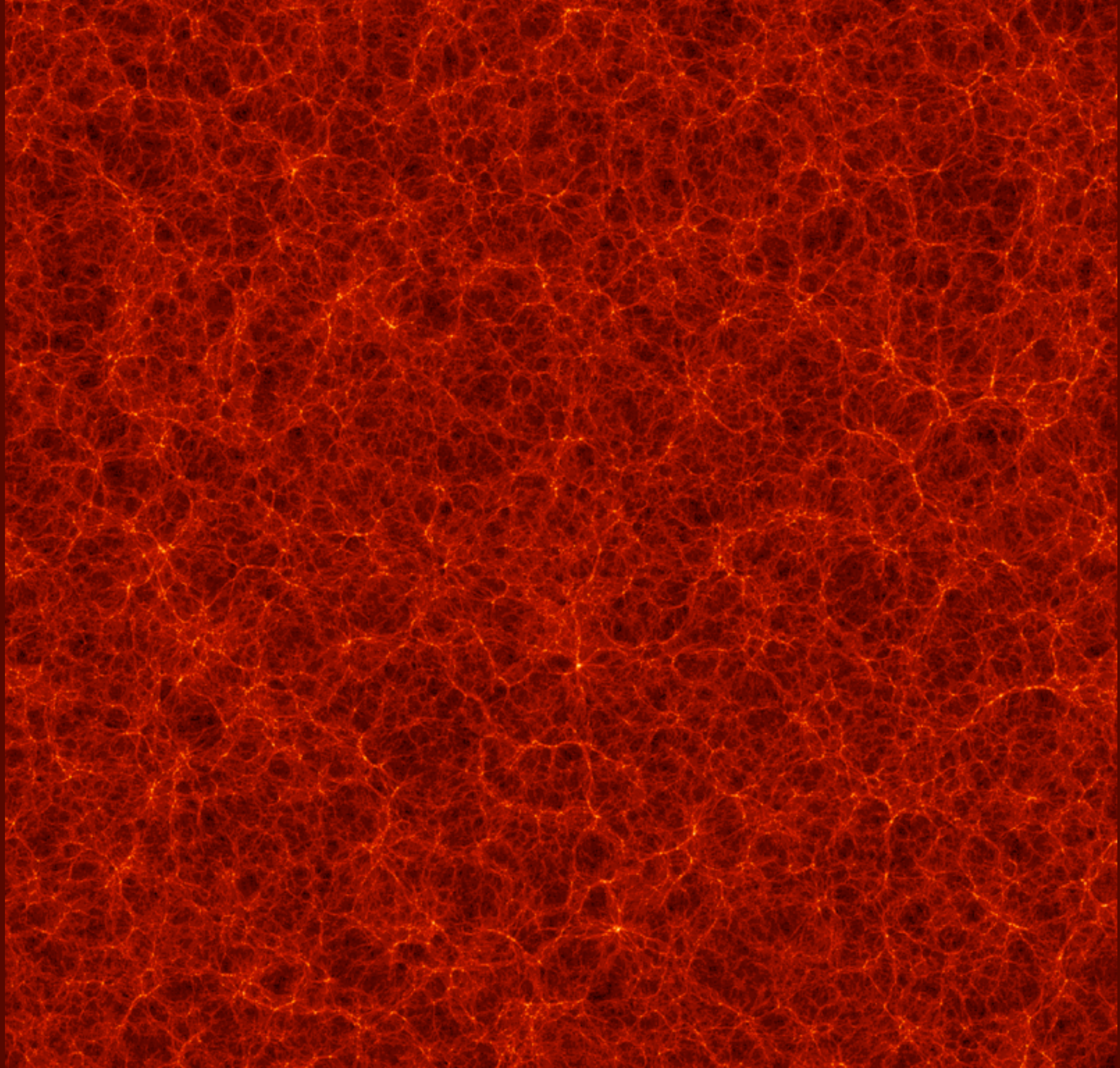
Nonlinear evolution of cosmological fluctuations

