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Filters: statistics of smoothed density field.

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1 FILTERS

Density field $\delta(\vec{x})$ is smoothed with filter $W(\vec{r})$:

$$\delta_{sm}(\vec{x}) = \frac{\int \delta(\vec{x}+\vec{r})W(\vec{r})d^3r}{\int W(\vec{r})d^3r}. \quad (1)$$

Here the denominator is the effective volume of the filter:

$$V_W = \int W(\vec{r})d^3r. \quad (2)$$

If W_k is the spectrum of function $W(\vec{r})$, then the power spectrum of smoothed field is the convolution of the power spectra:

$$\langle\left(\frac{\delta M}{M}\right)^2\rangle \equiv \langle\delta^2_{sm}(\vec{x})\rangle = \frac{V}{(2\pi)^3}\int\langle\delta^2_k\rangle W^2_k d^3k, \quad (3)$$

and

$$\langle\left(\frac{\delta M}{M}\right)^2\rangle = \frac{1}{2\pi^2}\int P(k)W^2_k k^2 dk. \quad (4)$$

Special case: top-hat filter with radius $r_0 = 8h^{-1}Mpc$.

$$\sigma_8 \equiv \langle\left(\frac{\delta M}{M}\right)^2\rangle = \frac{1}{2\pi^2}\int P(k)W^2_k(kr_0)k^2 dk. \quad (5)$$

2 EXAMPLES OF FILTERS

Top-hat filter:

$$W(r) = 1 \quad \text{for} \quad r < r_0, \qquad V_W = 4\pi r_0^3/3, \quad M_W = 4\pi\rho r_0^3/3, \quad (6)$$

$$W_k = \frac{3}{y^3}\left[\sin y - y\cos y\right], \quad y = kr_0 \quad (7)$$

Gaussian filter:

$$W(r) = \exp\left(-\frac{r^2}{2r_0^2}\right), \quad V_W = (2\pi)^{3/2}r_0^3, \quad (8)$$

and

$$W_k = \exp\left(-\frac{k^2r_0^2}{2}\right). \quad (9)$$