Galaxy Luminosity Function

Methods:
• $V_{\text{max}}$
• Max Likelihood

Observations of LF:
• Shape of LF
• Field LF
• LF in Groups and Clusters
Luminosity Function

- \( \Phi(M) \, dM \) is the number of galaxies per unit volume with absolute magnitudes in the range \((M, M+dM)\)

- \( \Phi(L) \, dL \) is the number of galaxies per unit volume with luminosities in the range \((L, L+dL)\)

- \( \int \Phi(M) \, dM = n \), where \( n \) is the number-density of all galaxies
\textbf{V}_{\text{max}} \text{ method}

• Find the largest distance at which a galaxy with observed abs magnitude $M_i$ can be found in order to have apparent magnitude equal to the limit of the sample $m_{\text{lim}}$

• Volume of the sample corresponding the distance is $V_{\text{max}}$. This is the volume available for the galaxy. The galaxy could have been anywhere inside the volume.

• Select all galaxies with abs magnitudes in the range $(M, M+dM)$. An estimate of the luminosity function is

\begin{equation}
\Phi(M)dM = \sum \left[1/V_{\text{max}}(i) \right]
\end{equation}
Parametric maximum-likelihood method of Sandage, Tammann, Yahil (1979)

Consider a galaxy i observed at redshift $z_i$ in a flux-limited sample. Apparent magnitude limits for the sample are $m_{\text{min}}$ and $m_{\text{max}}$. The differential luminosity function of the sample is $\phi(M)$, where $M$ is the absolute magnitude. The probability for a galaxy at redshift $z_i$ to be in the sample is $p_i$:

$$p_i \equiv p(M_i | z_i) = \frac{\phi(M_i)}{\int_{M_{\text{min}}(z_i)}^{M_{\text{max}}(z_i)} \phi(M) dM}$$

The likelihood function $\mathcal{L}$ for having a sample of $N$ galaxies with abs. magnitudes $M_i$ is the product of probabilities $p_i$:

$$\mathcal{L} = p(M_1, \ldots, M_N | z_1, \ldots, z_N) = \prod_{i=1}^{N} p_i$$

It is more convenient to deal with the ln of the function:

$$\ln \mathcal{L} = \sum_{i=1}^{N} \left\{ \ln \phi(M_i) - \ln \int_{M_{\text{min}}(z_i)}^{M_{\text{max}}(z_i)} \phi(M) dM \right\}$$

Assume a parametric form for $\Phi(M) = \Phi(M; p1, p2, \ldots)$. Maximize $\mathcal{L}$ with respect to those parameters. In practice, we use the Schechter function, which has three free parameters.
maximum-likelihood method

\[ \ln \mathcal{L} = \ln \mathcal{L}_{\text{max}} - \frac{1}{2} \Delta \chi^2 \]

Errors for parameters can be found by constructing contours around the maximum of likelihood function. Here \( \Delta \chi^2 \) is the change in the \( \chi^2 \) appropriate for desired confidence level. \( \chi^2 \) has two degrees of freedom.
Normalization

The overall normalization \( \bar{n} \) cannot be determined from this likelihood maximization procedure. We use the standard minimum variance estimator of Davis & Huchra (1982) to perform the normalization:

\[
\bar{n} = \frac{\sum_{j=1}^{N_{\text{gals}}} w(z_j)}{\int dV \, \phi(z) w(z)} ,
\]

(9)

where the integral is over the volume covered by the survey between the minimum and maximum redshifts used for our estimate. The weight for each galaxy is

\[
\phi(z) = \frac{\int_{L_{\text{min}}(z)}^{L_{\text{max}}(z)} dL \, \Phi(L, z)}{\int_{L_{\text{min}}}^{L_{\text{max}}} dL \, \Phi(L, z)} ,
\]

(11)

\[
w(z) = \frac{f_t}{1 + \bar{n} 10^{0.4 (z-z_0)} J_3 \phi(z)} ,
\]

where \( f_t \) is the galaxy sampling rate determined at each position of sky as the fraction of targets in each sector that were successfully assigned a classification. The integral of the correlation function is

\[
J_3 = \int_0^\infty dr \, r^2 \xi(r) = 10,000 \, h^{-3} \, \text{Mpc}^3 .
\]