z=0.53

1Gpc

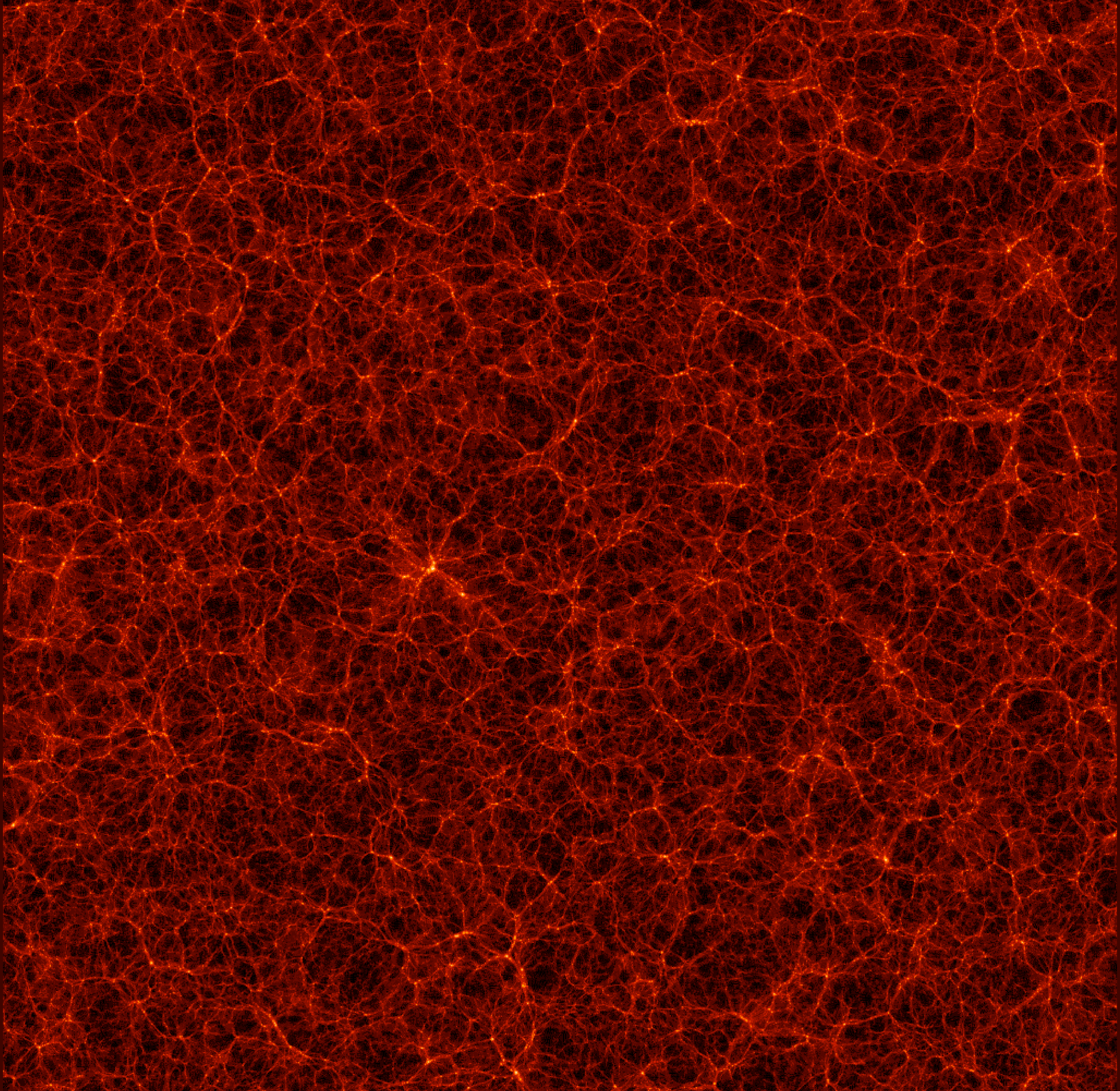
10Mpc slice

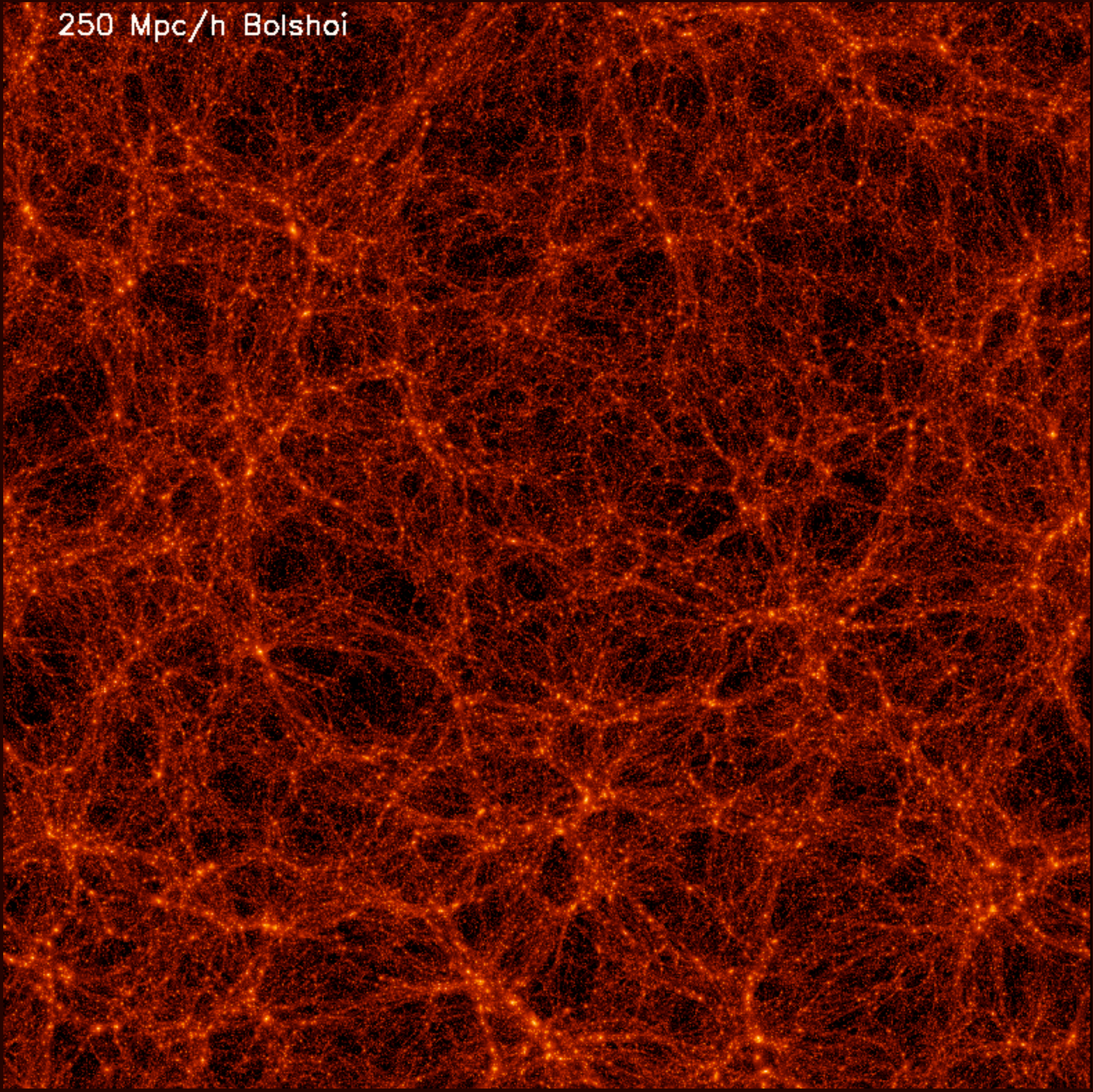


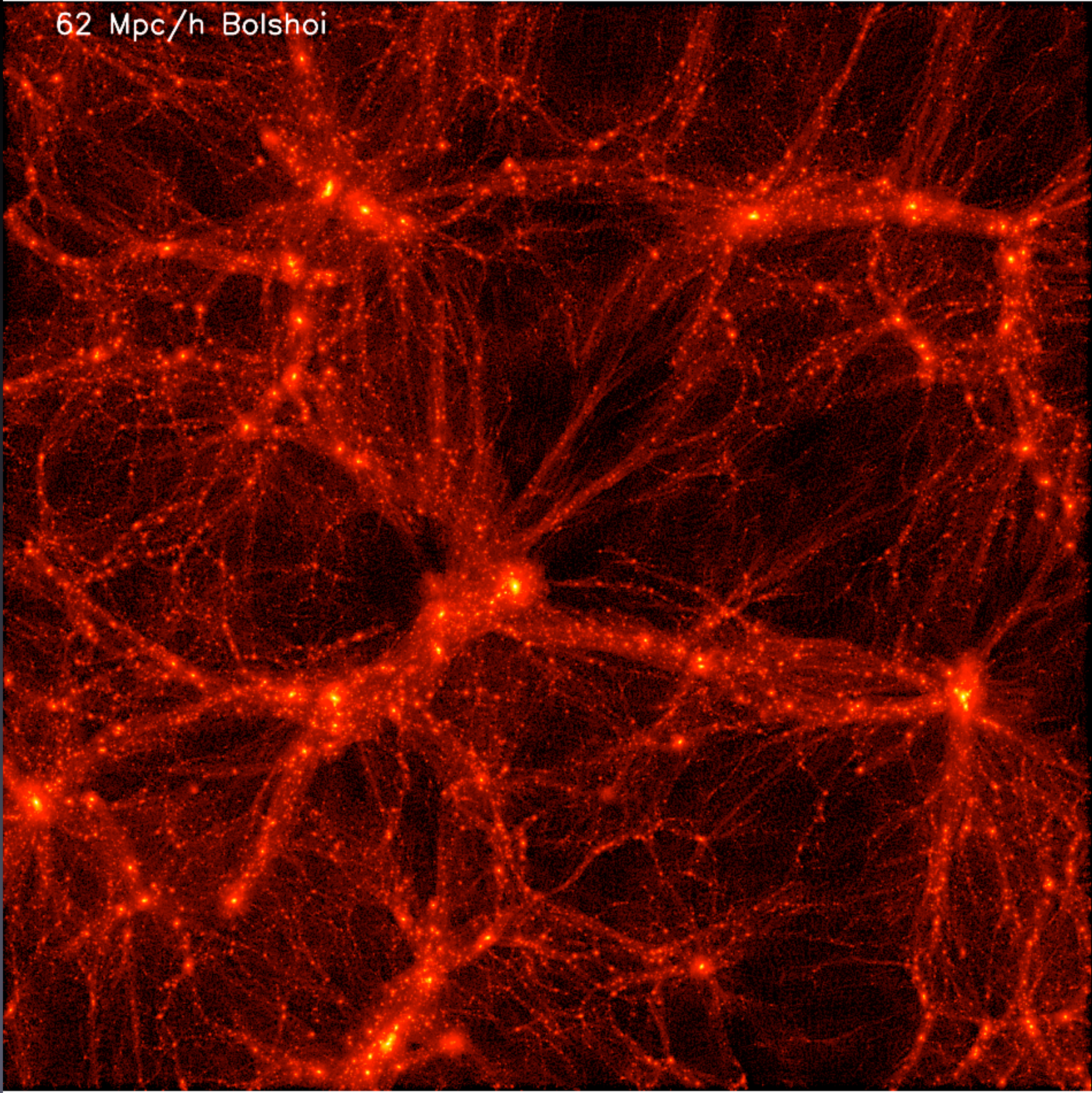
z = 0

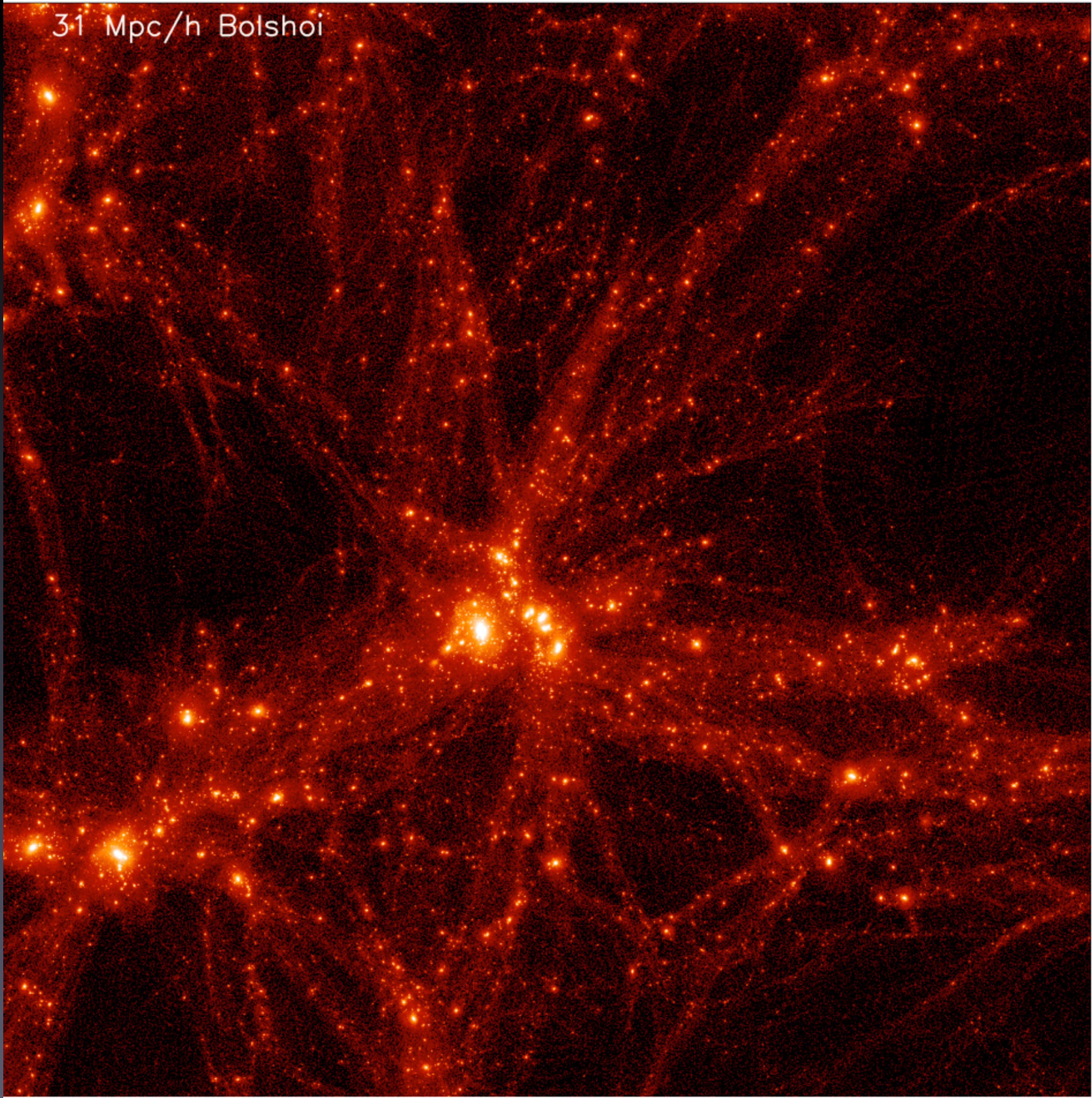
1Gpc

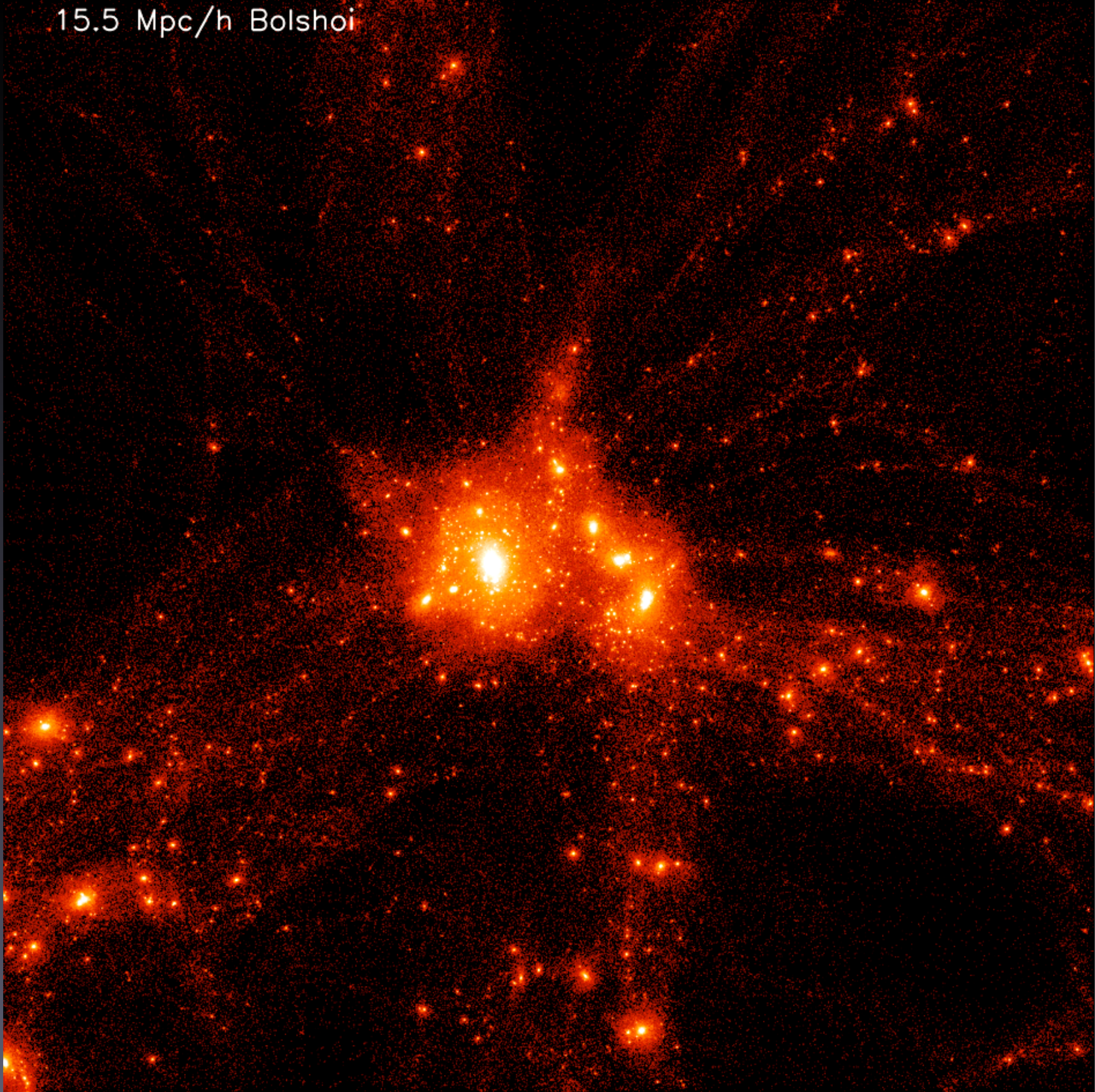
5Mpc slice



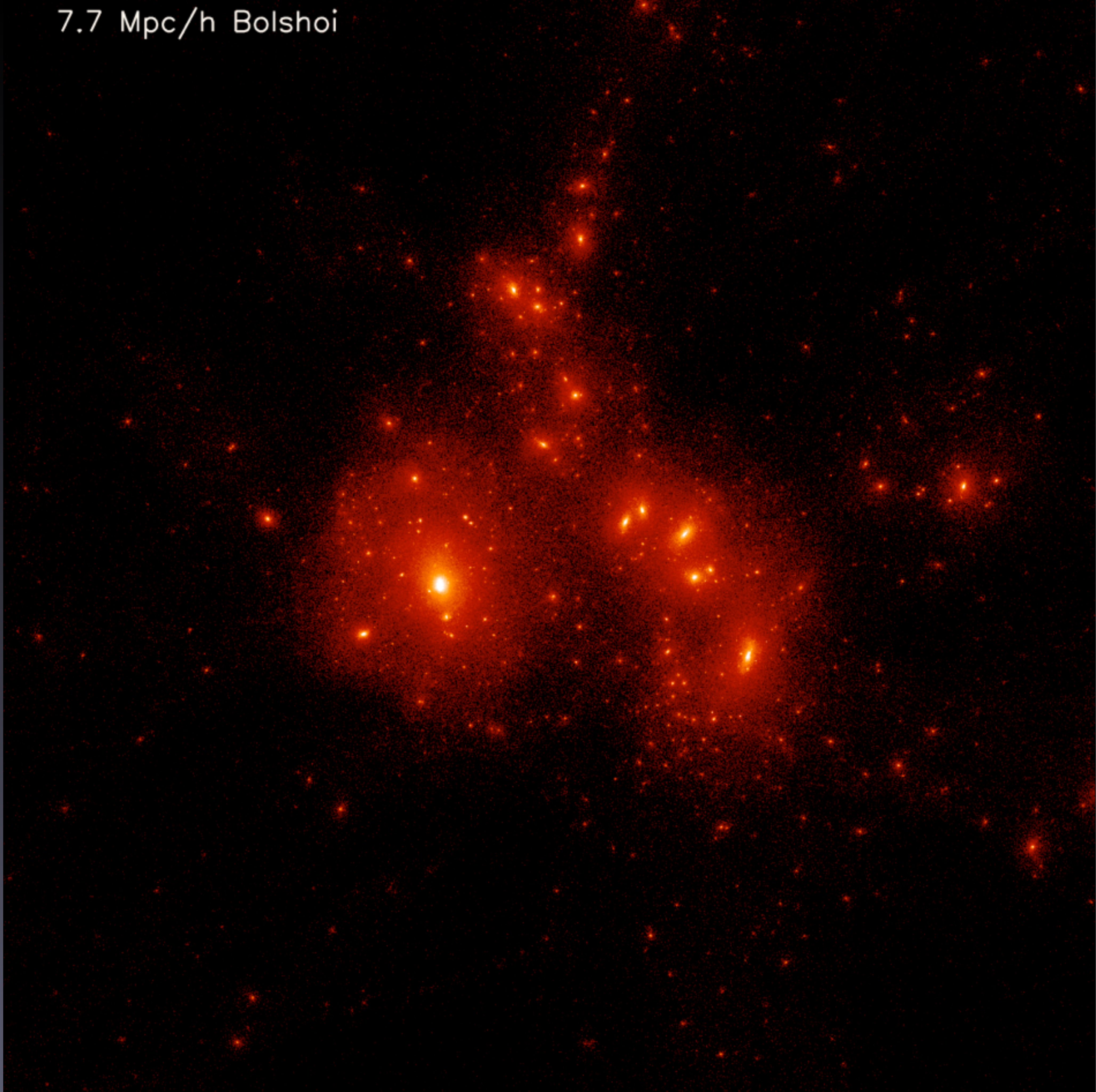

250 Mpc/h Bolshoi


62 Mpc/h Bolshoi


31 Mpc/h Bolshoi

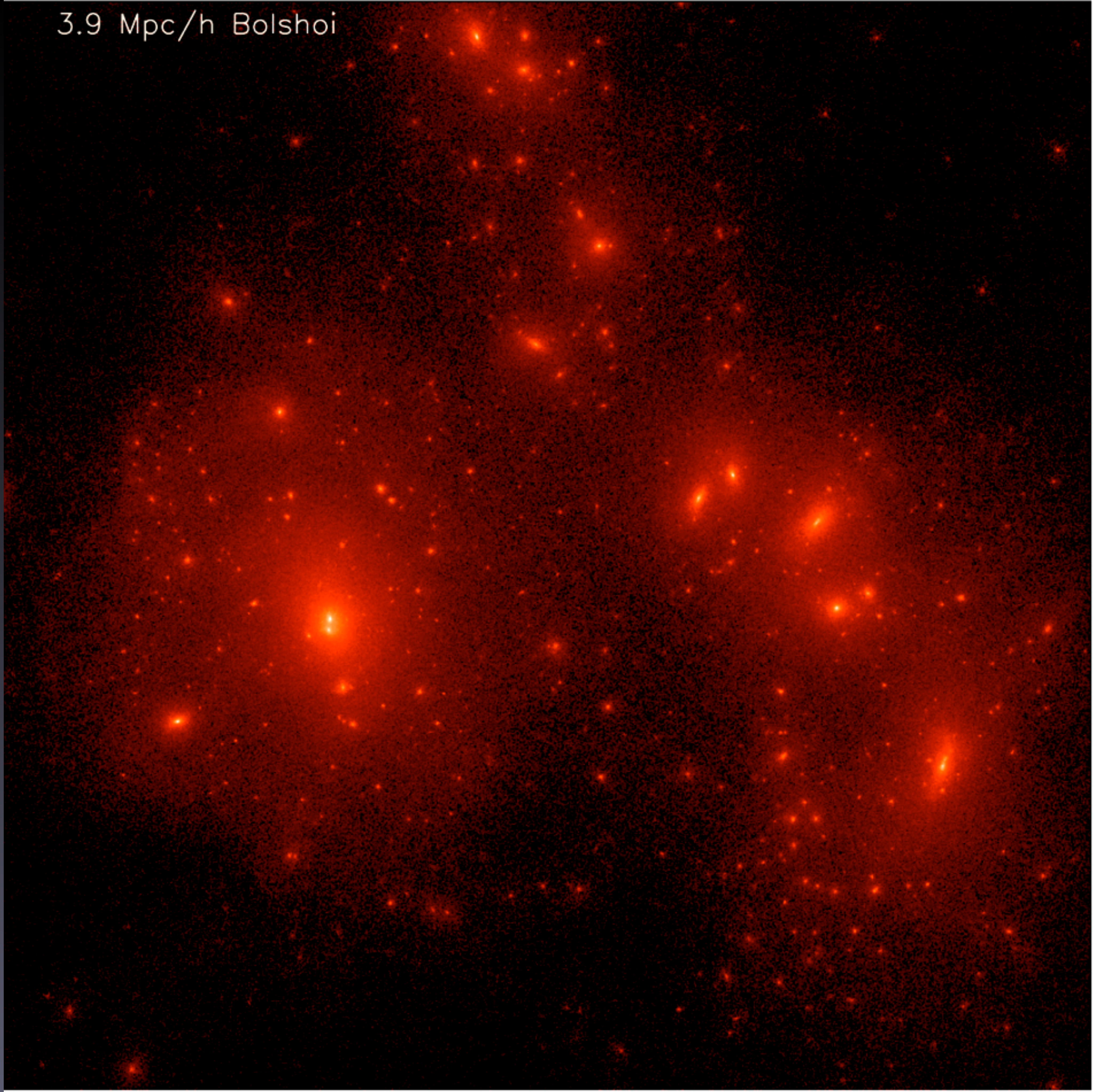

15.5 Mpc/h Bolshoi

Small Galaxy Group



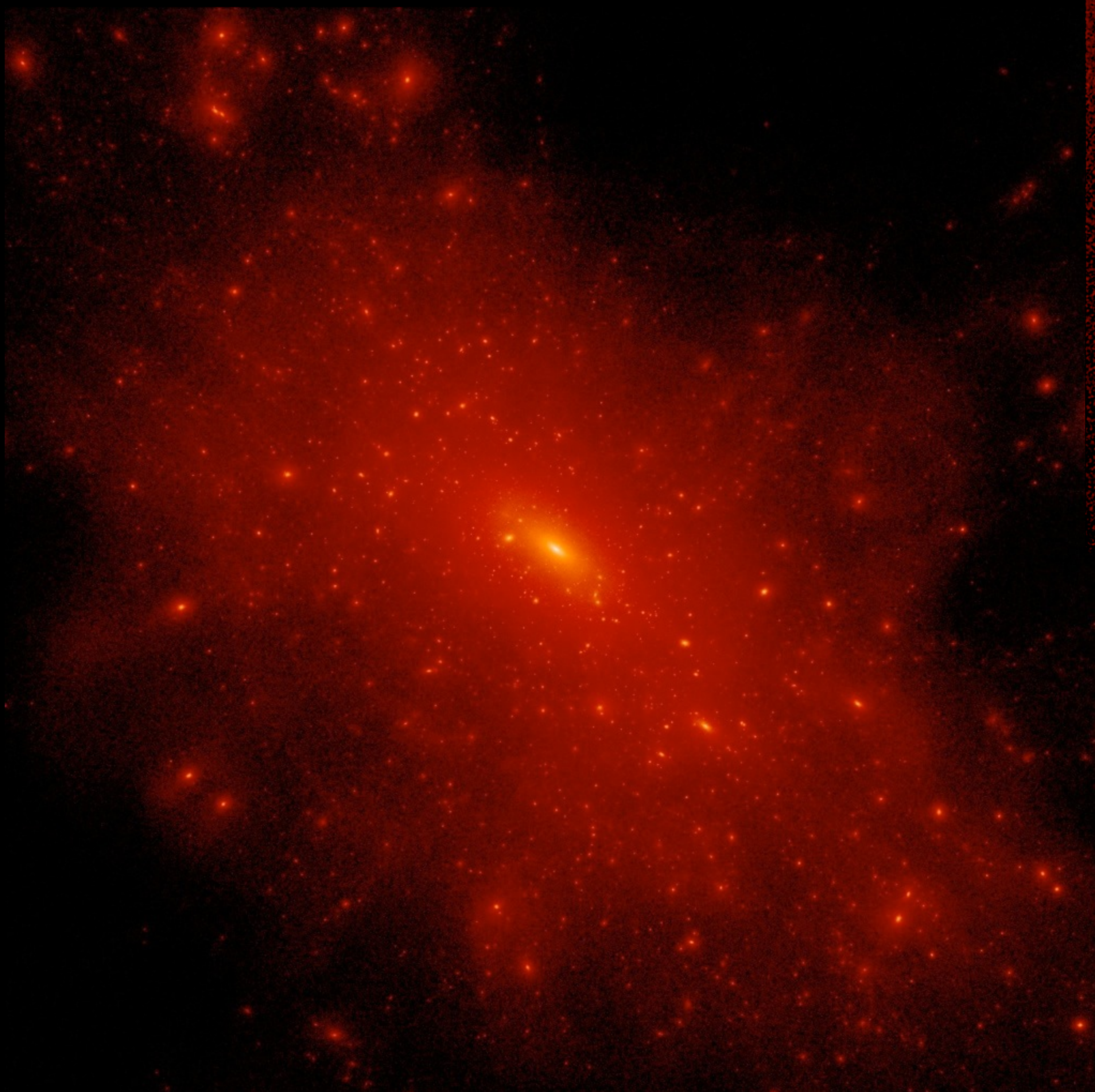
Small Galaxy Group

Central Region



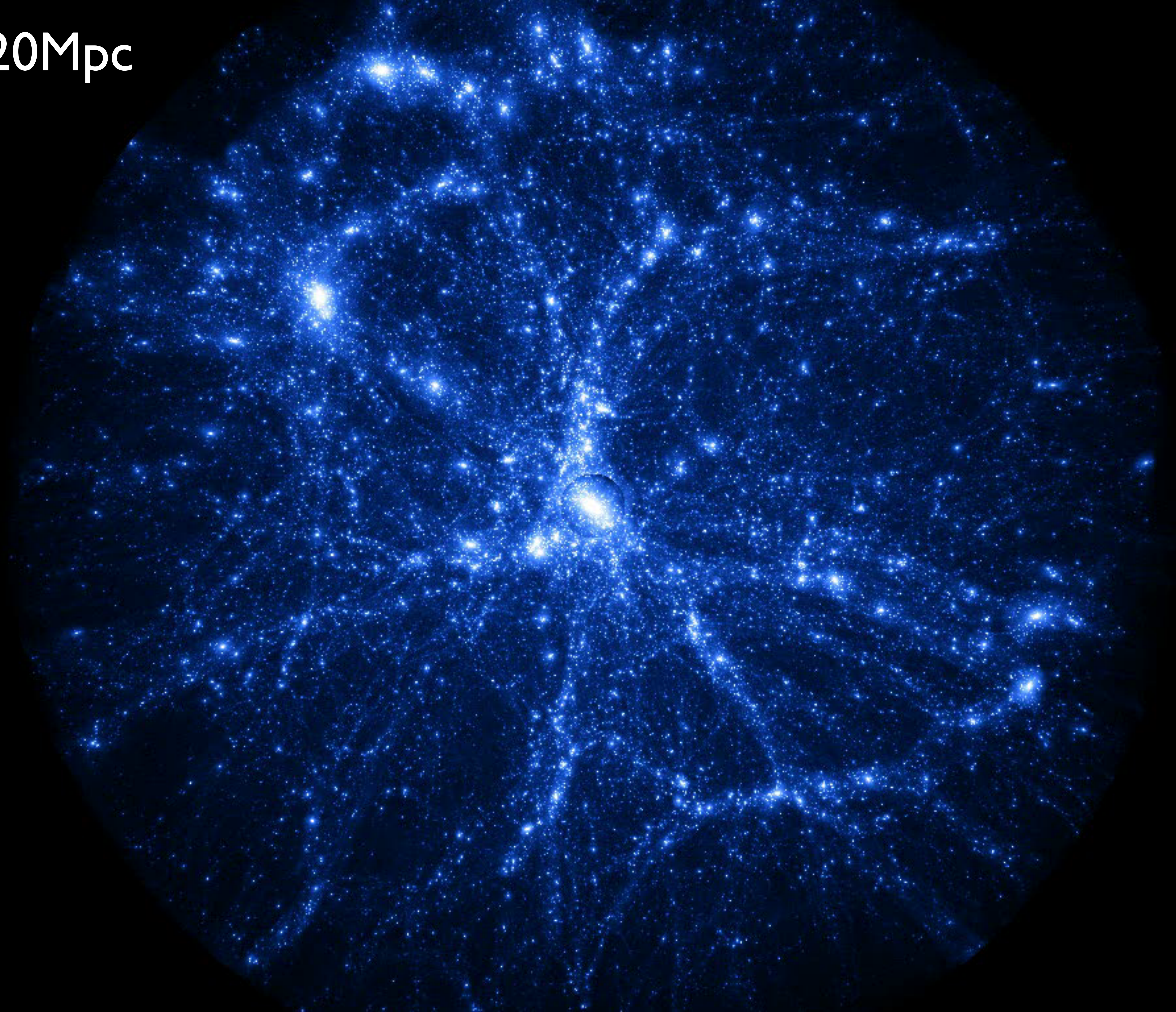
“Coma” cluster of galaxies

7.7Mpc



1.9Mpc

20Mpc



Power Spectrum

Consider a field of density perturbations in a volume V

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \rho_b}{\rho_b}$$

The average of the density contrast is equal to zero:

$$\langle \delta(\vec{x}) \rangle = 0$$

Let's find the dispersion of the density contrast:
Decompose the density contrast into the Fourier spectrum

$$\langle \delta^2(\vec{x}) \rangle$$

$$\delta(\vec{x}) = \frac{V}{(2\pi)^3} \int d^3k \, \delta_{\vec{k}} \, e^{-i\vec{k}\vec{x}}$$

$$\delta_{\vec{k}} = \frac{1}{V} \int d^3x \, \delta(\vec{x}) \, e^{i\vec{k}\cdot\vec{x}}$$

Find the dispersion:

$$\langle \delta^2(\vec{x}) \rangle = \frac{1}{V}\int d^3x\, \delta^2(\vec{x}) = \frac{1}{V}\int d^3x\, \delta(x)\,\delta(x) =$$

$$= \int \frac{d^3x}{V}\, V\int \frac{d^3K}{(2\pi)^3}\,\delta_{\vec{K}}\, e^{-i\vec{K}\vec{x}}\; V\int \frac{d^3K'}{(2\pi)^2}\,\delta^*_{\vec{K}'}\, e^{i\vec{K}'\vec{x}} =$$

$$\delta_{-\vec{K}} = \delta^*_{\vec{K}}$$

$$= \int \frac{d^3x}{V}\; V^2 \int \frac{d^3K\,d^3K'}{(2\pi)^6}\,\delta_{\vec{K}}\,\delta^*_{\vec{K}'}\, e^{-i(\vec{K}-\vec{K}')\vec{x}}$$

Change the order of integration:

$$\langle \delta^2 \rangle = \int \frac{d^3K\,d^3K'}{(2\pi)^6}\,\delta_{\vec{K}}\,\delta^*_{\vec{K}'}\; \underbrace{V^2 \int \frac{d^3x}{V}\, e^{-i(\vec{K}-\vec{K}')\vec{x}}}_{\delta(\vec{K}-\vec{K}')}$$

$$= V\int \frac{d^3K}{(2\pi)^3}\,|\delta_{\vec{K}}|^2 = \frac{V}{(2\pi)^3}\int_0^\infty 4\pi K^2 dK\,|\delta_K|^2$$

Thus, we get:

$$\langle \delta^2 \rangle = \frac{V}{2\pi^2}\int_0^\infty K^2 \delta_K^2\, dK$$

We define the **power spectrum** as

Here the averaging is done over all
Waves with given k and over the whole space

$$P(k) = \langle |\delta_{\vec{k}}|^2 \rangle$$

Correlation function is defined as

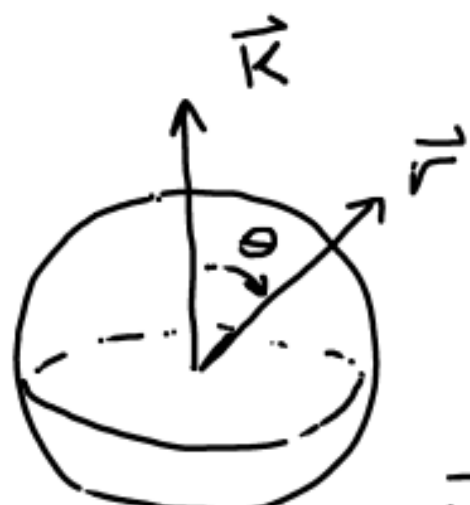
$$\xi(r) = \langle \delta(\vec{x})\, \delta(\vec{x}+\vec{r}) \rangle$$

The averaging is done for the whole
volume and over angles of vector $\vec{r}$

$$\xi(r) = \frac{1}{4\pi}\int d\Omega \int \frac{d^3x}{V} \int \frac{V d^3k}{(2\pi)^3}\, \delta_{\vec{k}}\, e^{-i\vec{k}\vec{x}} \int \frac{V d^3k'}{(2\pi)^3}\, \delta^*_{\vec{k}'}\, e^{i\vec{k}'(\vec{x}+\vec{r})} =$$

$$= \frac{1}{4\pi}\int d(\cos\theta)\, d\varphi \int \frac{V d^3k}{(2\pi)^3}\, \delta_{\vec{k}}\, e^{i\vec{k}\vec{r}}\, \delta^*_{\vec{k}} =$$

$$= \int \frac{V d^3k}{(2\pi)^3}\, \delta_{\vec{k}}\, \delta^*_{\vec{k}}\, \frac{1}{4\pi}\int d(\cos\theta)\, d\varphi\, e^{i\vec{k}\vec{r}}$$



imaginary

$$e^{i\vec{k}\vec{r}} = \cos(kr\cos\theta) + i\sin(kr\cos\theta)$$

$$\frac{1}{4\pi}\int_{-1}^{1} dx \int_0^{2\pi} d\varphi \cos(krx) = \frac{\sin(kr)}{kr}$$

Thus, we get the relation between the correlation function and the power spectrum:

$$\xi(r) = \frac{1}{2\pi^2} \int_0^\infty k^2 dk \, \frac{\sin kr}{kr} P(k)$$

There is an inverse relation:

$$P(k) = 4\pi \int r^2 dr \, \xi(r) \, \frac{\sin kr}{kr}$$

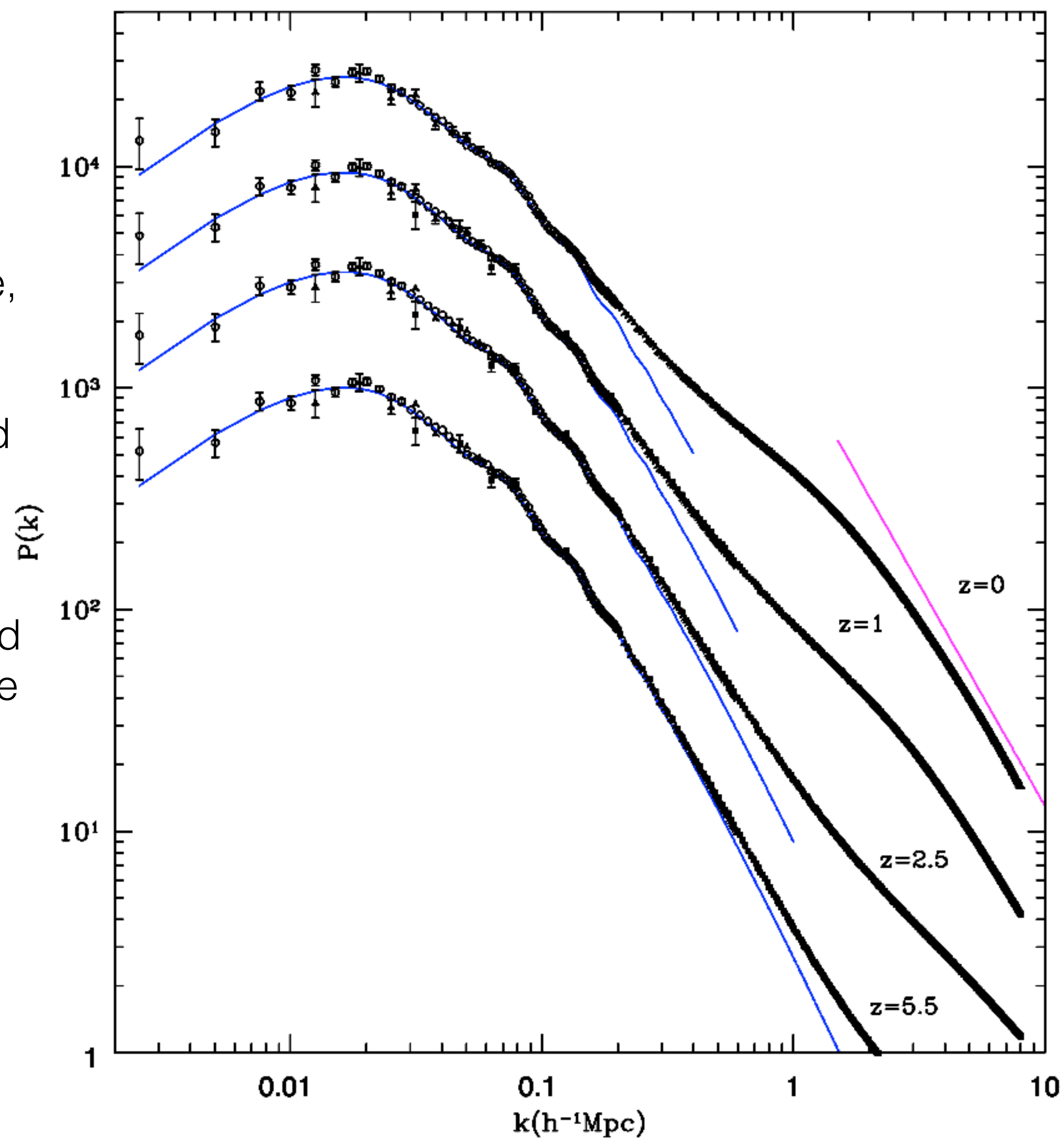
Evolution of the power spectrum of dark matter.

