Máster en Física Teórica

ASTRO/COSMOPARTICLE PHYSICS

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Date	Topic
25 April	Brief review of history of the Universe. Distance to the horizon. Moment of equality. Waves longer and shorter than the horizon. Shape of the power spectrum. Effects baryons.
26 April	Nonlinear evolution of fluctuations: overview. Evolution of power spectrum. Biases. Redshift distortions. Observations: power spectrum and correlation function. Dependence on galaxy morphology and stellar mass.
May 3	Large-scale structure: distribution of galaxies in space. Dark matter halos and galaxies: ways to connect. Halo mass - stellar mass relation.
May 17	Abundance of dark matter halos and sub halos. Evolution of galaxy luminosity function and star formation rates over time.
May 23	Halo properties: density and velocity profiles, concentrations. Milky Way and Local Group: dark matter and baryons
May 24	Galaxy formation. Dark matter - baryons connection. Adiabatic compression. Flattening of dark matter cusps. Too-big-to-fail problem. Abundance of dwarf galaxies.

Arrow of time

Main events in the history of the Universe

Energy/Temp	Time	Event/Epoch
10^{19}GeV	10^{-43}sec	Planck Time
		Inflation
10^{14}GeV	10^{-35}sec	end of Inflation. Reheating. Beginning of Big Bang
	10^{-34}sec	end of grand unification. Baryogenesis: formation of matter-antimatter asymmetry
300GeV	10^{-12}sec	end of electroweak unification
1GeV	10^{-5}sec	Normal physics. Composition of the Universe: n, p, e^-,e^+, γ, ν
1MeV	1sec	Neutrino decoupling. Neutrino do not interact with the rest of matter
0.5MeV		Electron-positron annihilation. Composition: n, p, e^-, γ, ν
0.1MeV	100sec	Big Bang Nucleosynthesis: formation of elements He,D, Li
10^5K	10^3yrs	Equality of matter and radiation: $\rho_{matter} = \rho_{rel.particles}$
3000K =0.3eV	10^5yrs	Recombination and Decoupling. Composition: H,He, γ, ν
	1Gyr (z=10)	First galaxies. QSO quickly form.
	z=3	Galaxy formation
	z=1-2	Formation of clusters and superclusters. Acceleration of the Universe.
	13Gyrs	Now

Probing different epochs with observations

Epoch	Phenomenon	Test
Inflation	Spectrum of perturbation on very long scales	•Large-scale CMB anisotropies •Large-scale spectrum of perturbation in distribution of galaxies
Moment of equality	Position of maximum in the spectrum of perturbations	Distribution of galaxies: Spectrum, sizes of large voids, Superclusters.
BBN	abundance of light elements: He, D, Li	ISM, stellar atmospheres, spectra of high-z galaxies
Recombination	Small-scale structure of CMB	CMB anisotropies on armin -degree scales
Acceleration of the Universe	Distances depend on the rate of expansion	Distances to SNI
	Dark matter	•Rotation curves of galaxies •Possible annihilation signal from centers of galaxies •X-ray emission from clusters of galaxies •Lensing of galaxies

Distance to the horizon Question: what fraction of the Universe can be possibly in causal contact? We need to find the proper distance at z=0 for a point, from which we receive light for the first time. This will be the distance to the horizon.

We chose a frame, which is most convenient for integration. We are at the origin and the point, from which we receive the light is along the radius.

We start with FRW:

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

Fix time and find proper distance to an object with coordinate distance r_H

$$d_H(t) = a(t)\int_0^{r_H} \frac{dr}{\sqrt{1-kr^2}}$$

In order to take the integral, we need to know the coordinate distance to the point, from which we receive the light for the first time in the history of the Universe. We find this by putting ds=0 into FRW and integrating it from t=0 till present:

$$ds = 0 \rightarrow c\,dt = a(t)\frac{dr}{\sqrt{1-kr^2}} \rightarrow \int_0^t \frac{c\,dt}{a(t)} = \int_0^{r_H} \frac{dr}{\sqrt{1-kr^2}}$$

Thus

$$d_H(t) = a(t)\int_0^t \frac{c\,dt}{a(t)}$$

Thus

$$d_H(t) = a(t)\int_0^t \frac{c\,dt}{a(t)}$$

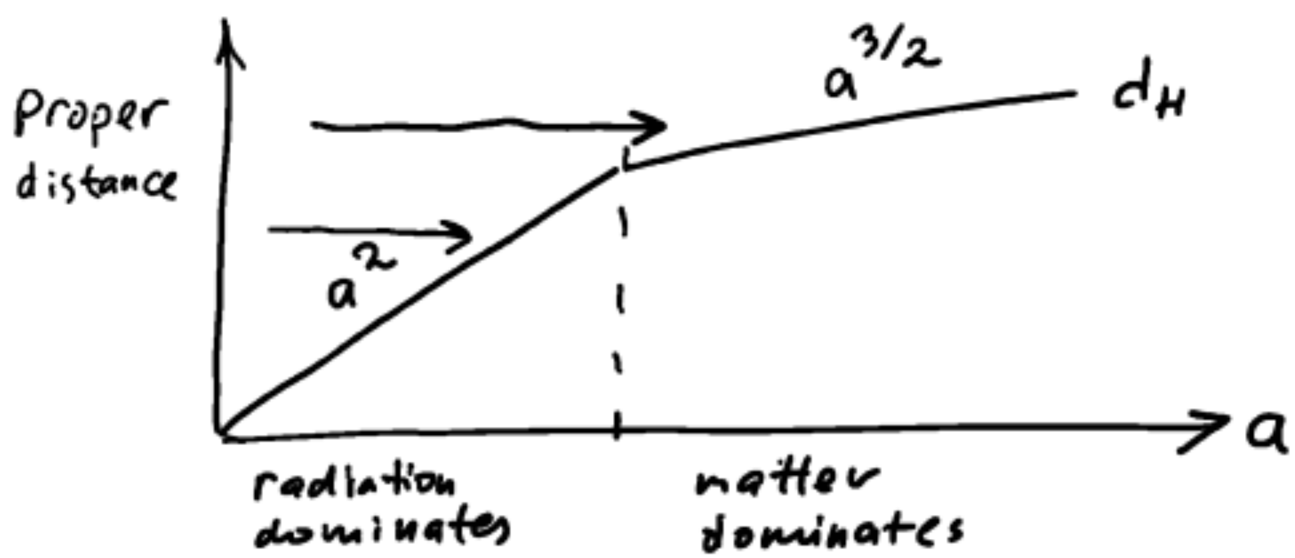
For flat universe dominated by non-relativistic particles $a(t) \propto t^{2/3}$