(12)
The Schechter luminosity function

\[ \phi(M) = (0.4 \ln 10) \phi^* \left[ 10^{0.4(M^* - M)} \right]^{1+\alpha} \exp[-10^{0.4(M^* - M)}] \]

\[ \Phi(L) = \left( \frac{\Phi^*}{L^*} \right) \left( \frac{L}{L^*} \right)^\alpha \exp \left( -\frac{L}{L^*} \right) \]

A convenient approximation to the luminosity function was suggested by Paul Schechter in 1976.

In this expression:
- \( \phi^* \) is a normalization factor which defines the overall density of galaxies (number per cubic Mpc)
- \( L^* \) is a characteristic galaxy luminosity. An \( L^* \) galaxy is a bright galaxy, roughly comparable in luminosity to the Milky Way. A galaxy with \( L < 0.1 L^* \) is a dwarf.
- \( \alpha \) defines the `faint-end slope' of the luminosity function. \( \alpha \) is typically negative, implying large numbers of galaxies with low luminosities.
Schechter Function

\[ \Phi(L) = \left( \frac{\Phi^*}{L^*} \right) \left( \frac{L}{L^*} \right)^\alpha \exp \left( -\frac{L}{L^*} \right) \]

\[ l_{\text{tot}} = \int_0^{\infty} L \Phi(L) \, dL = \Phi^* L^* \Gamma(2 + \alpha) \]
Fig. 7.—Redshift distribution of the $0.1_r$-band sample, for each quartile (weighted by number) in absolute magnitude. The thick line represents the data; the thin line is a Monte Carlo representation of the model, including the selection effects in the survey. In this figure and in Figs. 8–12, the model is a decent representation of the data, but not a perfect one; much of the difference is likely to be due to large-scale structure, but it is possible that further complications of our evolution model or our error model might be necessary to fully reproduce the data.
**SDSS**  \( z=0.1 \)

**Figure 2.** Redshift distributions in all SDSS photometric bands.

**Figure 1.** Redshift completeness versus apparent magnitude in the \( r \) band. At \( r \lesssim 14 \) completeness decreases sharply. Errors have been estimated by propagating the Poissonian uncertainties to the redshift completeness.
Fig. 15.—Best-fit Schechter function of Blanton et al. (2001), based on the sample of ~10,000 galaxies in sample5 (solid line), and a fit using the same method to the current sample of ~150,000 galaxies in sample10 (dotted line). These two results are in remarkable agreement, showing that the differences between our results and those of Blanton et al. (2001) are not due to cosmic variance. The dashed line shows a Schechter fit to the current sample allowing for luminosity evolution (finding a best fit of $Q = 2.06$). When evolution is allowed for, the faint-end slope becomes shallower and the overall luminosity density decreases. [See the electronic edition of the Journal for a color version of this figure.]
We present number counts, luminosity functions (LFs) and luminosity densities of galaxies obtained using the Sloan Digital Sky Survey Sixth Data Release (SDSS DR6) in all SDSS photometric bands. Thanks to the SDSS DR6, galaxy statistics have increased by a factor of ≈7 in the $u$ band and by a factor between ≈3 and ≈5 in the rest of the SDSS bands with respect to the previous work of Blanton et al. In addition, we have ensured a high redshift completeness in our galaxy samples, mainly by minimizing the effect of brightness-dependent incompleteness. With these advances, we have estimated very accurate SDSS DR6 LFs at both the bright and the faint end. In the $^0.1r$ band, our LF is well fitted by a Schechter LF with parameters $\Phi_* = 0.93 \pm 0.07$, $M_* - 5\log_{10} h = -20.71 \pm 0.04$ and $\alpha = -1.26 \pm 0.02$. As compared with previous results, we find some notable differences. At the bright end of the $^0.1u$-band LF we find a remarkable excess, of ≈1.7 dex at $M_{^0.1u} \simeq -20.5$, with respect to the best-fitting Schechter LF. This excess weakens in the $^0.1g$ band, fading away towards the very red $^0.1z$ band. A preliminary analysis on the nature of this bright-end bump reveals that it is composed of quasi-stellar objects/type I Seyferts (≈60 per cent), starbursts and star-forming galaxies (≈20 per cent) and normal galaxies (≈20 per cent). It seems, therefore, that an important fraction of this excess luminosity may come from nuclear activity. At the faint end of the SDSS DR6 LFs, where we can reach 0.5–1 mag deeper than the previous SDSS LF estimations, we obtain a steeper slope that increases from the $^0.1u$ band, with $\alpha = -1.05 \pm 0.05$, to the very red $^0.1z$ band, with $\alpha = -1.26 \pm 0.03$. We have also investigated the effect of galaxy evolution on our LFs. These state-of-the-art results may be used to constrain a variety of aspects of star formation histories and/or feedback processes in galaxy formation models.
Figure 7. The $^{0.1}\text{r}$-band SDSS DR6 LF. The SWML LF estimate is shown in diamonds. The dashed line represents the best-fitting Schechter function and the solid line, the $^{0.1}\text{r}$-band LF from Blanton et al. (2003a). Best-fitting values of Schechter parameters $\alpha$, $M_*$, and $\Phi_*$ are also shown in the figure. Shaded regions represent the 1σ uncertainty calculated using a bootstrapping technique.
Steepening of LF at faint mags in dense environments