three regimes:

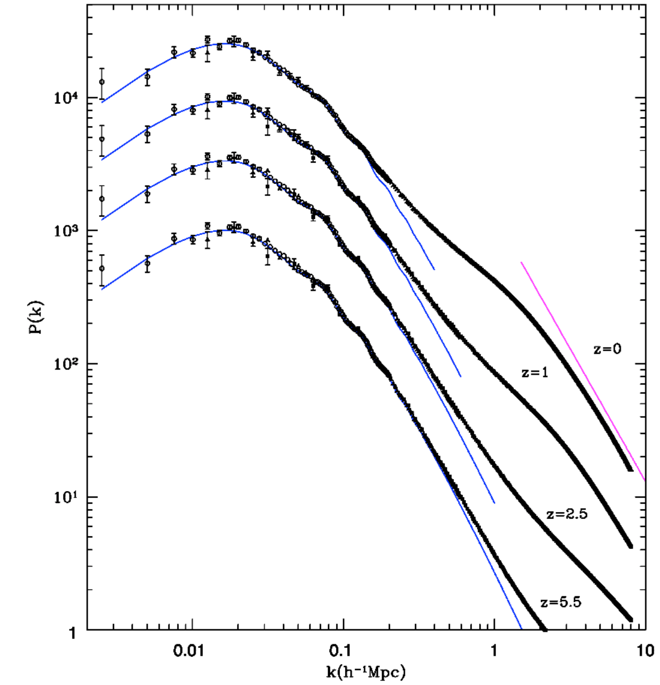
- linear regime: fluctuations increase the amplitude, but shape of P(k) is the same

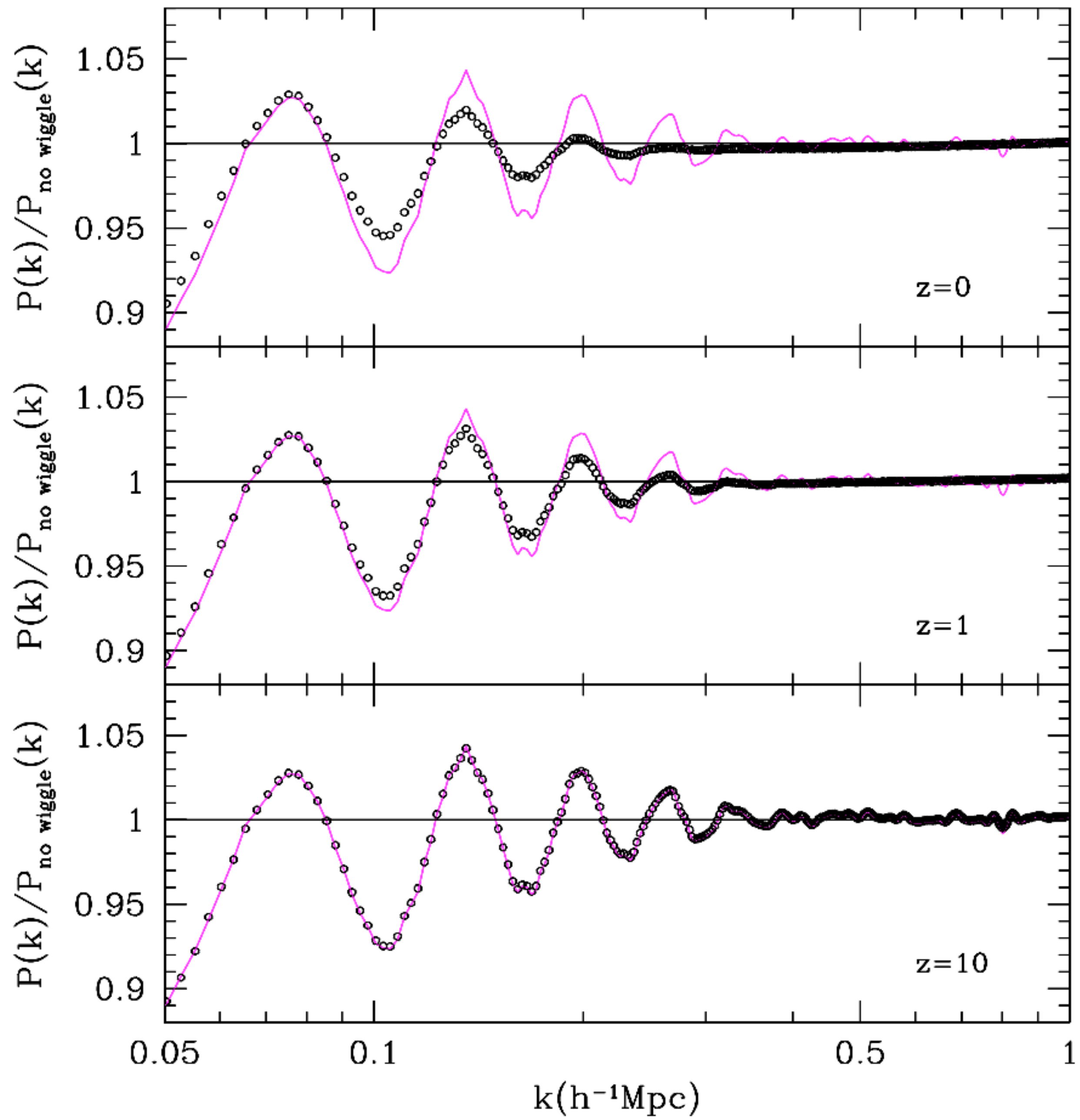
- mildly nonlinear regime: fluctuations collapse and amplitude grows faster than in linear regime. Shape evolves with time.

- Deep nonlinear regime: dark matter has collapsed into virtualized dark matter halos that do not change their physical interior mass. In comoving coordinates they get smaller as universe expands.



Evolution of BAO wiggles: dumping and shift due to (weak) non-linear gravitational coupling of modes. Effect is small, but is important when we use BAO features to estimate parameters of the Universe.





Magenta: initial conditions (z=100)
circles - results of n-body simulations

Correlation function: definition

This is usually quantified using the ***2-point correlation function***, ***ξ* (*r*),** defined as an “excess probability” of finding another galaxy at a distance ***r*** from some galaxy, relative to a uniform random distribution; averaged over the entire set:

$$dN(r) = \rho_0 \big(1 + \xi(r)\big) dV_1 dV_2$$

Correlation function is often approximated with a power law:

$$\xi(r) = \left(r / r_0\right)^{-\gamma}$$

Parameter r_0 is called the correlation length

Estimators of the correlation function

- Simplest estimator: count the number of data-data pairs, $\langle DD \rangle$, and the equivalent number in a randomly generated (Poissonian) catalog, $\langle RR \rangle$:

$$\xi(r)_{est} = \frac{\langle DD \rangle}{\langle RR \rangle} - 1$$

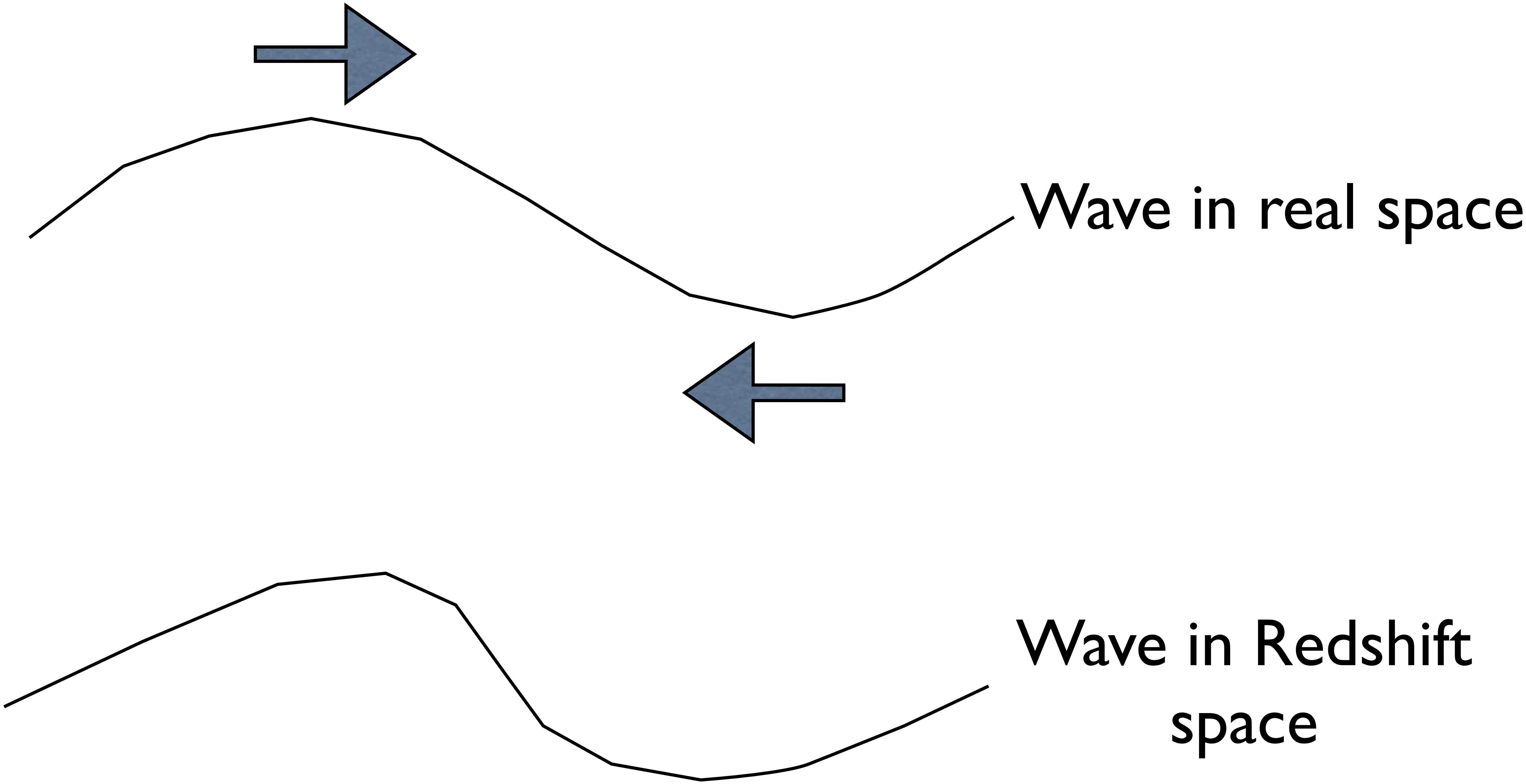
- A better (Landy-Szalay) estimator is:

$$\xi(r)_{est} = \frac{\langle DD \rangle - 2\langle RD \rangle + \langle RR \rangle}{\langle RR \rangle}$$

where $\langle RD \rangle$ is the number of data-random pairs

- This takes care of the edge effects, where one has to account for the missing data outside the region sampled, which can have fairly irregular boundaries