In this case

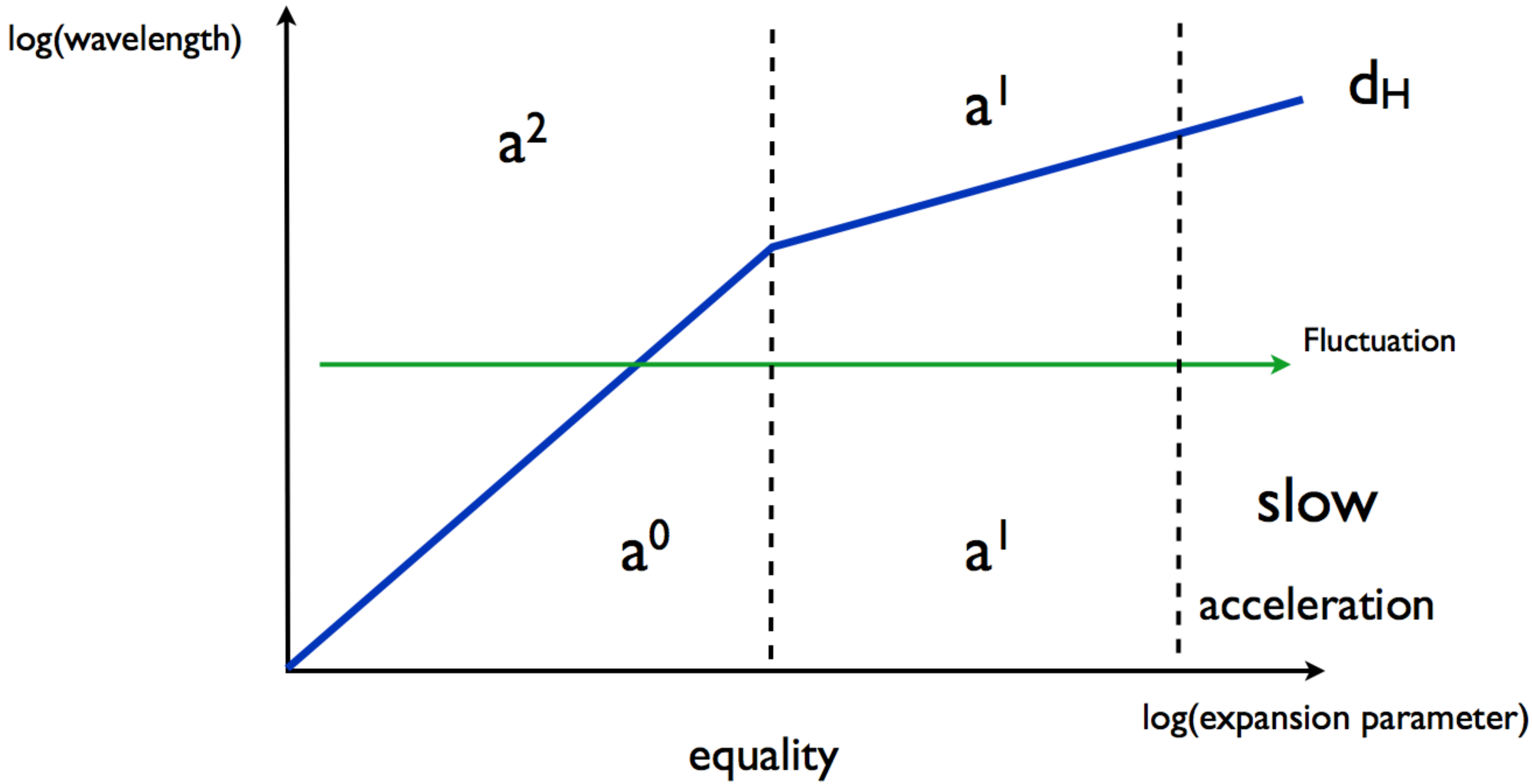
$$d_H(t) = \frac{2c}{H_0} a^{3/2} = 3ct$$

For flat universe dominated by relativistic particles $a(t) \propto t^{1/2}$

And $d_H \propto a^2 \quad (d_H = 2ct)$



Important notion: distance to the horizon grows faster than wavelengths of waves, which expand together with the Universe. Thus, free waves were outside of the horizon at early moment and cascade inside the horizon at later moments.



Chessboard of growth of adiabatic perturbations

The Universe is not uniform. We have ignored this when we talked about FRW metric and when we discuss physics of early Universe such as Big Bang Nucleosynthesis or neutrino freeze-out (when neutrino decouple from the rest of the matter). There is a reason why we treated the Universe as homogeneous: the deviations from homogeneity are small and they were even smaller in the past.

There numerous issues related with perturbations. Big Bang itself cannot explain how the fluctuations formed. There is a natural source of fluctuations: statistical fluctuations in a medium, which consists of discrete particles. The amplitude of those fluctuations is roughly 1/sqrt(N), where N is the number of particles in some given volume. There are two problems with those fluctuations. First, their amplitude is very small. For example, consider a cluster of galaxies with mass about 1e15 Solar mass. Calculate the number of protons and take square root of it. This is what we expect from statistical mechanics. Second, the fluctuations (small or very small) grow relatively slow. This is due to the expansion of the Universe. In the absence of expansion the fluctuations grow exponentially:

$$\delta \propto e^{t/t_{dyn}}, \qquad t_{dyn} \simeq \frac{1}{\sqrt{4\pi G\rho}}$$

Here ρ is the density and t_{dyn} is the dynamical time scale. Unfortunately, the fluctuations grow much slower: only as a power-law $\delta \propto a^{\gamma}$, $\gamma \simeq 1-2$ and statistical fluctuations do not play any role as an origin of fluctuations. Thus, we need something else. *So far, the only explanation for the origin of fluctuations is coming from Inflation.*

Regardless their origin, fluctuations can be decomposed into MODES (components). In addition, we need to pay attention to different physical components: perturbations in dark matter, in gas, or radiation are different and they evolve differently.

Modes:

This is curvature ≡ adiabatic perturbations

$$\rho(\vec{x}) = \rho_{background} + \delta\rho(\vec{x})$$

$$\delta \equiv \frac{\delta\rho}{\rho_{backgr}}$$

In this mode the metric is perturbed and The amplitude of perturbations initially is the same for all different mass component.

