Anisotropies in the CMBR

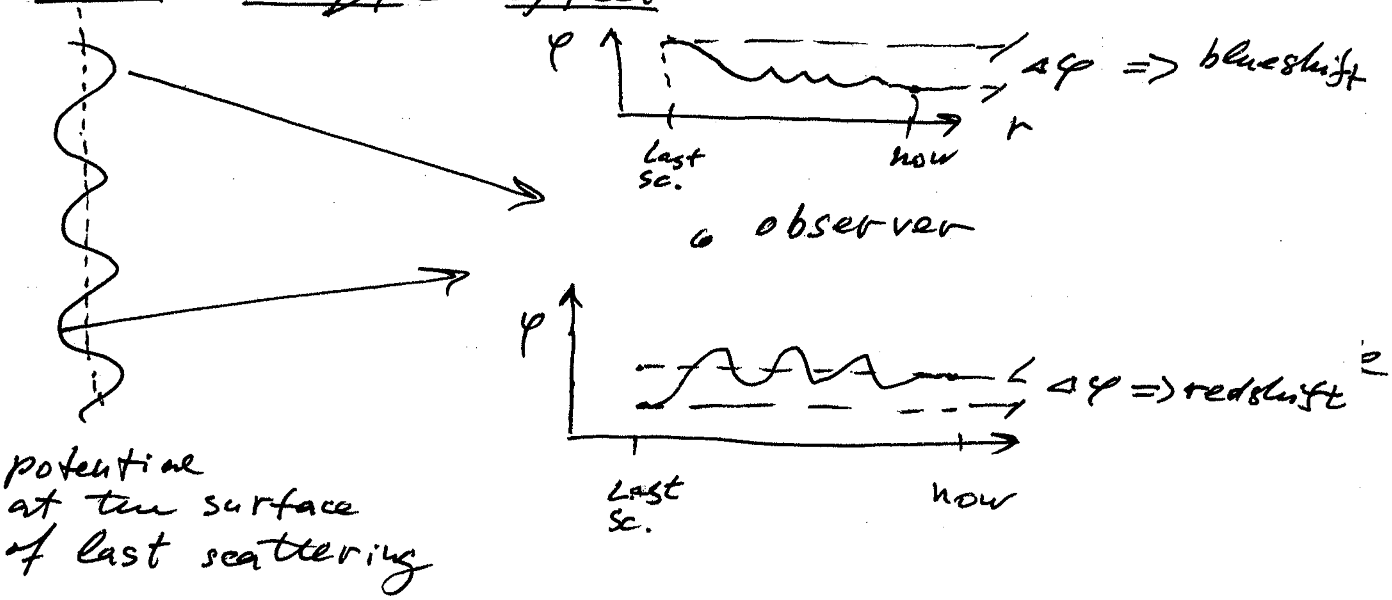
angular scales: distance to the horizon at decoupling subtends an angle of

$$\theta_{dec} = 0.^\circ 87\, \Omega_0^{1/2} \left(\frac{z_{dec}}{1100}\right)^{-1/2}$$

for $\Omega_0 = 1$, $z_{dec} = 1100$ this gives $\theta_{dec} \sim 1^\circ$

This angle naturally separates large scales ($\theta > 1^\circ$) where fluctuations in the gravitational potential produce largest contribution to the fluctuations in the temperature of cosmic microwave background radiation (CMBR), and small scales ($\theta < 1^\circ$), where we have many different competing effects

(I) Sachs-Wolfe effect



$$\frac{\delta T}{T}(\vec{x}) = \frac{1}{3}\varphi_{\substack{\text{last}\\\text{scattering}}}(\vec{x}) \sim \left(\frac{\delta\rho}{\rho}\right)_{\substack{\text{at last}\\\text{scattering}\\\text{on horizon}}}$$

$\vec{x}$ = direction on the sky

It is often convenient to expand $\frac{\delta T}{T}$ in spherical harmonics:

$$\frac{\delta T}{T}(\vec{x}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \varphi)$$

monopole ($\ell=0$) = constant temperature = = not observable

dipole ($\ell=1$) = constant motion = can not distinguish from local motion

<u>Relation between $a_{\ell m}$ and power spectrum of fluctuations of density $P(k)$</u>

$$\boxed{\langle |a_{\ell m}^2| \rangle = \frac{4}{R_H^3} \int_0^{\infty} \frac{dx}{x^3} P(k) J^2_{\ell+1/2}(x)}, \quad x \equiv k \cdot R_H,$$

where R_H is the distance to the Horizon,

J_n is the Bessel function

Q_ℓ rms fluctuations on the sky due to all harmonics with the same ℓ (so only m is different) is called multipole:

There are $(2\ell+1)$ harmonics, which contribute to multipole ℓ

$$Q_\ell^2 \equiv \frac{(2\ell+1)}{4\pi} \langle a_{\ell m}^2 \rangle \quad \Big\| \quad C_\ell \equiv \langle a_{\ell m}^2 \rangle_{\text{over } m}$$

for Herrison-Zeldovich spectrum $P(k) = A k^1$

$$\boxed{Q_\ell^2 = \frac{(2\ell+1)}{4\pi} \langle a_{\ell m}^2 \rangle = \frac{(2\ell+1)}{\pi^2} \frac{1}{\ell(\ell+1)} \frac{A}{R_H^4}}$$

Thus, if observations give us value of a multipole Q_ℓ $\Rightarrow$ normalization of the spectrum A $\Rightarrow$ spectrum $P(k) = A \cdot k\, T(k,t)$ $\Rightarrow$ predictions for different statistics (like, σ_8, V_{50}, number of galaxy clusters ...)