Redshift distortions: long waves



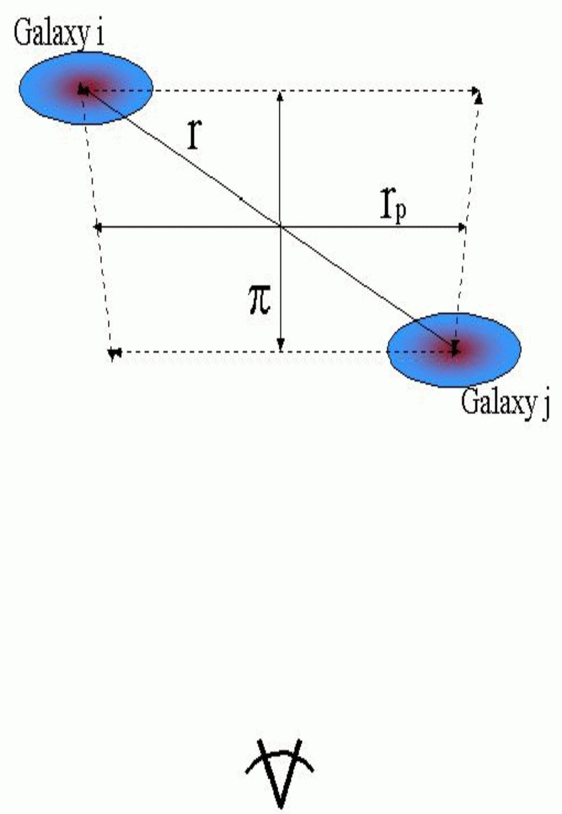
$V = Hr + V_{pec}$

Angular and 3D correlation functions

$$w(r_p) = 2\int_{r_p}^{\infty} \xi(r)\left(r^2 - r_p^2\right)^{-1/2} dr$$

r_p: projected distance between pairs of galaxies,

Π: distance parallel to the line of sight



$$w(r_p) = \int_{-\delta\pi}^{+\delta\pi} \xi(r_p, \pi)\, d\pi$$

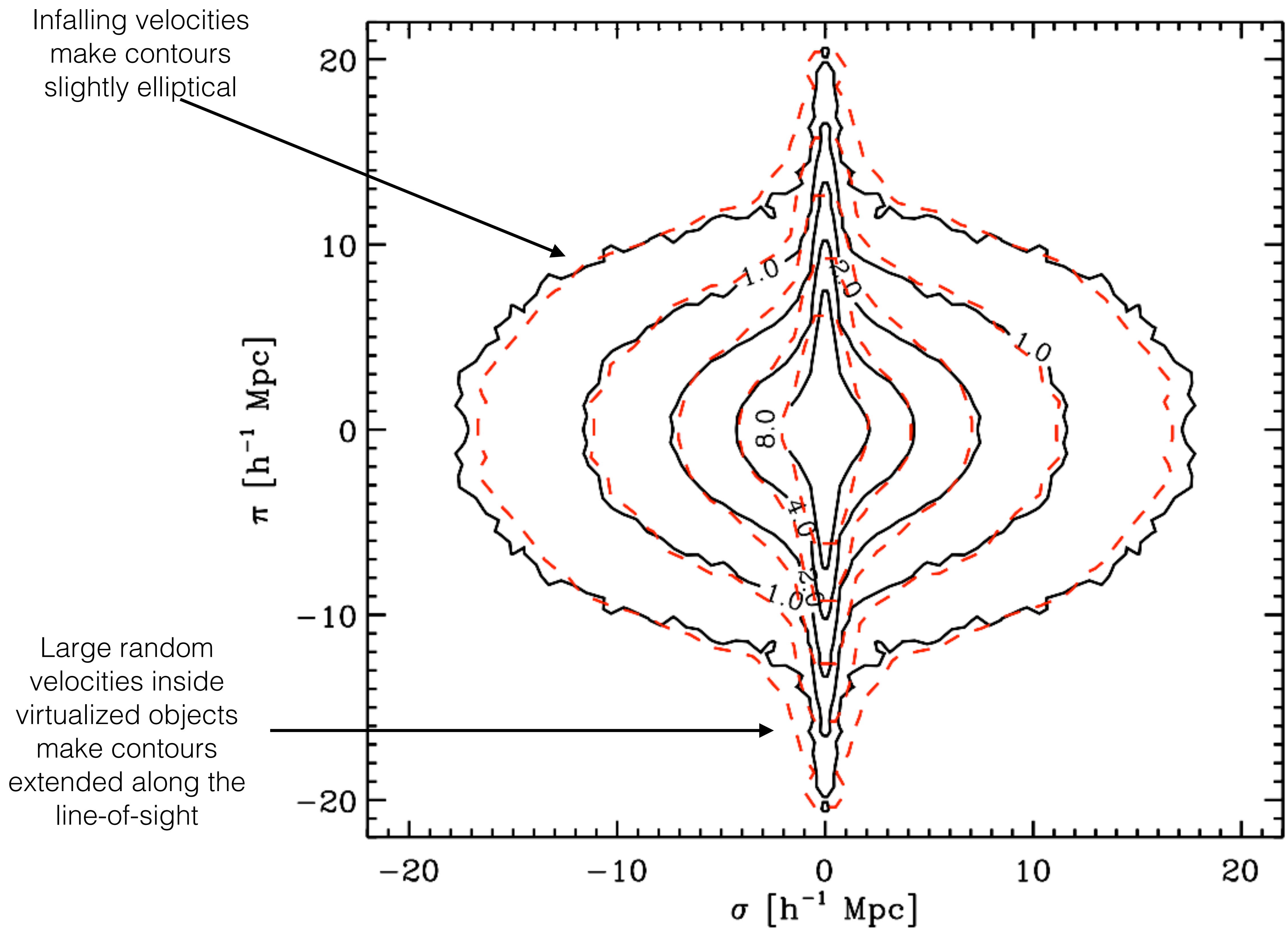
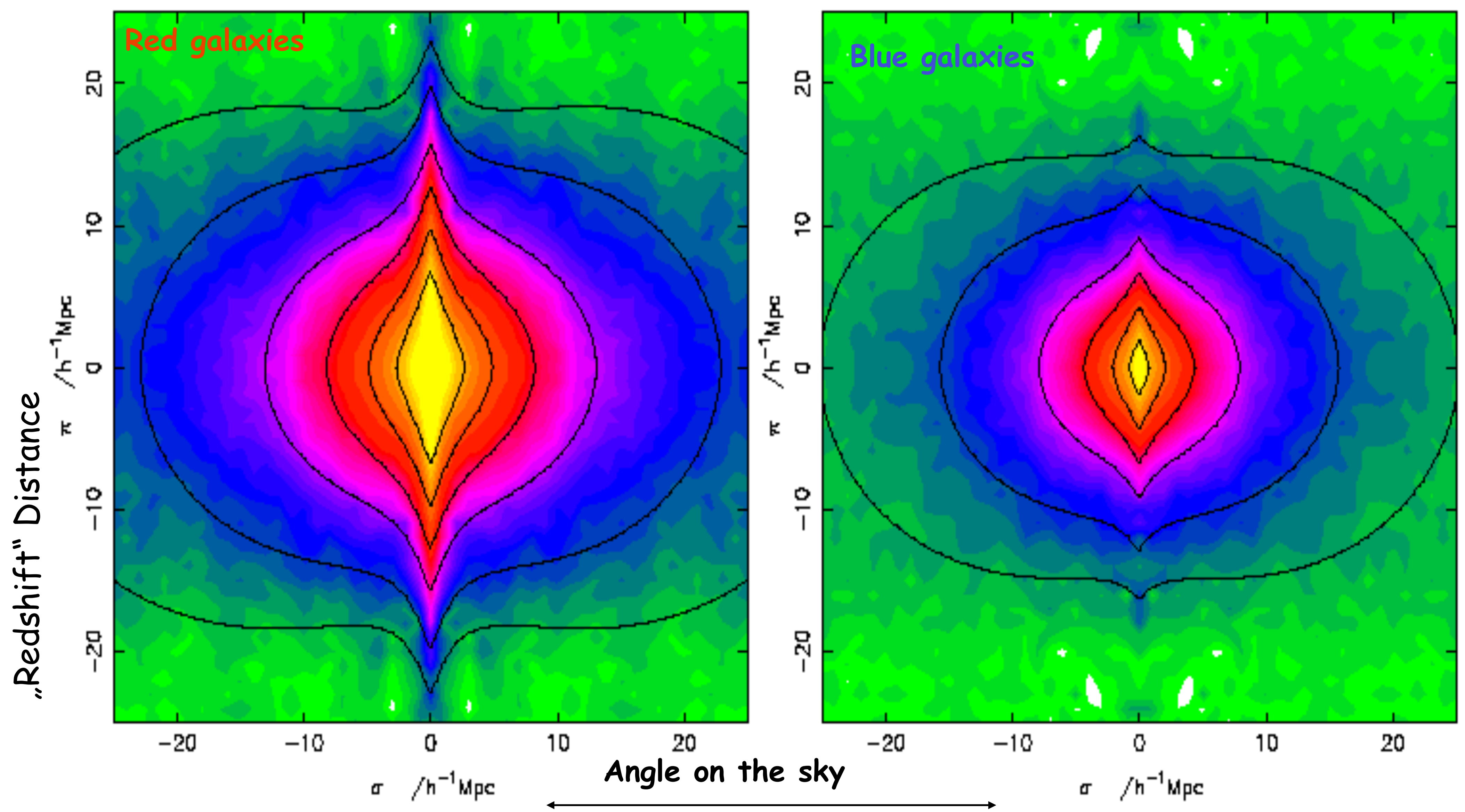


Figure 7. Contours of the two-dimensional correlation function $\xi(\sigma, \pi)$ estimated from the two-year BOSS-CMASS North galaxy sample (dashed line) at $0.4 < z < 0.7$ and for our MultiDark halo catalog constructed using the HAM technique at $z = 0.53$.

Redshift distortions: ‘finger-of-god’ effect on small scales



Angular correlation function $w_p(r_p)$

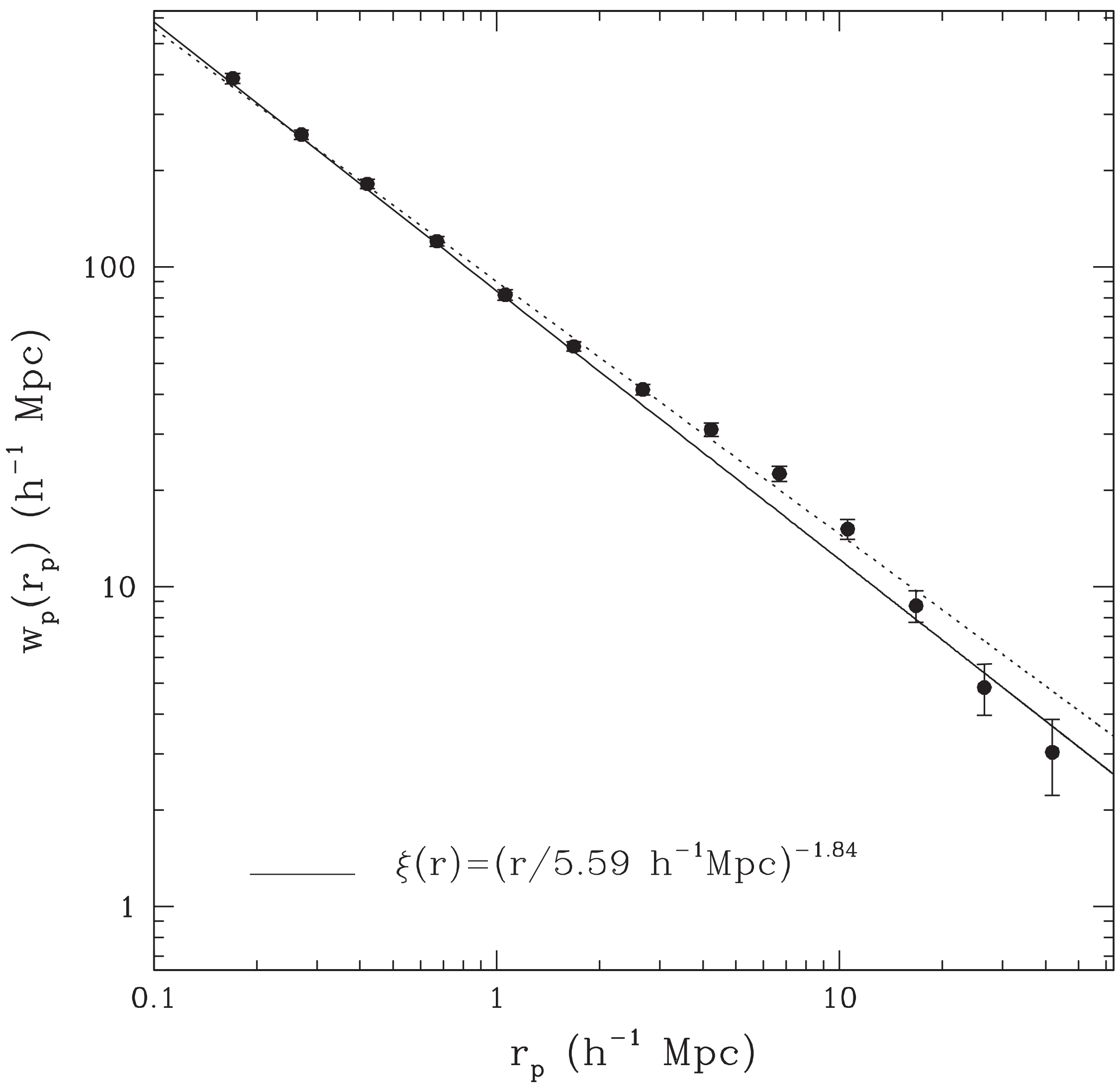


FIG. 6.—Projected galaxy correlation function $w_p(r_p)$ for the flux-limited galaxy sample. The solid line shows a power-law fit to the data points, using the full covariance matrix, which corresponds to a real-space correlation function $\xi(r) = (r/5.59\ h^{-1}\ \mathrm{Mpc})^{-1.84}$. The dotted line shows the fit when using only the diagonal error elements, corresponding to $\xi(r) = (r/5.94\ h^{-1}\ \mathrm{Mpc})^{-1.79}$. The fits are performed for $r_p < 20\ h^{-1}$ Mpc.

If only 2-D positions on the sky are known, then use angular separation $\boldsymbol{\theta}$ instead of distance $\boldsymbol{r}$:

$$\boldsymbol{w(\theta) = (\theta/\theta_0)^{-\beta}, \quad \beta = \gamma - 1}$$

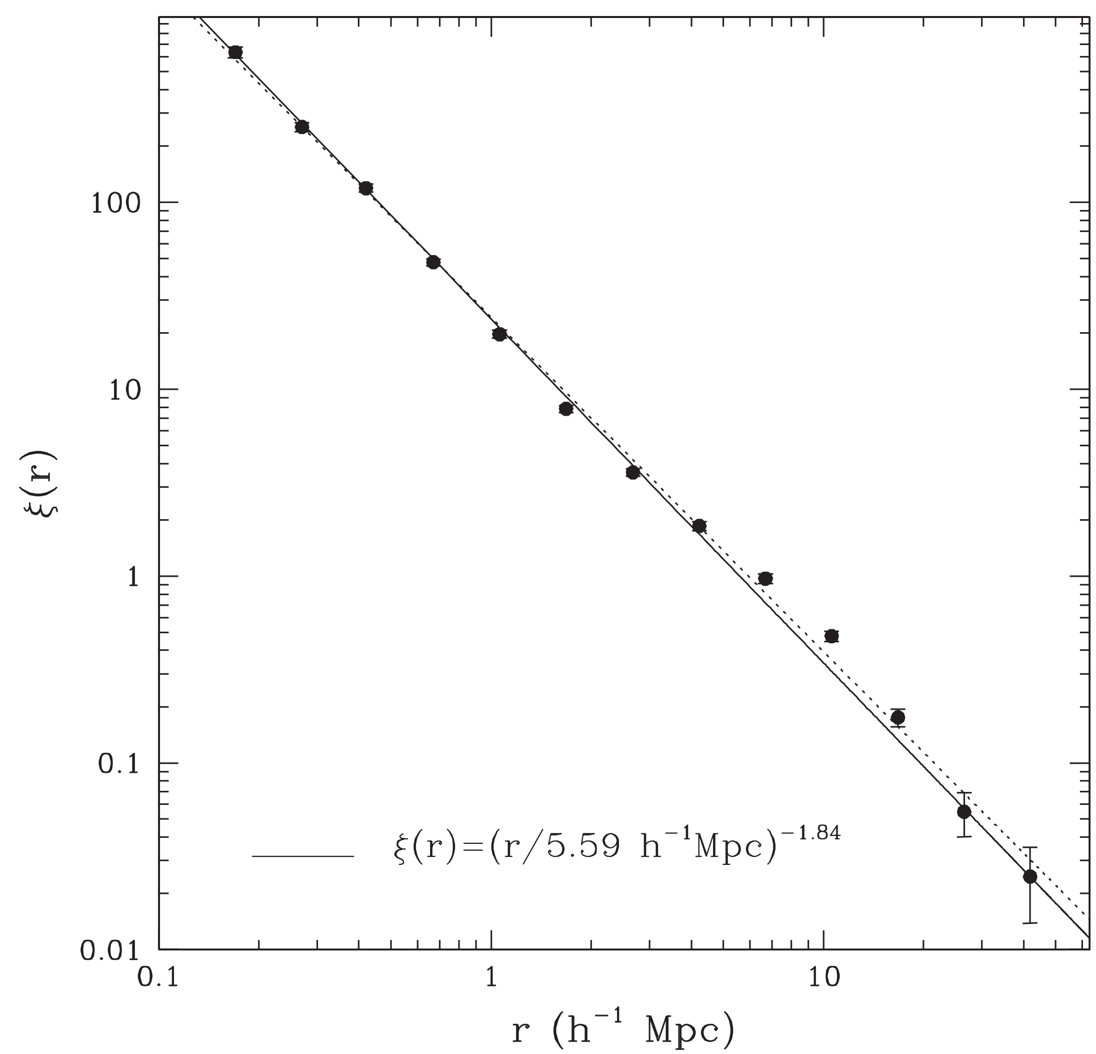
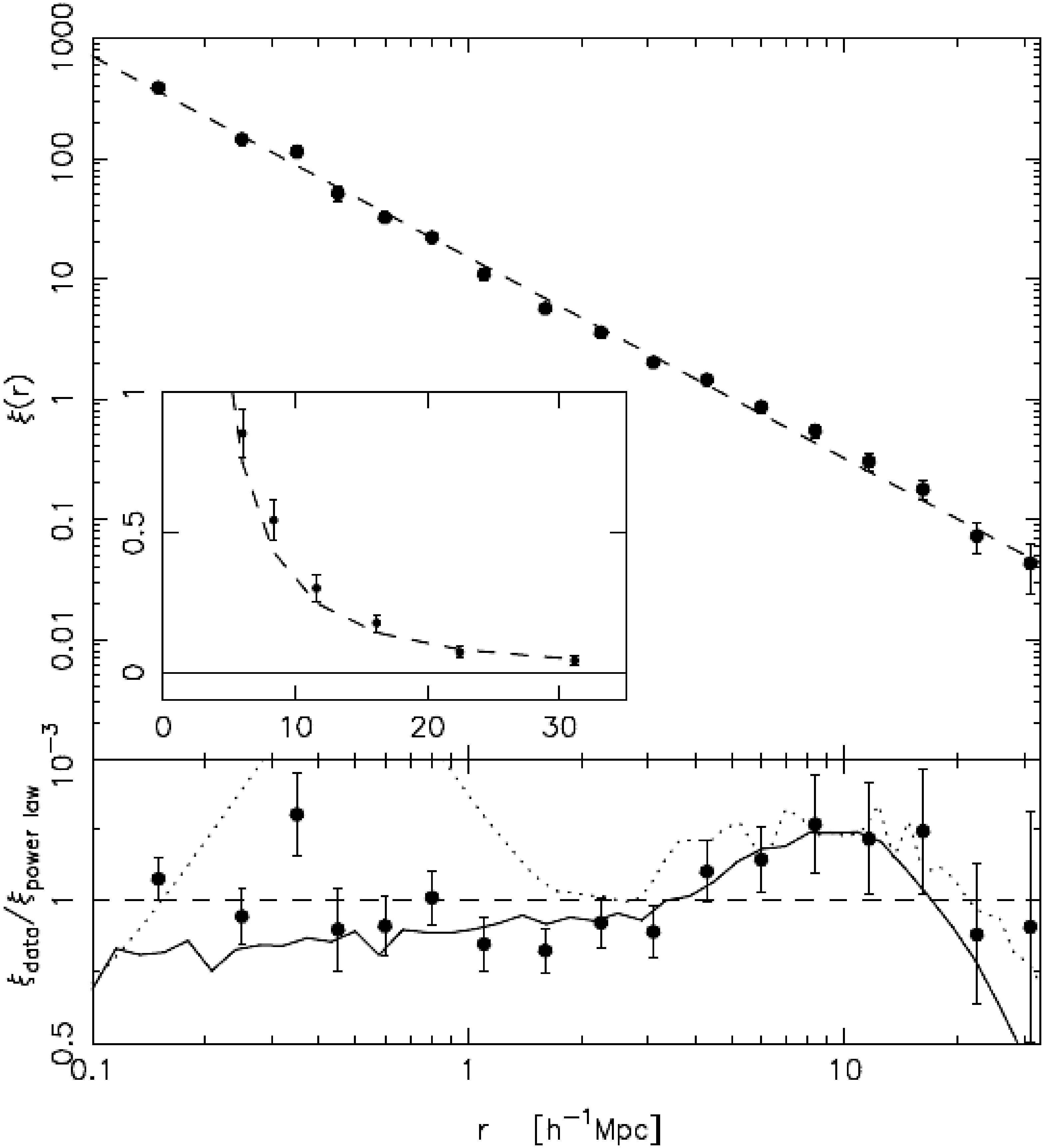


FIG. 7.—Real-space correlation function $\xi(r)$ for the flux-limited galaxy sample, obtained from $w_p(r_p)$ as discussed in the text. The solid and dotted lines show the corresponding power-law fits obtained by fitting $w_p(r_p)$ using the full covariance matrix or just the diagonal elements, respectively.