Isocurvature or isothermal fluctuations: total density initially is not perturbed, but each component is perturbed:

$$\delta\rho_{total} = 0$$

tensor ≡ gravity waves

$$\Rightarrow \delta v = 0, \quad \delta\rho_{total} = 0, \quad g_{\alpha\beta} = \bar{g}_{\alpha\beta} + h_{\alpha\beta}$$

$$h_{\alpha\beta} \propto 1/a$$

Vector perturbations:

$$\mathrm{div}\, \vec{v} = 0, \quad \delta\rho_{total} = 0$$

$$v \propto 1/a$$

Different notions:

Decaying and growing modes. Equations of evolution of perturbations are second order ODE. Those have two solutions: increasing amplitude and declining amplitude. We ignore the decaying modes almost everywhere except during transitions between different regimes. Here we need to match solutionsfrom one regime to another. This involves both modes.

Perturbations inside and outside the horizon evolve typically differently. We will consider two limiting regimes: much longer and much shorter than the distance to the horizon.

Perturbations in matter grow differently before and after the epoch of equality (moment when density of relativistic and nonrelativistic particles equals).

Around the moment of recombination there interesting physical processes (e.g., Silk dumping), which affect perturbations in baryons.

When the Universe starts to accelerate at late stages (due to the cosmological constant or dark energy), fluctuations start to grow very slowly.

The rigorous approach to the evolution of perturbations is to trace the evolution of perturbations in metric caused (and coupled) by perturbations in the energy-stress tensor. Here is a very short story.
We impose small perturbations in metric: $g_{\mu\nu} = g^{0}_{\mu\nu} + h_{\mu\nu}$

Where $h_{\mu\nu}$ are small perturbations.

Since the metric is a symmetric 4x4 matrix, we have 10 independent functions. There is a freedom of using the gauge: some variations in metric are just coordinate transformations, not real physical perturbations. There are 4 free functions for those non-perturbative adjustments. Of the remaining 6 independent functions:

2 describe scalar perturbations (trace of the metric and the spatial curvature)
1 describes vector (rotation) perturbation (2 components)
1 tensor (gravity wave) (2 components)

We are not interested in the vector modes: they die out as the Universe expands. The gravity waves lose their energy in the same way as the radiation. Once the grav.waves enter the horizon, they start to die out.

We are left with two scalar components. One of them is growing and another is decaying.

This is still a lot because we have different physical components (radiation, dark matter, gas) and we have different regimes of evolution.

In the linear stage of evolution different modes evolve independently. Because of the different physical components, we need to solve a system of coupled linear differential equations. There are ways of doing this. Solving the Boltzman equation is the best way. There are analytical approximations, which produce remarkably accurate results when compared with the direct Boltzman solutions.

We will be interested in understanding different physical processes and in putting together the whole picture. When time comes, we will use results of accurate modeling.
Equations, which we will derive are accurate for physical processes and regimes, which we will use. For example, the growth of perturbations well inside the horizon give a very accurate description how fluctuations grow at late stages of evolution before they become non-linear.

Very long waves

Wednesday, September 12, 2007
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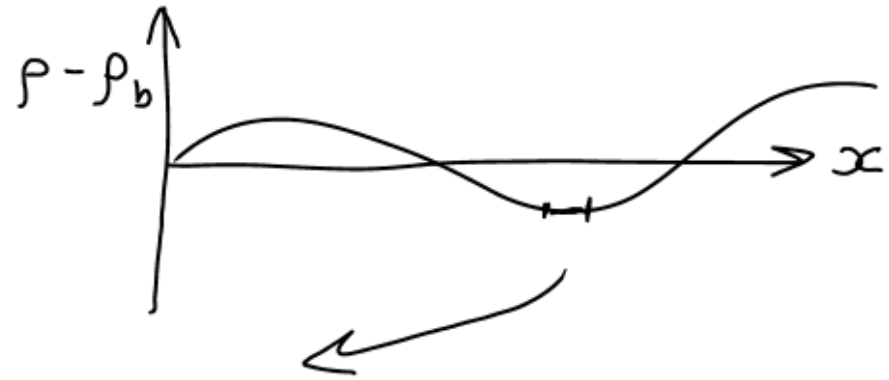
Adiabatic modes with wavelength much longer than the distance to the horizon. There are two modes: growing and decaying. Here is the derivation for the growing mode:



$\lambda \gg ct$

Each fragment of the wave has size of the horizon at that moment of time. Thus, different parts of the long wave cannot "communicate" with other parts. Each fragment evolves independently as a Friedmann universe with slightly different density, but with the same Hubble constant:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_b \quad , \quad \rho_b = \text{unperturbed density}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \quad , \quad \rho = \text{perturbed density}, \quad \rho = \rho_b + \delta\rho$$

Because the Hubble constant is the same (this selects the fastest growing mode), We get:

$$\frac{8\pi G}{3}\rho - \frac{K}{a^2} = \frac{8\pi G}{3}\rho_b \Rightarrow$$

$$\delta = \frac{\delta\rho}{\rho} = \frac{K}{a^2}\frac{3}{8\pi G}\frac{1}{\rho_b} \propto \frac{1}{a^2\rho_b}$$

For matter-dominated Universe we get:

$$\rho_b \propto a^{-3} \Rightarrow \quad \delta \propto a \propto t^{2/3}$$

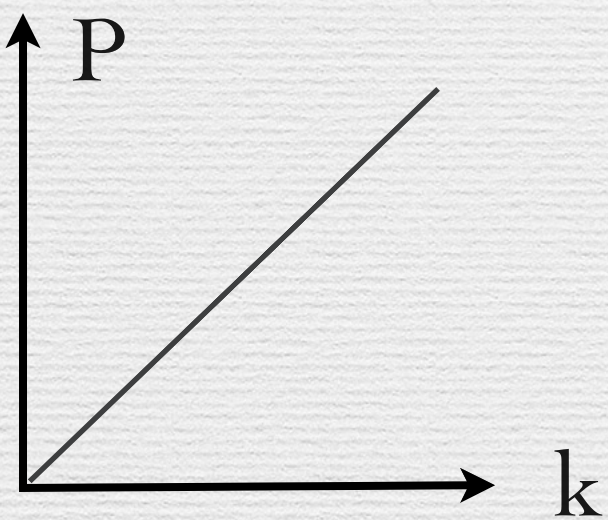
For radiation-dominated Universe:

$$\rho_b \propto a^{-4} \Rightarrow \quad \delta \propto a^2 \propto t$$

Evolution of perturbations at early times:
linear growth

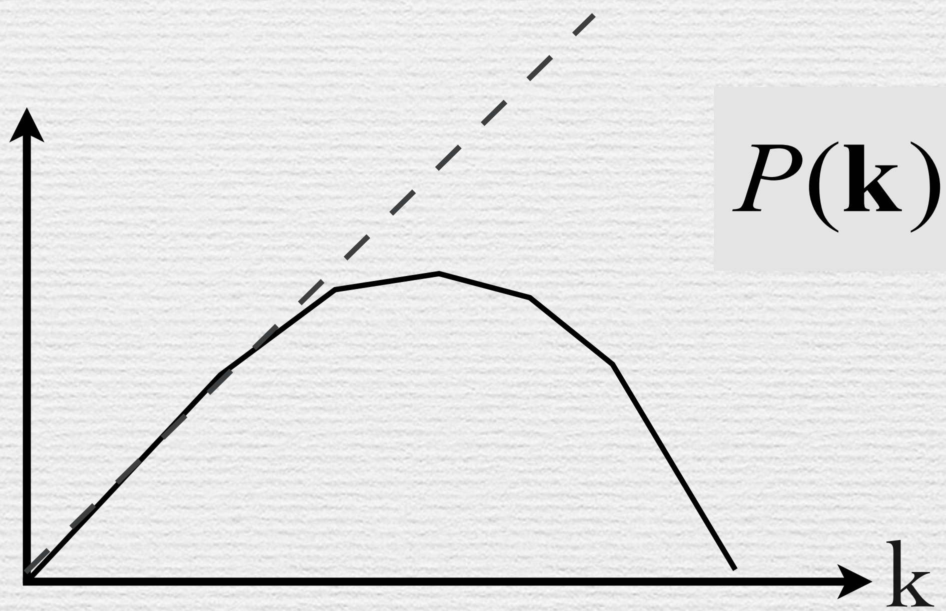
Inflation provides very a simple spectrum of fluctuations: gaussian fluctuations in metrics (=gravitational potential):
$(\Delta\phi)$

This gives the power spectrum of fluctuations in the density
$P(k)$



After Inflation

$$P(\mathbf{k}) = |\delta(\mathbf{k})|^2$$



After moment of equality

General equation for growth of perturbations is:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho_b\delta + \frac{\nabla^2 P}{\rho_b a^2}$$

Particular cases. The simplest case is ***the flat Universe, wavelength shorter than the horizon, waves longer than the Jeans mass.*** This is also the case of the cold dark matter (negligible random velocities):

$$P=0,\ \Omega_m=1,\ \rho_b=\frac{1}{6\pi G t^2}\propto a^{-3}$$

Friedmann equation is: $\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G}{3}\rho=\left(\frac{2}{3t}\right)^2$

ow the equation for the density contrast can be written as:

$$\ddot{\delta}+\frac{4}{3}\frac{\dot{\delta}}{t}=\frac{2}{3}\frac{\delta}{t^2}$$

olution of this equation is found in the form $\delta=At^n$

his gives: $n(n-1)+\frac{4}{3}n=\frac{2}{3} \rightarrow n=\frac{2}{3}$ or $n=-1$

hus, the general solution is:

$$\delta(x,t)=A(x)t^{2/3}+B(x)t^{-1}$$

$A(x)t^{2/3}$ ↓ growing

$B(x)t^{-1}$ ↓ decaying

Case: **waves inside the horizon, relativistic particles dominate**
Growth of perturbations in non-relativistic matter. Fluctuations in the relativistic matter are wiped out by the free streaming.

$$H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\gamma)$$

$$\rho_m \propto a^{-3}, \quad \rho_\gamma \propto a^{-4}$$

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G \rho_m \delta$$

$$\delta = \frac{\rho_m - \langle \rho_m \rangle}{\langle \rho_m \rangle}$$

Note that delta is the density contrast in matter, not in the total density

Introduce new variable:
Change variable t->y

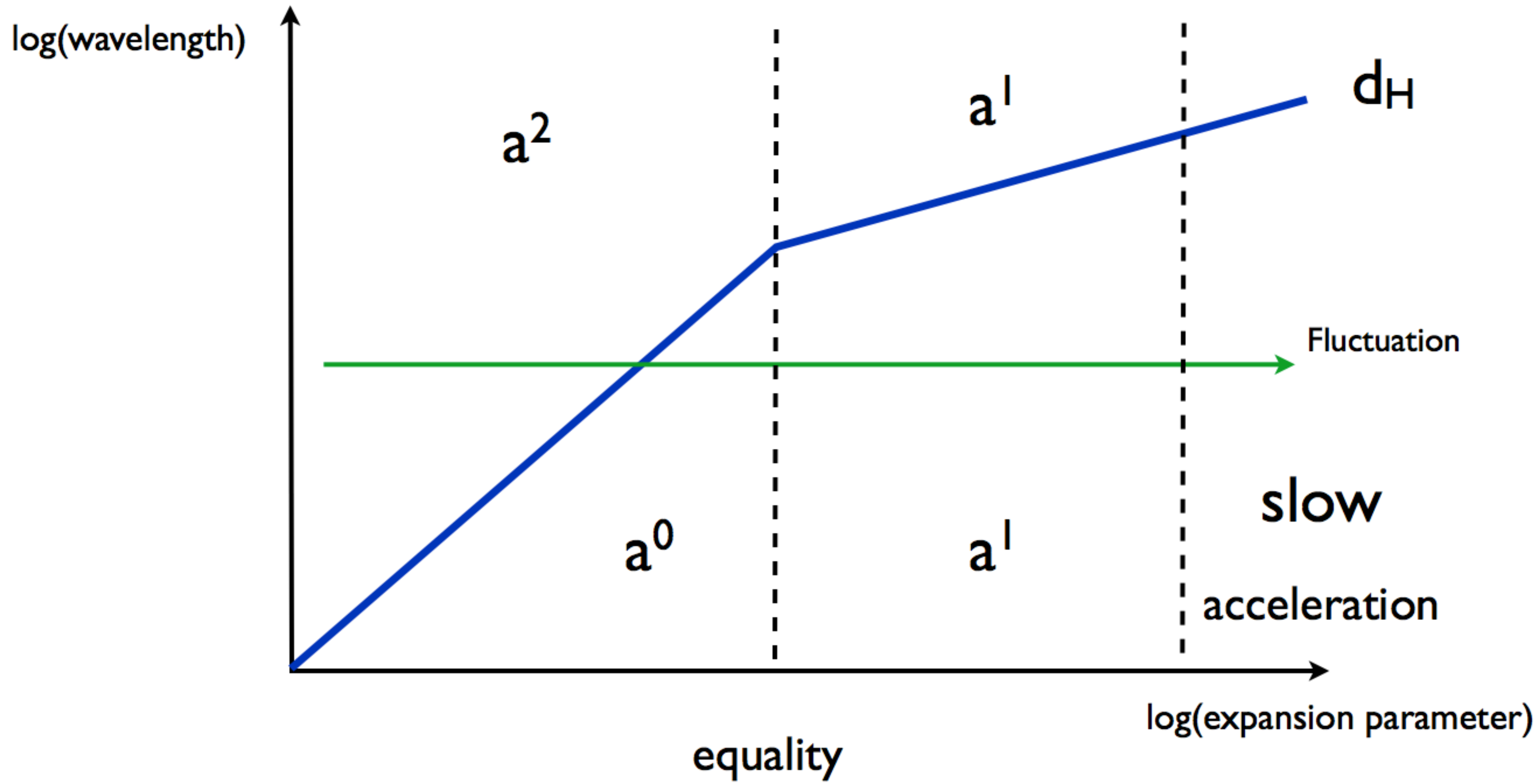
$$y \equiv \frac{\rho_m}{\rho_\gamma} = \frac{a}{a_{eq}}$$