*Figure 3.* The nearby group and cluster luminosity functions (left) and 1-, 2- and 3-σ error ellipses for a Schechter function fit to either the full luminosity range ($-22 < M <$ various, near right) or the range comparable to the field range ($-22 < M - 5\log h < -17$, far right), all fits are to the solid data points. Local group data: P ritchet and van den Bergh, 1999 (solid); Mateo, 1998 (open), Local sphere data: Jerjen, Binggeli, Freeman, 2000 (solid); Karachentsev et al., 2002 (open). Virgo data: Trentham and Hodgkin, 2002 (solid); Fornax data: Ferguson, 1989 + Deady et al., 2002 (solid); Coma data: Mobasher et al., 2003 (solid); Beijersbergen et al., 2002 (open triangles), Trentham, 1998 (open circles) and Andreon and Culliandre, 2002 (open squares). The solid line shows, in each case, the optimal Schechter function fit and the luminosity range to which it was fitted. The data point and errorbars on the far right panels shows the location of the adopted global luminosity function.
The faint-end of the galaxy luminosity function in groups

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We compute the galaxy luminosity function in spectroscopically selected nearby groups and clusters. Our sample comprises 728 systems extracted from the third release of the Sloan Digital Sky Survey in the redshift range $0.03 < z < 0.06$ with virial mass range $10^{11} \, M_\odot < M_{\text{vir}} < 2 \times 10^{14} \, M_\odot$. To compute the galaxy luminosity function, we apply a statistical background subtraction method following usually adopted techniques. In the $r$ band, the composite galaxy luminosity function shows a slope $\alpha = -1.3$ in the bright–end, and an upturn of the slope in the faint–end, $M_r \approx -18 + 5 \, \log (h)$, to slopes $-1.9 < \alpha < -1.6$. We find that this feature is present also in the $i, g$ and $z$ bands, and for all explored group subsamples, irrespective of the group mass, number of members, integrated color or the presence of a hot intra-cluster gas associated to X-ray emission.
Fig. 2. $r$-band composite galaxy LF for the total sample of groups calculated within $0.5 \, h^{-1}$ Mpc from group centres. The solid line corresponds to the best two Schechter function fits obtained from a maximum likelihood estimator with an upturn limit at $M_r = -18$ (see 2 for parameter values). For comparison we show with an arbitrary normalization Popesso et al. (2005) and Blanton et al. (2005) LF determinations of galaxies in X-ray clusters and in the field, respectively.

Fig. 3. Composite galaxy LF in the $u, g, i, z$ photometric bands calculated for the total group sample within $0.5 \, h^{-1}$ Mpc from group centres.
**Fig. 4.** Composite $r$-band galaxy LF for different group centric distance ranges.