Galaxy Correlation Function

As measured by the 2dF redshift survey

Deviations from the power law:

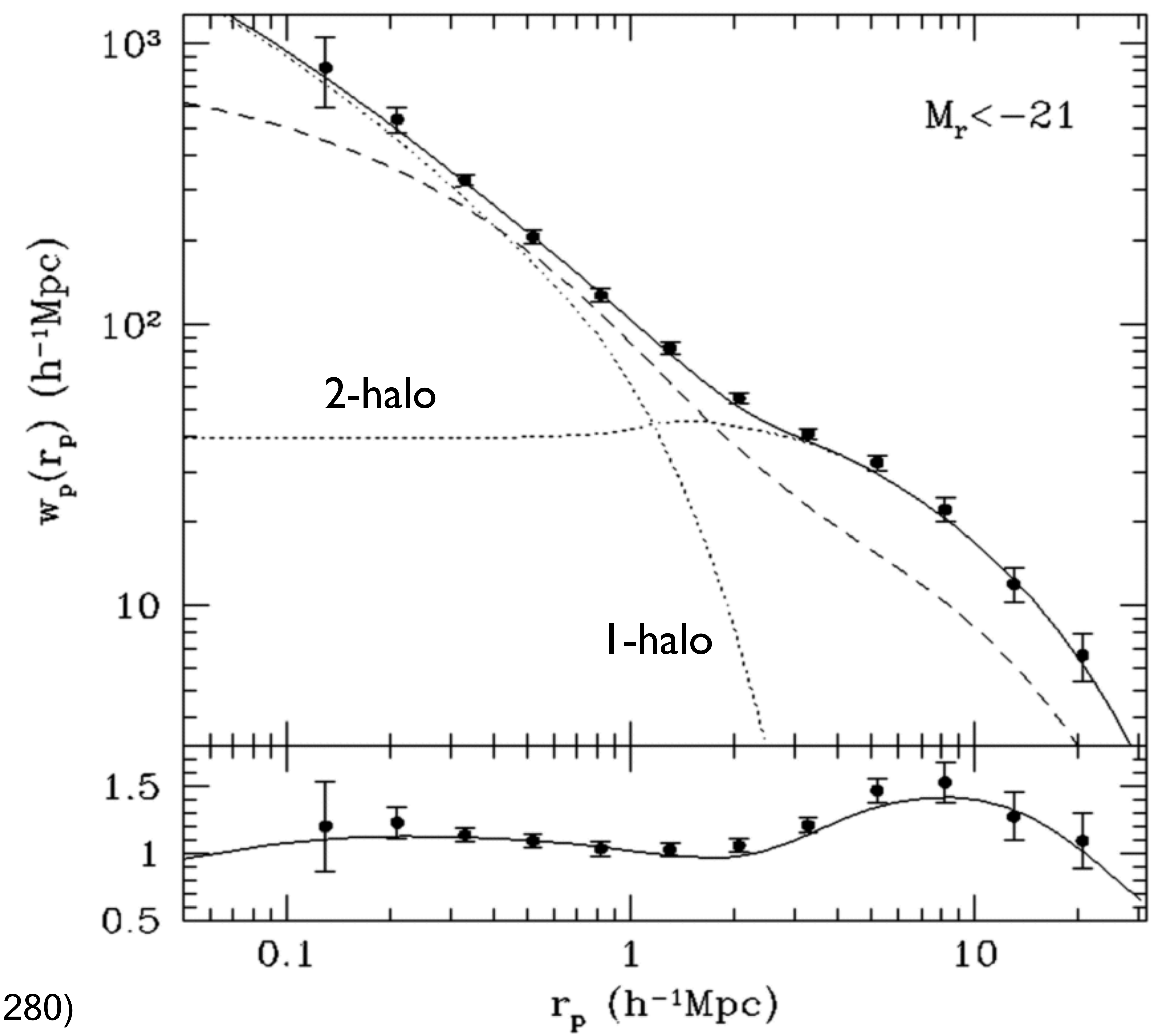


Angular correlation function: SDSS results

Two contributions:

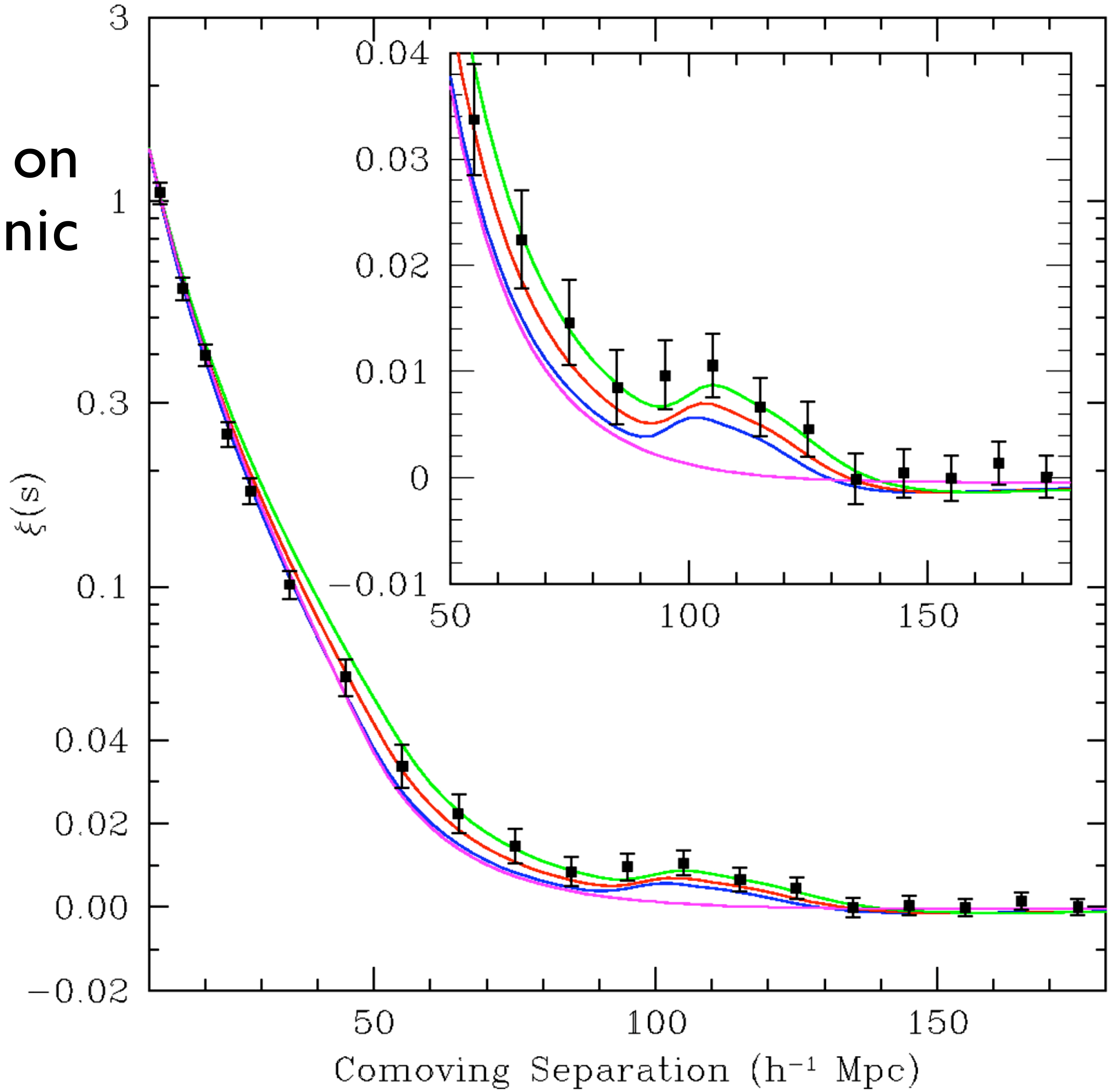
- number-density profile of galaxies inside the same halo

- clustering of halos



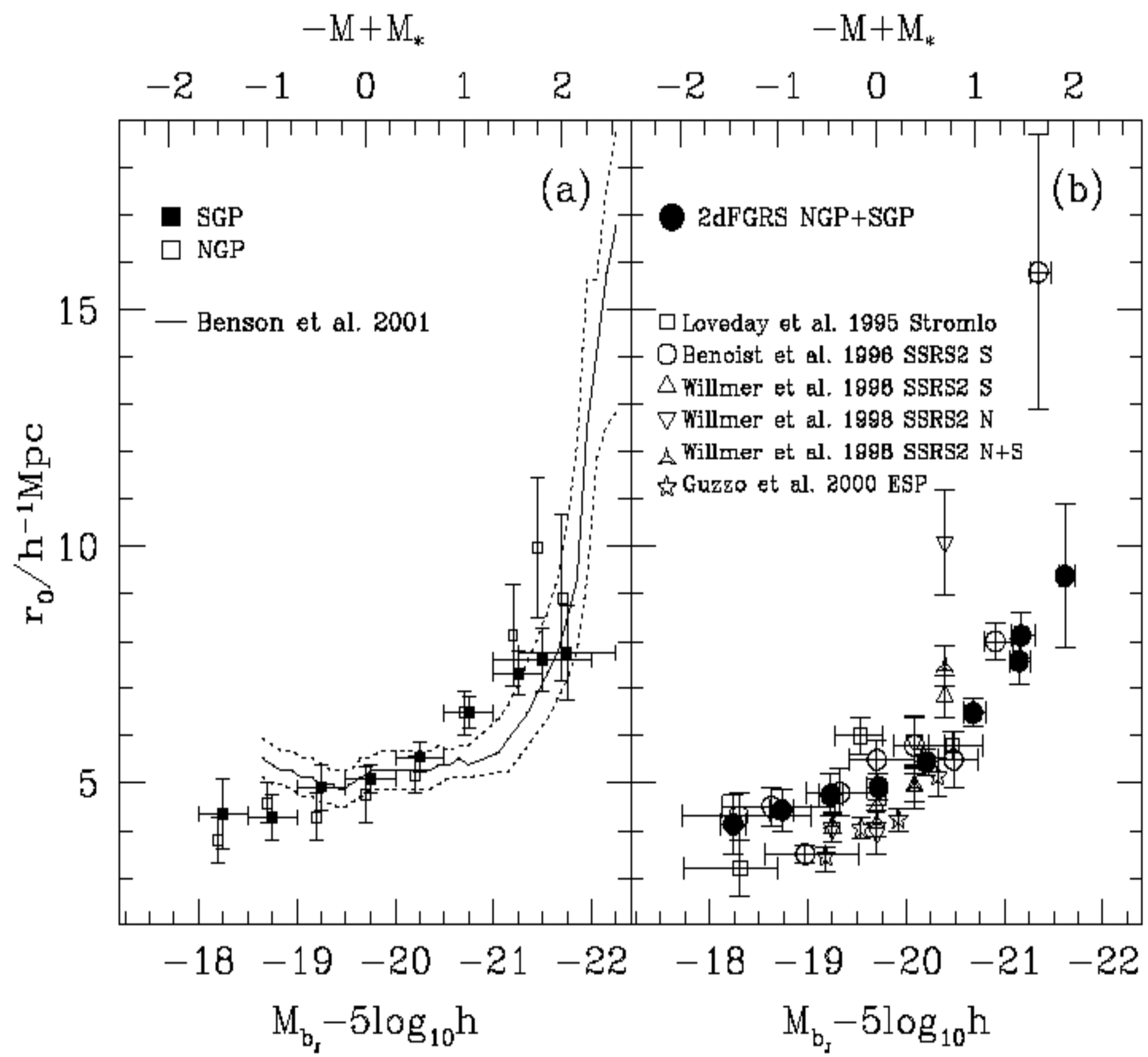
Zehavi et al. (astro-ph/0301280)

Correlation function on large scales: baryonic oscillations



SDSS (Eisenstein et al.)