For flat Universe dominated by cold particles, the quadrupole $(\ell = 2)$ is

$$\left(\Delta T\right)_{quadrupole} = 19\,\mu k \Rightarrow$$

COBE

$$\Rightarrow \left(\frac{\Delta T}{T}\right)_2 = 7 \cdot 10^{-6}$$

[for ΛCDM models $\left(\frac{\Delta T}{T}\right)_2$ is slightly larger $\sim 20-21\,\mu k$]

II ISW For Universe, which does not expand as $a \propto t^{2/3}$, there is additional effect, which is also due to variations of the gravitational potential. The effect is called Indused Sachs-Wolfe effect and is typically $\sim 10\%$ of SW effect

III Fluctuations of the radiation field at the surface of large scattering

$$\rho_\gamma = a_1 T^4 \Rightarrow \frac{\delta T}{T} = \frac{1}{4}\frac{\delta \rho_\gamma}{\rho_\gamma}$$

For coupled baryons and photons $\delta_\gamma = \frac{4}{3}\delta_{Bar}$

$$\boxed{\frac{\delta T}{T} = \frac{1}{3}\frac{\delta \rho_{bar}}{\rho_{bar}}}$$

IV Doppler effect

$$\frac{\delta T}{T} = v_{OBS.} - v_{emitted}$$

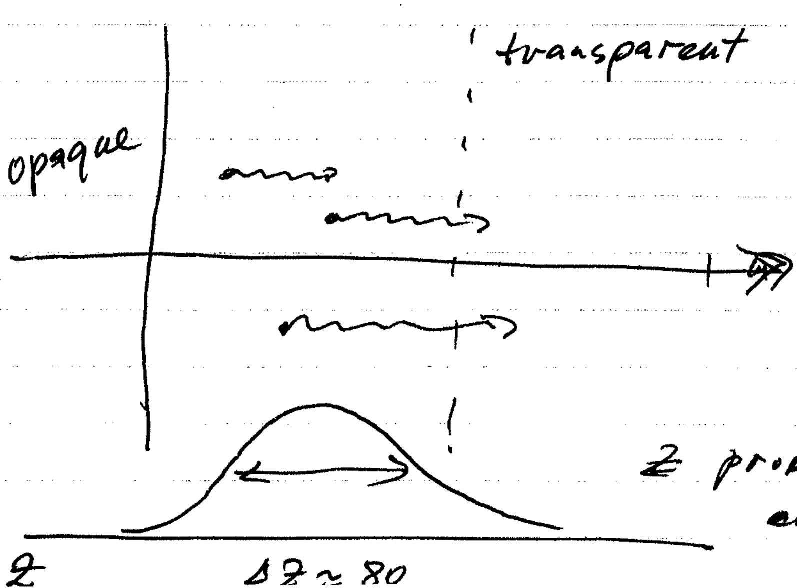
$$\left(\frac{\delta T}{T}\right)^2_k \propto \frac{\delta_k^2}{k^2} d^3k \propto \delta_k^2\, dk$$

$$\Rightarrow \frac{\delta T}{T} \propto \begin{cases} \theta \,, & \theta < \theta_{Horizon} \\ 1/\theta \,, & \theta > \theta_{Horizon} \end{cases}$$

This contribution is small on large scales, but it dominates on scales of the horizon $\sim 1^\circ$

V Damping of fluctuations on small scales

$\Rightarrow$ finite thickness of recombination

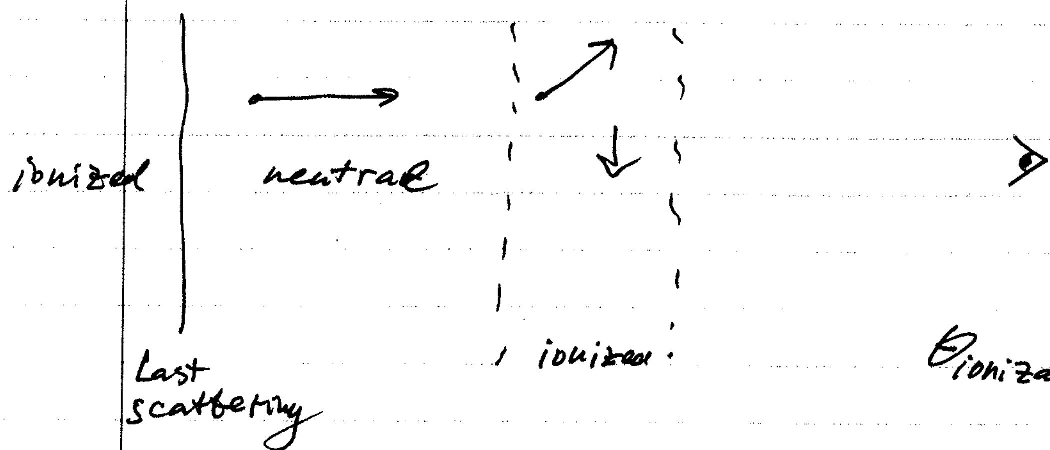


$$\Delta z \Rightarrow \boxed{\Delta \ell \approx \frac{15}{\sqrt{\Omega_0}}\ \mathrm{Mpc}}$$

$$\boxed{\Delta\theta \approx 8'\sqrt{\Omega_0}\cdot h}$$

z probability to receive a photon emitted at z

$\Rightarrow$ secondary reionization



$$\theta_{ionization} \simeq 3^{\circ}\, \Omega^{1/2} \left(\frac{100}{z_{ion}}\right)^{1/2}$$

maximum value $\boxed{\theta_{ion,max} = 7.^{\circ}4 \left(\Omega_{Bar}\, \Omega_0\, h\right)^{1/3}}$

secondary ionization wipes out fluctuations coming from the surface of last scattering, but creates its own fluctuations due to motion of baryons at the epoch of the secondary ionization.