The equation for the growth rate takes the form:

$$\frac{d^2\delta}{dy^2} + \frac{2+3y}{2y(1+y)}\frac{d\delta}{dy} - \frac{3\delta}{2y(1+y)} = 0$$

The growing solution of this equation can be found by trying: $\ddot{\delta} = 0$
This gives:

$$\delta_{grow} = 1 + \frac{3}{2}y = 1 + \frac{3}{2}\frac{a}{a_{eq}}$$



Chessboard of growth of adiabatic perturbations

Case: ***flat Universe with cosmological constant***

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda c^2}{3}$$

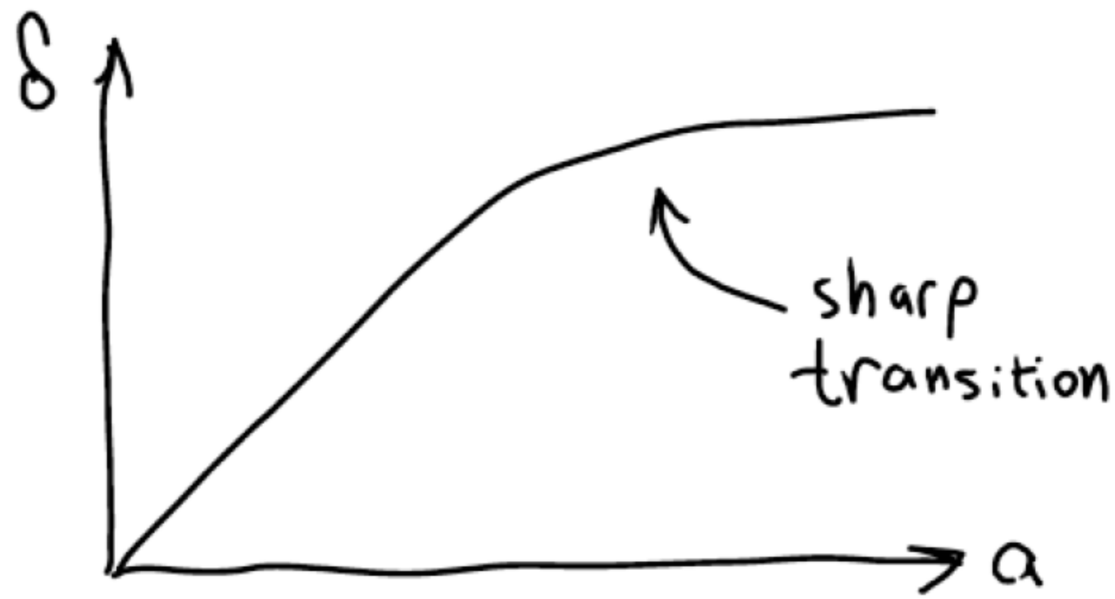
Solution for the growth of small perturbations is

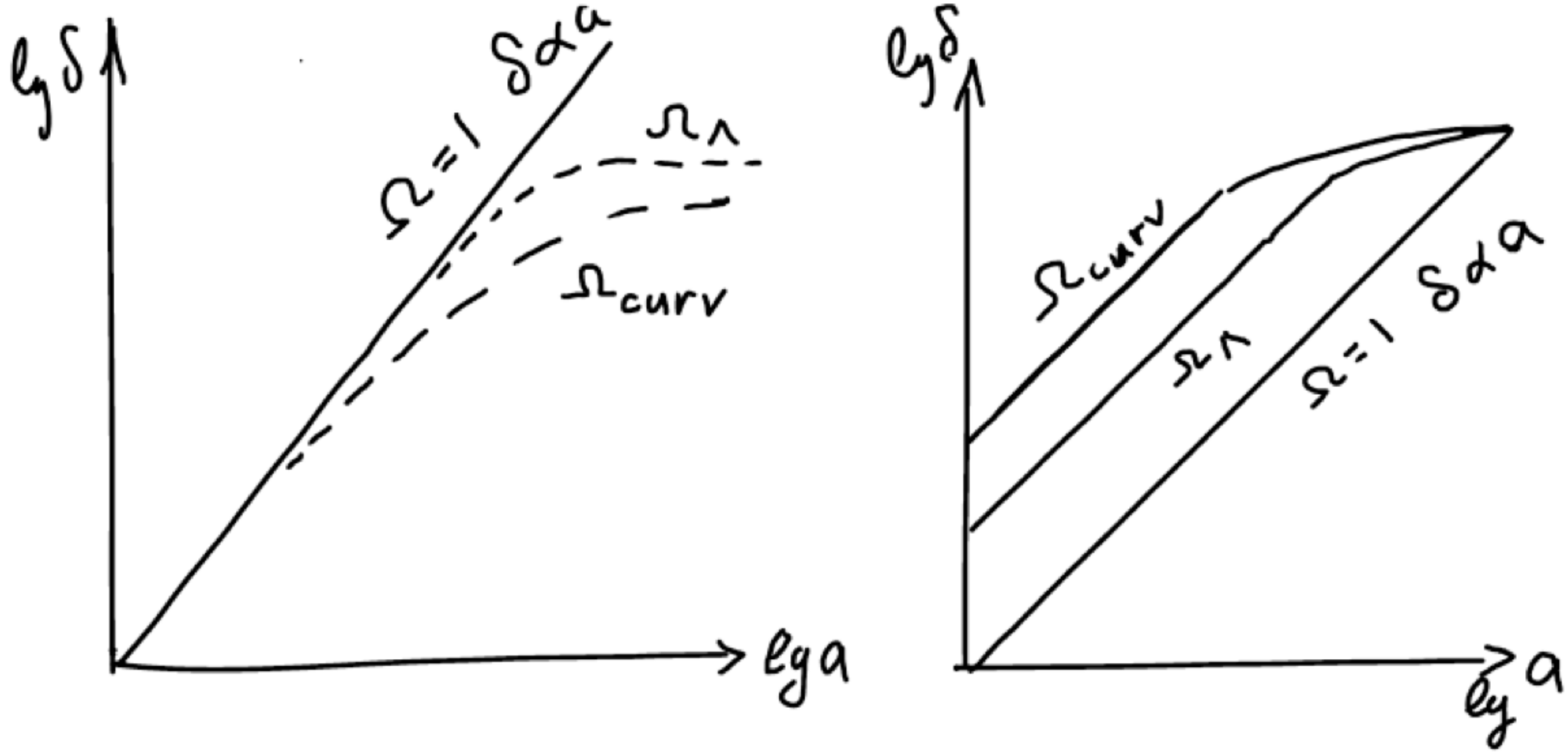
$$\delta = \frac{1}{X_0} \frac{\sqrt{1+x^3}}{x^{3/2}} \int_0^x \frac{x^{3/2}\,dx}{[1+x^3]^{3/2}}$$