Clustering of different galaxies



More luminous/massive galaxies are more strongly clustered

lustering of different galaxies

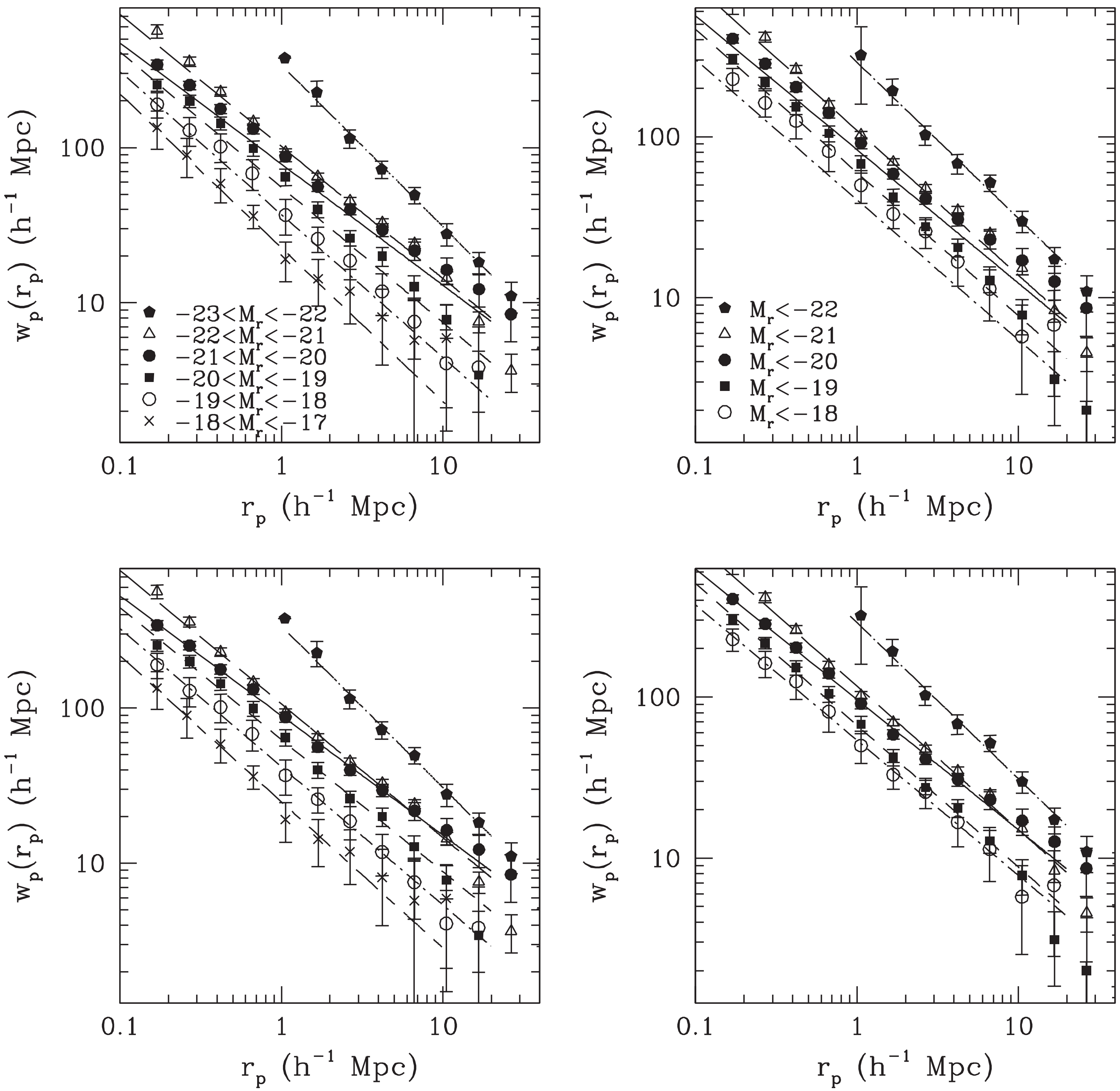


FIG. 8.—*Top left*: Projected galaxy correlation functions $w_p(r_p)$ for volume-limited samples with the indicated absolute magnitude and redshift ranges. Lines show power-law fits to each set of data points, using the full covariance matrix. *Top right*: Same as top left, but now the samples contain all galaxies brighter than the indicated absolute magnitude; i.e., they are defined by luminosity thresholds rather than luminosity ranges. *Bottom panels*: Same as the top panels, but now with power-law fits that use only the diagonal elements of the covariance matrix. [*See the electronic edition of the Journal for a color version of this figure.*]

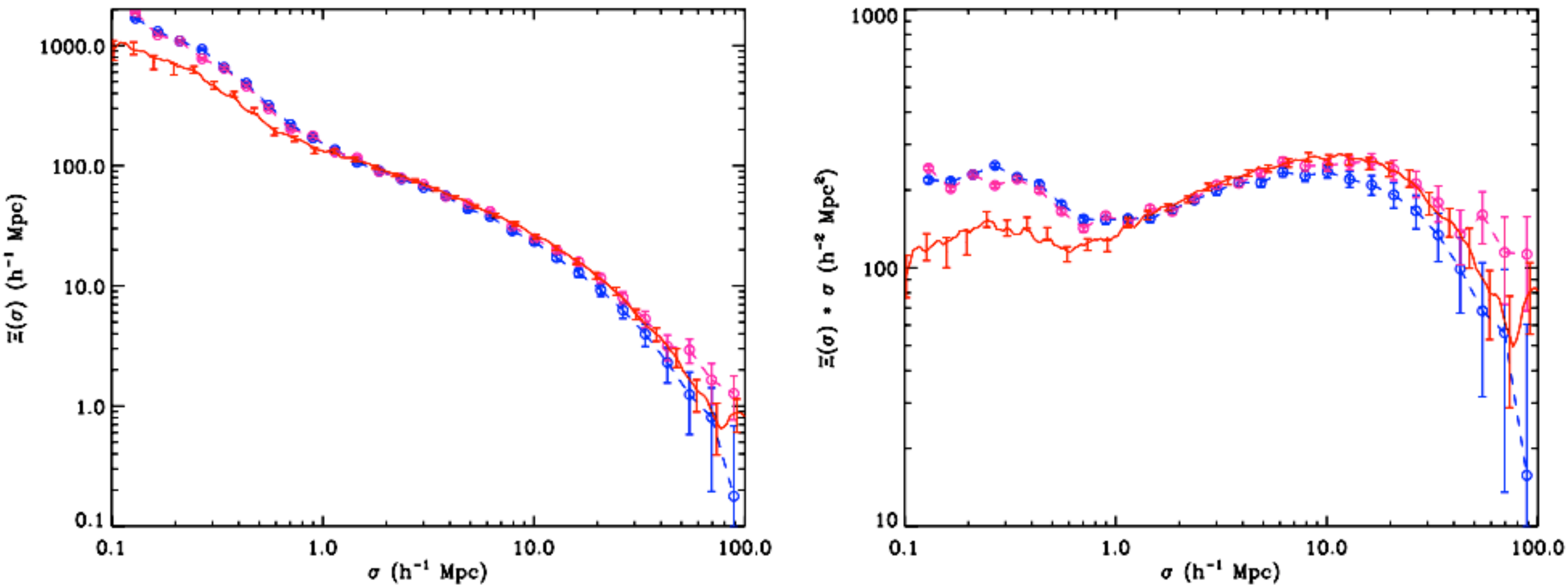


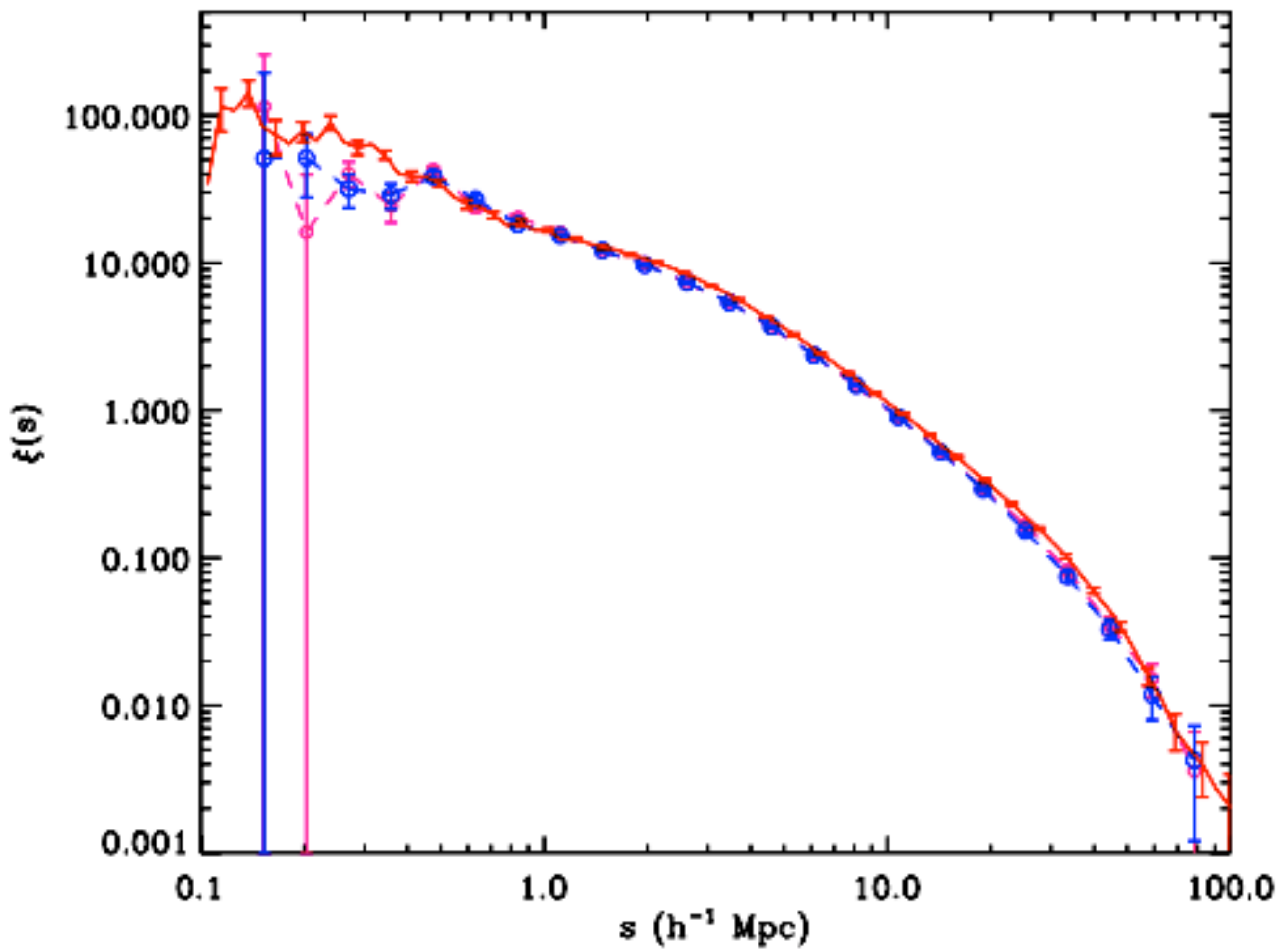
Figure 8. *Left panel:* Projected correlation function for the $0.4 < z < 0.7$ two-year BOSS-CMASS North and South galaxy samples (blue and magenta open circles respectively) and the MultiDark catalog selected with the HAM procedure at $z = 0.53$ (solid line). Error bars for MultiDark give an estimate of the cosmic variance magnitude. BOSS-CMASS error bars were estimated using an ensemble of 600 PTHalos mock galaxies. The transition between the 1st and 2nd halo terms can be seen at $\sim 1\ h^{-1}$ Mpc. Flattening of the signal at intermediate scales and bending at large scales are also evident features. *Right panel:* Detailed differences between our ΛCDM model and BOSS clustering measures is better seen when plotting the quantity $\Xi(\sigma)\,\sigma$ as a function of projected distance (see text).

$$\Xi(\sigma) = 2\int_0^\infty \xi(\sigma,\pi)\,\mathrm{d}\pi. \tag{2}$$

In practice, we integrate out to $\pi_{\rm max} = 200\,h^{-1}\,\mathrm{Mpc}$.

We compute the full correlation functions $\xi(\sigma,\pi)$ using the Landy & Szalay (1993) estimator

$$\xi(\sigma,\pi) = \frac{\mathrm{DD} - 2\mathrm{DR} + \mathrm{RR}}{\mathrm{RR}} \tag{3}$$



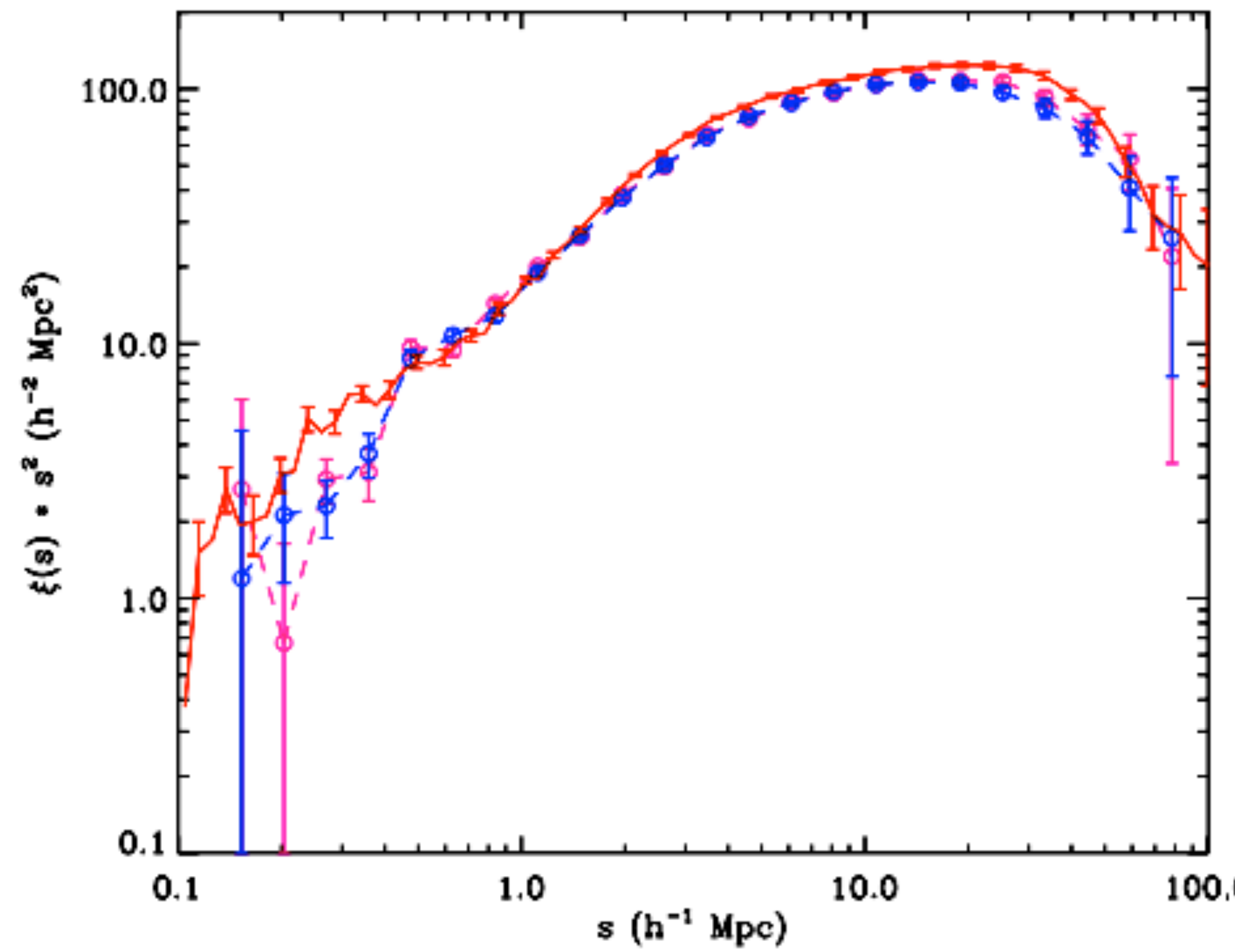
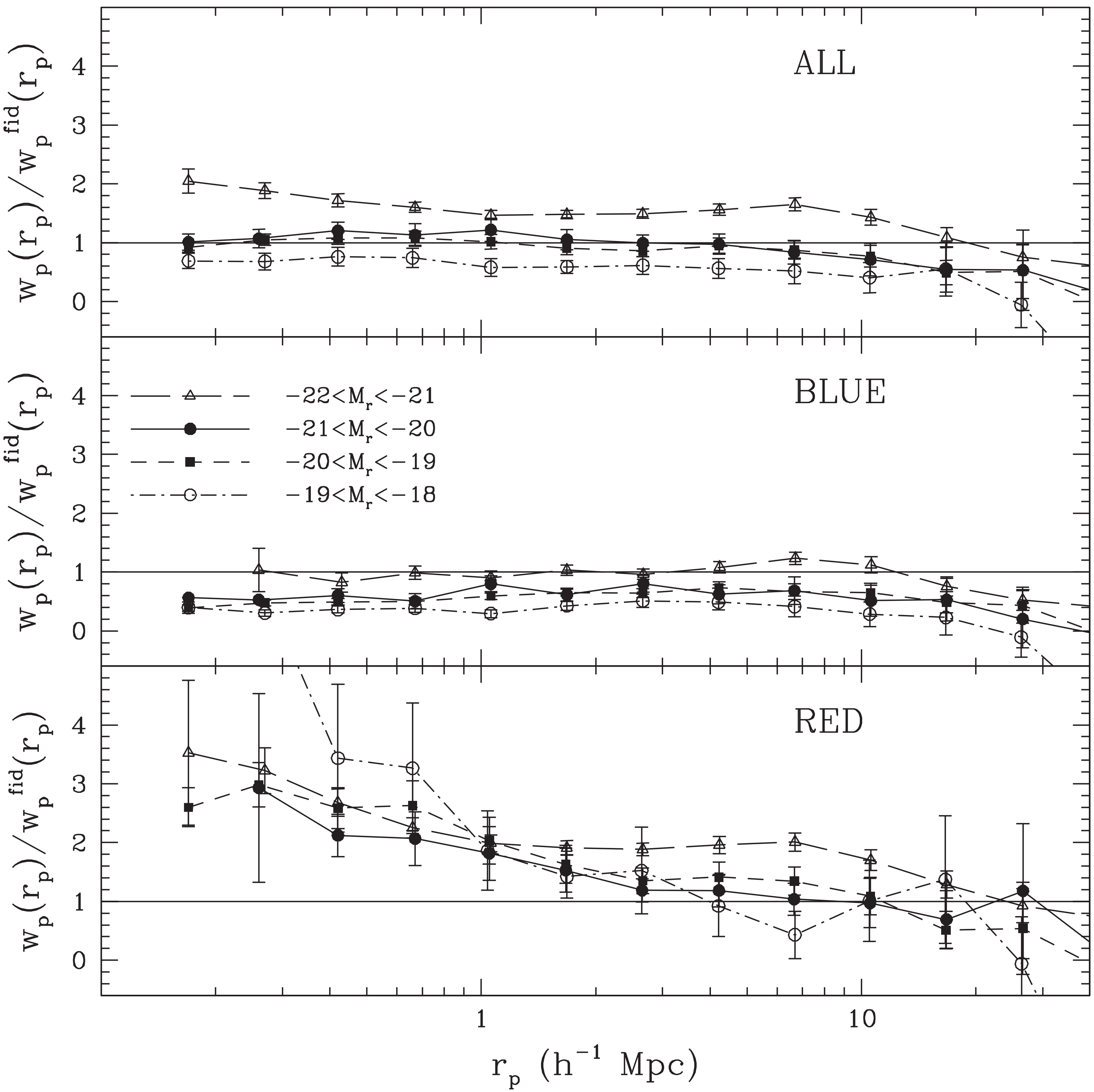


