_{gas}} = \frac{\Delta n_\gamma}{n_\gamma}. \tag{6}$$

The number of photons scales with the temparature as $n_\gamma \propto T^3$. That implies that $\Delta n_\gamma/n_\gamma = 3\Delta T/T$. Thus, fluctuations in gas density and in gas temperature are related:

$$\frac{\Delta T}{T} = \frac{1}{3}\frac{\Delta\rho_{gas}}{\rho_{gas}}. \tag{7}$$

There is an additional effect that acts only on small scales. It is related with diffusion of photons. It is called "Silk dumping" (after Joe Silk). It will be discussed separately.

We can add a correction due to "normal" gas pressure to find a more accurate approximation:

$$v_s = \frac{c}{\sqrt{3}}\left(\frac{3}{4}\frac{\rho_{gas}}{\rho_\gamma} + 1\right)^{-1/2} \tag{8}$$

2 EVOLUTION OF FLUCTUATIONS

Equation of growth of linear fluctuations δ in baryons can be written as:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho_b\left[\Omega_{bar}\delta + \Omega_{dm}\delta_{dm}\right] + \frac{\nabla^2 P}{\rho_{bar} a^2}. \tag{9}$$

Here the second term in the brackets $\Omega_{dm}\delta_{dm}$ appears only after the recombination and is absent before. This happens because before the recombination the waves in gas are traveling acoustic waves and the waves in dark matter are standing waves. A traveling wave averages out the effect of fluctuations in the dark matter.

After the recombination the radiation moves freely and does not provide pressure support for the gas. Pressure and sound speed drop many orders of magnitude after the recombination.

Before the recombination we search for the solution in the form of plane waves:

$$\delta(x,t) = \delta_k(t)e^{-i\boldsymbol{kx}}, \tag{10}$$

where k is the comoving wave number.

Perturbations in the pressure are $\Delta P = v_s^2 \Delta\rho$. Thus,

$$\frac{\nabla^2 P}{\rho_{bar}} = v_s^2 \nabla^2 \delta = -v_s^2 k^2 \delta_k. \tag{11}$$

Now the equation (9) can be written as:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = \left[4\pi G\rho_{bar} - \left(\frac{kv_s}{a}\right)^2\right]\delta_k. \tag{12}$$

The critical wavenumber k_J (Jeans wavenumber) is found by equating the term in the brackets to zero:

$$k_J^2 = \frac{4\pi G\rho_{bar}}{v_s^2}a^2. \tag{13}$$

Note that was the comoving wavenumber. The proper Jeans wavelength is

$$\lambda_J = \frac{2\pi}{k_J}a = v_s\sqrt{\frac{\pi}{G\rho_{bar}}}. \tag{14}$$

The Jeans mass:

$$M_J = \frac{4\pi}{3}\rho_{bar}\left(\frac{\lambda_J}{2}\right)^3 = \frac{\pi^{5/2}}{6}\rho_{bar}\left(\frac{v_s}{\sqrt{G\rho_{bar}}}\right)^3, \tag{15}$$

and the ratio of the Jeans mass M_J to the mass of baryons inside the horizon M_H:

$$\frac{M_J}{M_H} \approx 26\frac{4\pi}{3}\rho_{bar}\left(\frac{\lambda_J}{2}\right)^3 = \frac{\pi^{5/2}}{6}\rho_{bar}\left(\frac{v_s}{\sqrt{G\rho_{bar}}}\right)^3. \tag{16}$$

For the Jeans mass just before the recombination we find:

$$M_J \approx 1.2 \times 10^{16}\left(\Omega_{b,0}h^2\right)^{-2} M_\odot \tag{17}$$

After the recombination the sound speeds drops dramatically to:

$$v_s = \left(\frac{\gamma kT}{\mu m_H}\right)^{1/2} \approx 10\text{km/sec} \tag{18}$$

and comoving Jeans length and mass become small:

$$M_J \approx 1.5 \times 10^5\left(\Omega_{b,0}h^2\right)^{-1/2} M_\odot \tag{19}$$

$$\lambda_J \approx 0.01\left(\Omega_{b,0}h^2\right)^{-1/2}\text{Mpc} \tag{20}$$