$$\Omega_\Lambda = \Omega_{\Lambda,0} \frac{H_0^2}{H^2}$$

$$x = x_0 a = \left(\frac{\Omega_{\Lambda,0}}{\Omega_0}\right)^{1/3} a$$

$$x_0 = \left(\frac{\Omega_{\Lambda,0}}{\Omega_0}\right)^{1/3}$$





Three models: 1) Flat, matter only
2) Flat +Cosmological constant
3) Open, no cosmological constant

Left panel: the same amplitude of fluctuations at early times

Right panel: The same amplitude at z=0

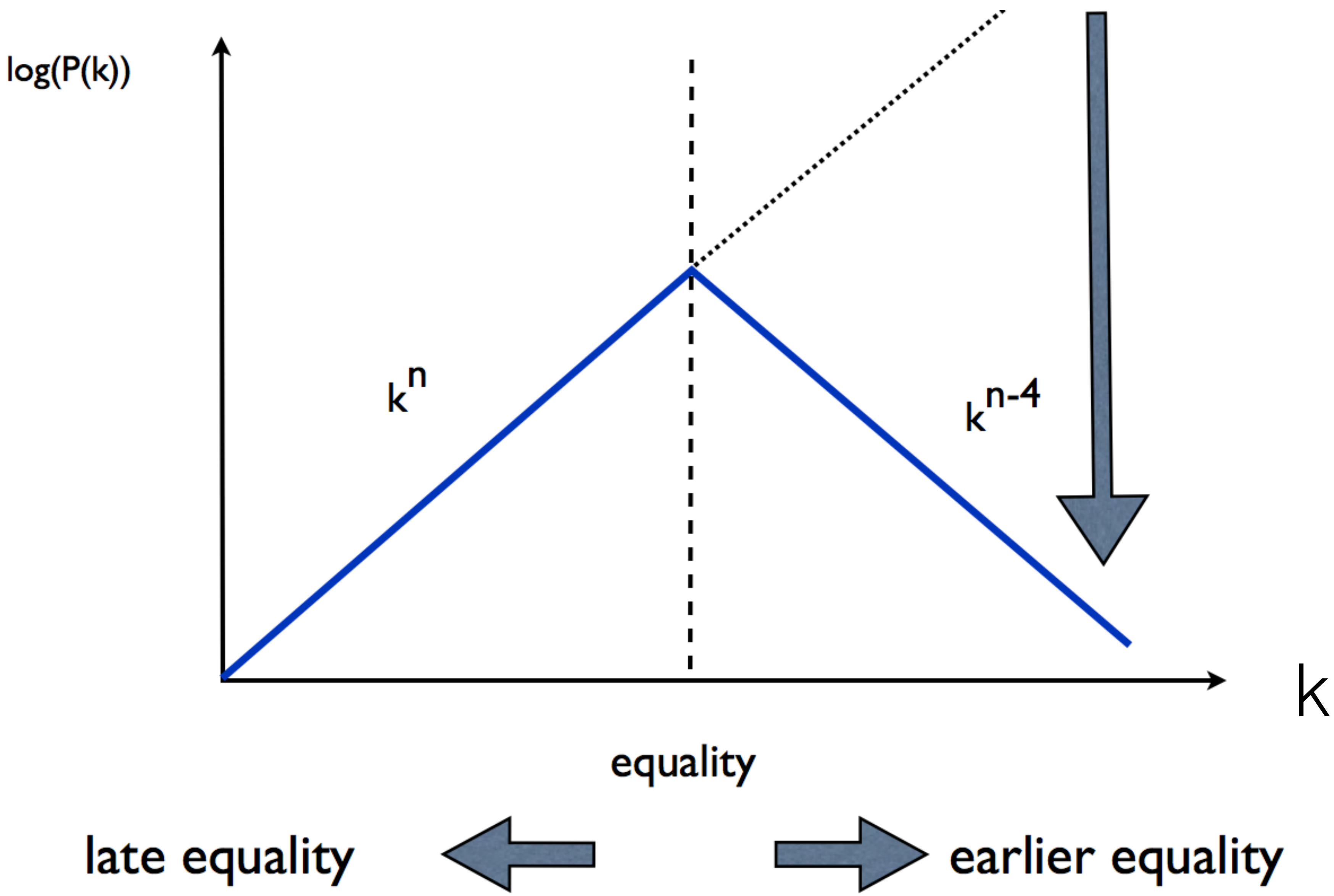
We define the **power spectrum** as

Here the averaging is done over all
Waves with given k and over the whole space

$$P(k) = \langle |\delta_{\vec{k}}|^2 \rangle$$

Correlation function is defined as

$$\xi(r) = \langle \delta(\vec{x}) \delta(\vec{x}+\vec{r}) \rangle$$



Power spectrum evolution

Friedmann equation:

$$H^2 = H_0^2\left(\frac{\rho_m/a^3 + \rho_r/a^4}{\rho_c} + \frac{k}{H_0^2 a^2} + \frac{\Lambda}{3H_0^2}\right)$$
$$= H_0^2(\Omega_m/a^3 + \Omega_r/a^4 + \Omega_k/a^2 + \Omega_\Lambda)\,,$$

$$\Omega_r \approx 4.2 \times 10^{-5} h^2$$

Moment of equality:

$$1 + z_{\rm eq} = \Omega_m/\Omega_r \approx 23800\Omega_m h^2.$$

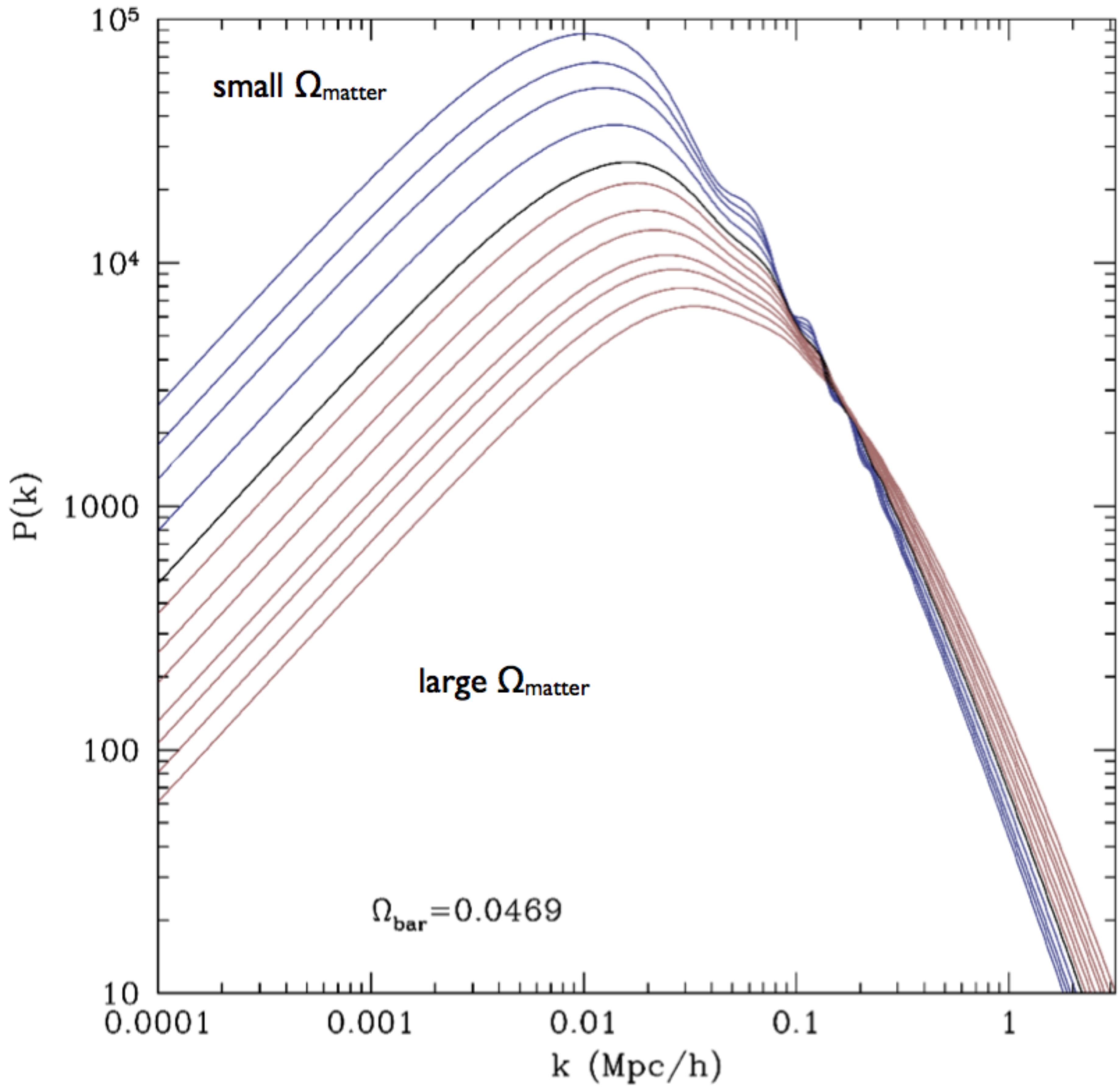
$$\rho_r = \left(1 + (7/8) R_\nu^4 N_\nu\right)\rho_\gamma \qquad R_\nu \equiv T_\nu/T_\gamma\,, \qquad R_\nu^0 = (4/11)^{1/3}.$$

R_ν = neutrino-to-photon temperature ratio

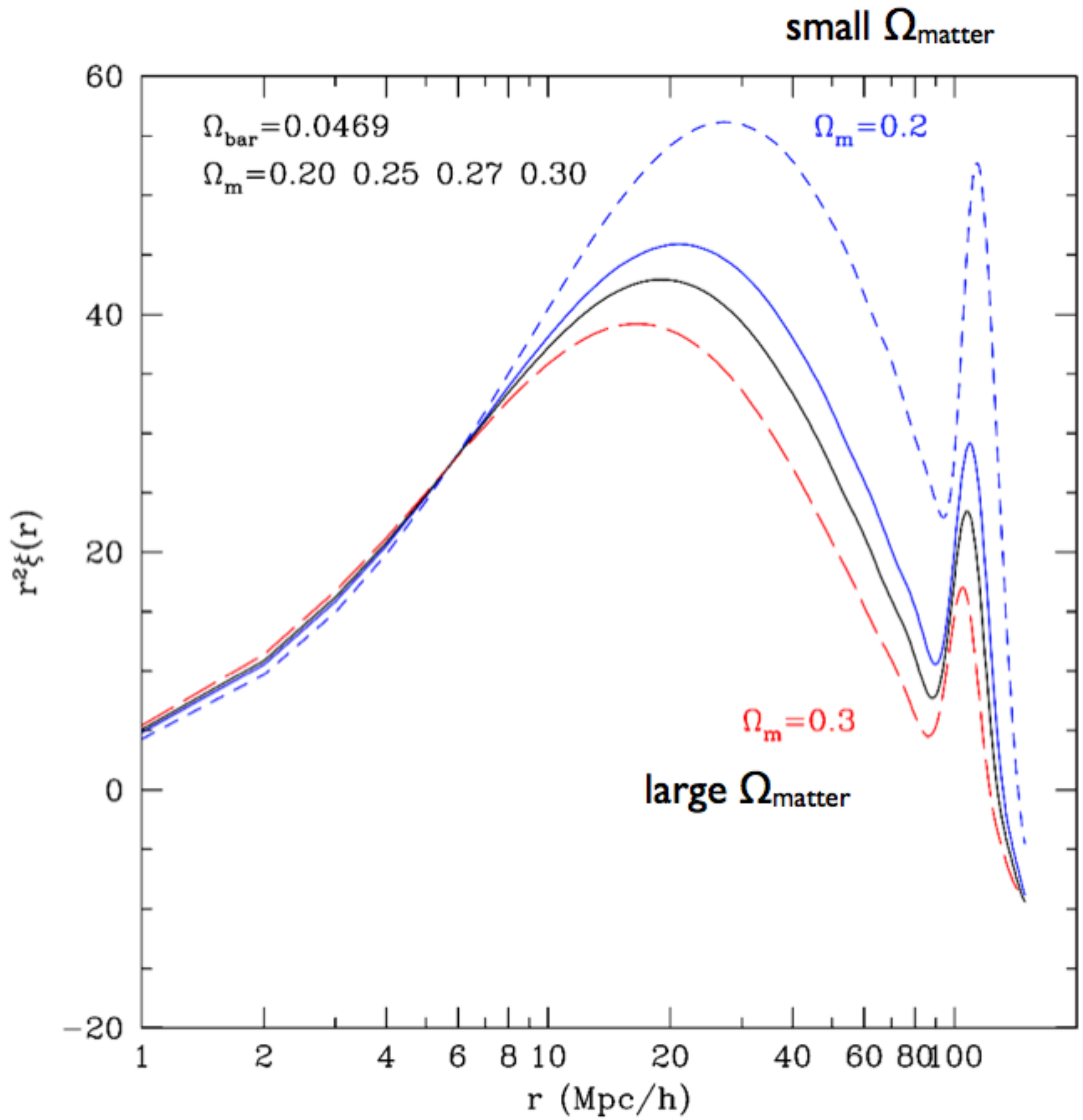
$N_\nu = 3.62 \pm 0.25$ (CMB+H_0)