Figure 9. *Left panel:* Redshift-space correlation function both for the tow-year BOSS-CMASS North and South galaxy samples at $0.4 < z < 0.7$ (blue and magenta open circles respectively) and the MultiDark catalog selected with the HAM procedure at $z = 0.53$ (solid line). Error bars are obtained in tha same way as in Fig. 8. *Right panel:* Shown is the quantity $\xi(s)\, s^2$ which better reflects the differences between our ΛCDM model and BOSS clustering measures.

Clustering: galaxy morphology



Bias $\quad b^2 = W(r, sample1)/W(r, sample2)$

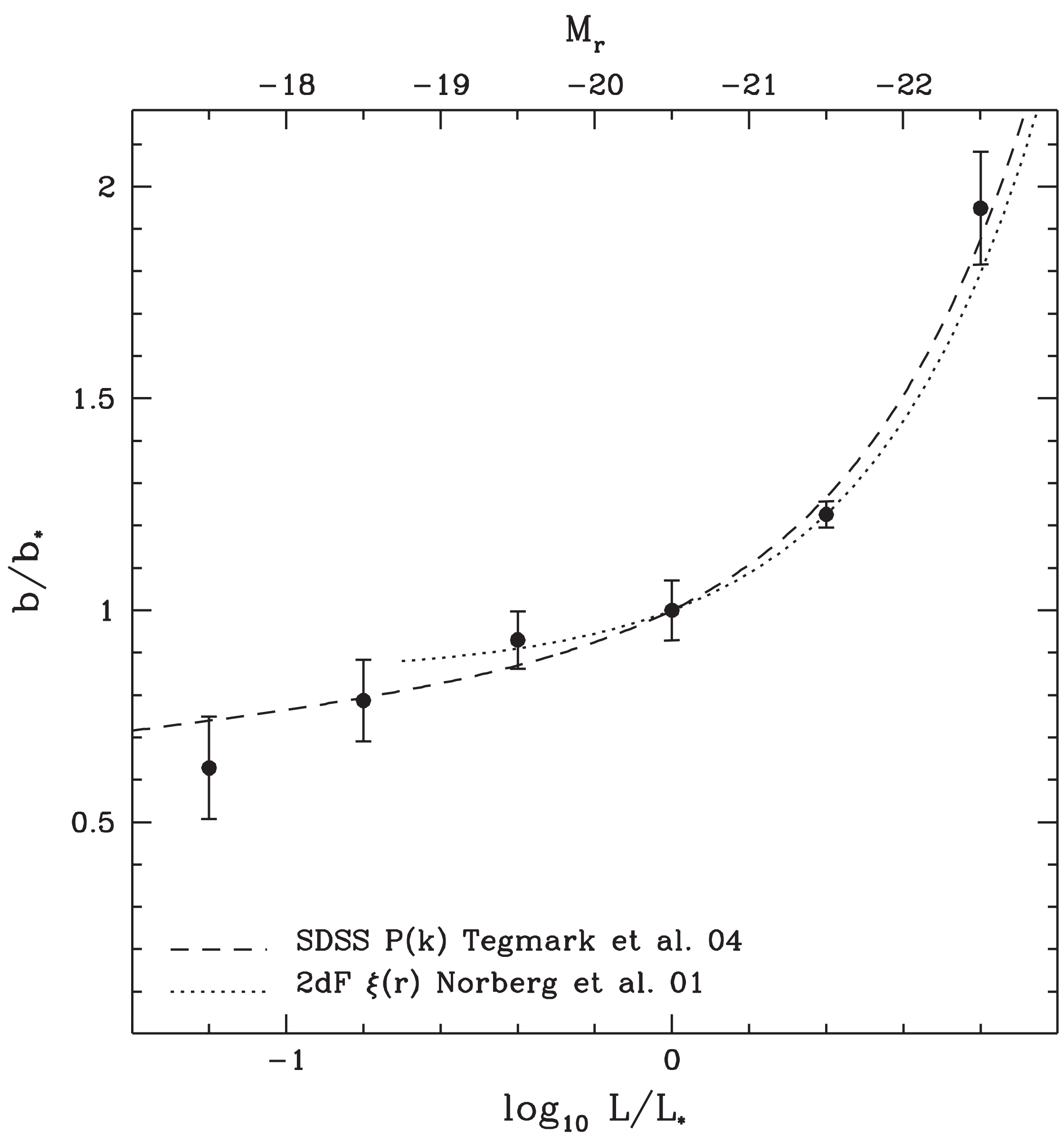
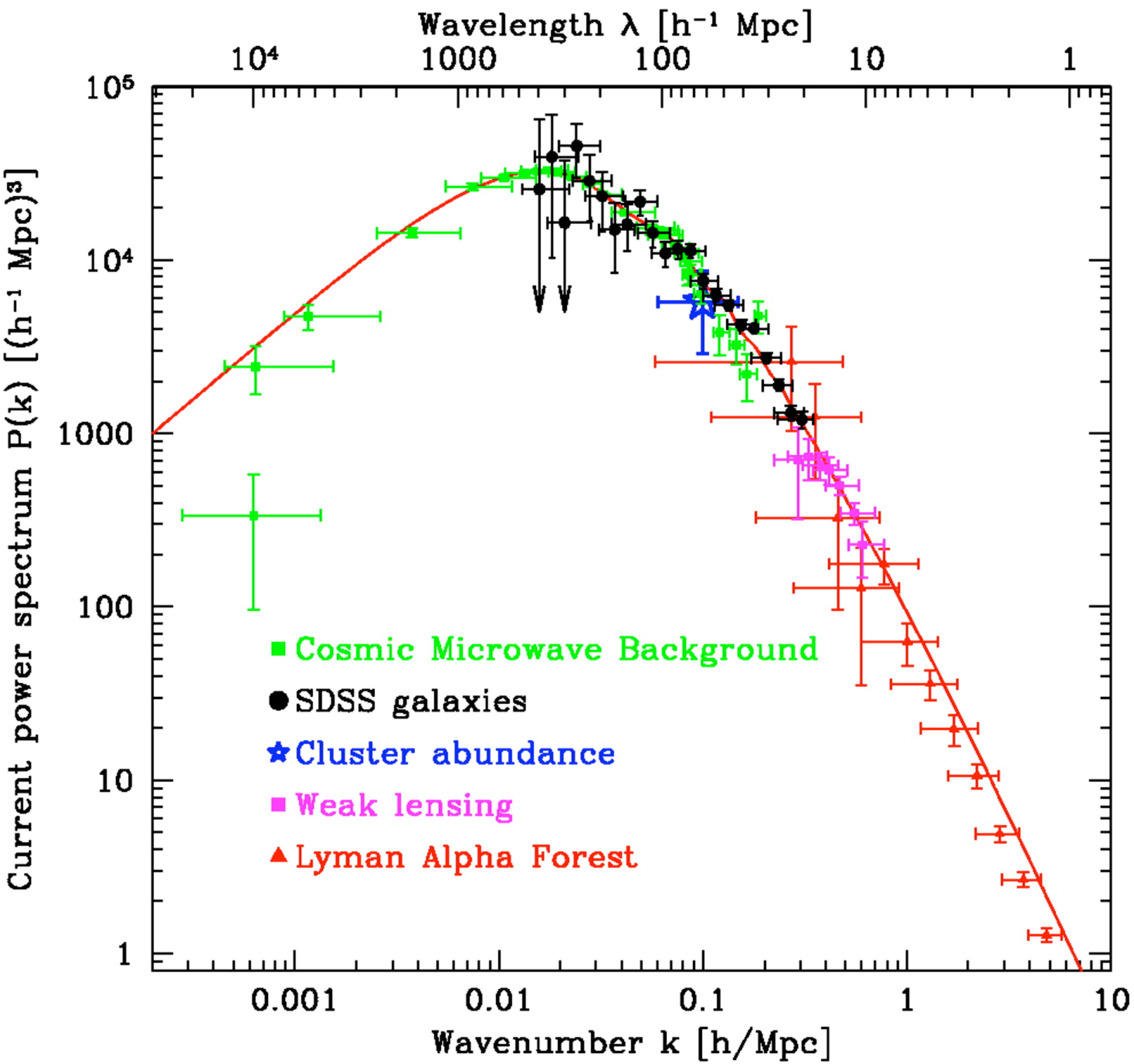


FIG. 11.—Relative bias factors for samples defined by luminosity ranges. Bias factors are defined by the relative amplitude of the $w_p(r_p)$ estimates at a fixed separation of $r_p = 2.7\ h^{-1}$ Mpc and are normalized by the $-21 < M_r < -20$ sample ($L \approx L_*$). The dashed curve is a fit obtained from measurements of the SDSS power spectrum, $b/b_* = 0.85 + 0.15L/L_* - 0.04(M - M_*)$ (Tegmark et al. 2004a), and the dotted curve is a fit to similar $w_p(r_p)$ measurements in the 2dF survey, $b/b_* = 0.85 + 0.15L/L_*$ (Norberg et al. 2001).

The Observed Power Spectrum



(Tegmark et al.)

Baryonic acoustic oscillations: Power spectrum

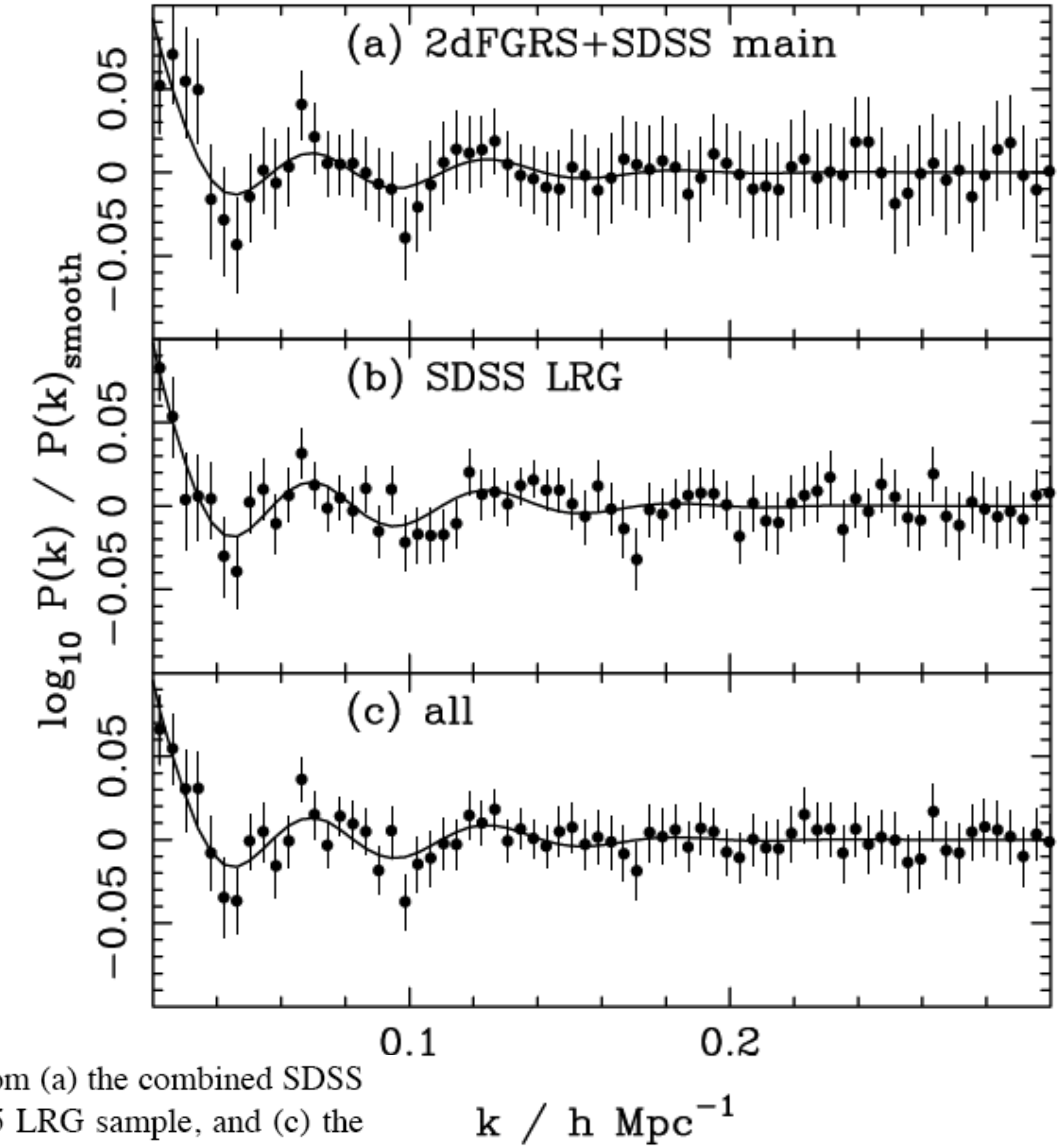


Figure 2. BAOs in power spectra calculated from (a) the combined SDSS and 2dFGRS main galaxies, (b) the SDSS DR5 LRG sample, and (c) the combination of these two samples (solid symbols with 1σ errors). The data are correlated and the errors are calculated from the diagonal terms in the covariance matrix. A standard ΛCDM distance–redshift relation was assumed to calculate the power spectra with $\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$. The power spec-

Percival etal 2007