$N_\nu = 3.30 \pm 0.27$ (CMB + BAO),

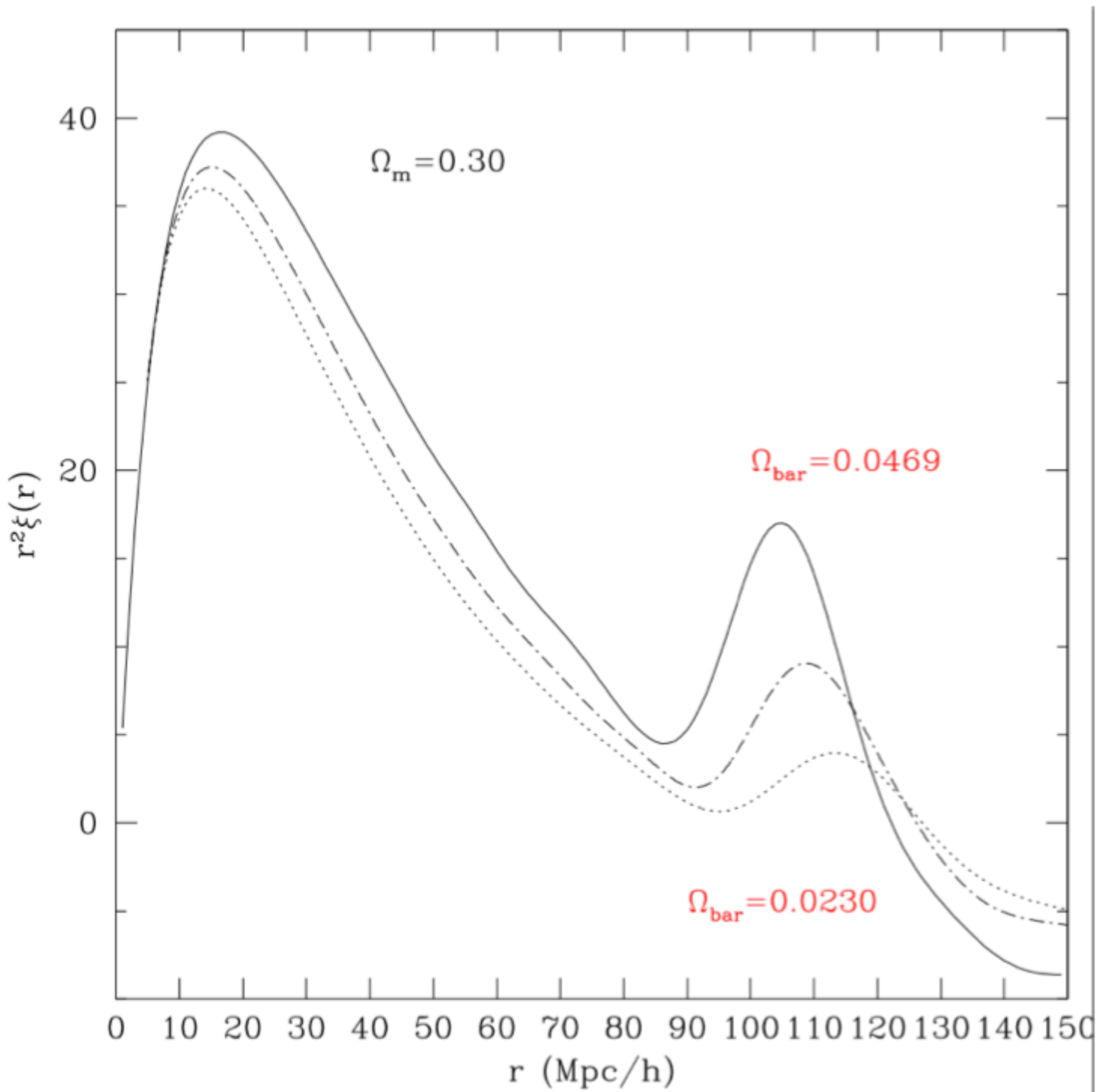
Dependance of P(k) on Ω_{matter}
Amplitude of fluctuations and $\Omega_{baryons}$ are fixed.



Dependance of Correlation function on Ω_{matter}
Amplitude of fluctuations is fixed at 5Mpc/h



Dependance of Correlation function on $\Omega_{mbaryon}$
Amplitude of fluctuations is fixed at 5Mpc/h



Warm dark matter

Random ("thermal") velocities of dark matter particles suppress fluctuations in dark matter: free-streaming effect. Details depend on particular particle model of wdm candidates.

the wavenumber at which the linear WDM suppression reaches 50% in terms of matter power, $k_{1/2}$, w.r.t. the ΛCDM case can be approximated as:

$$k_{1/2} \sim 6.5 \frac{h}{\mathrm{Mpc}} \left(\frac{m_{\mathrm{WDM}}}{1\mathrm{keV}}\right)^{1.11} \left(\frac{\Omega_{DM}}{0.25}\right)^{-0.11} \left(\frac{h}{0.7}\right)^{1.22},$$

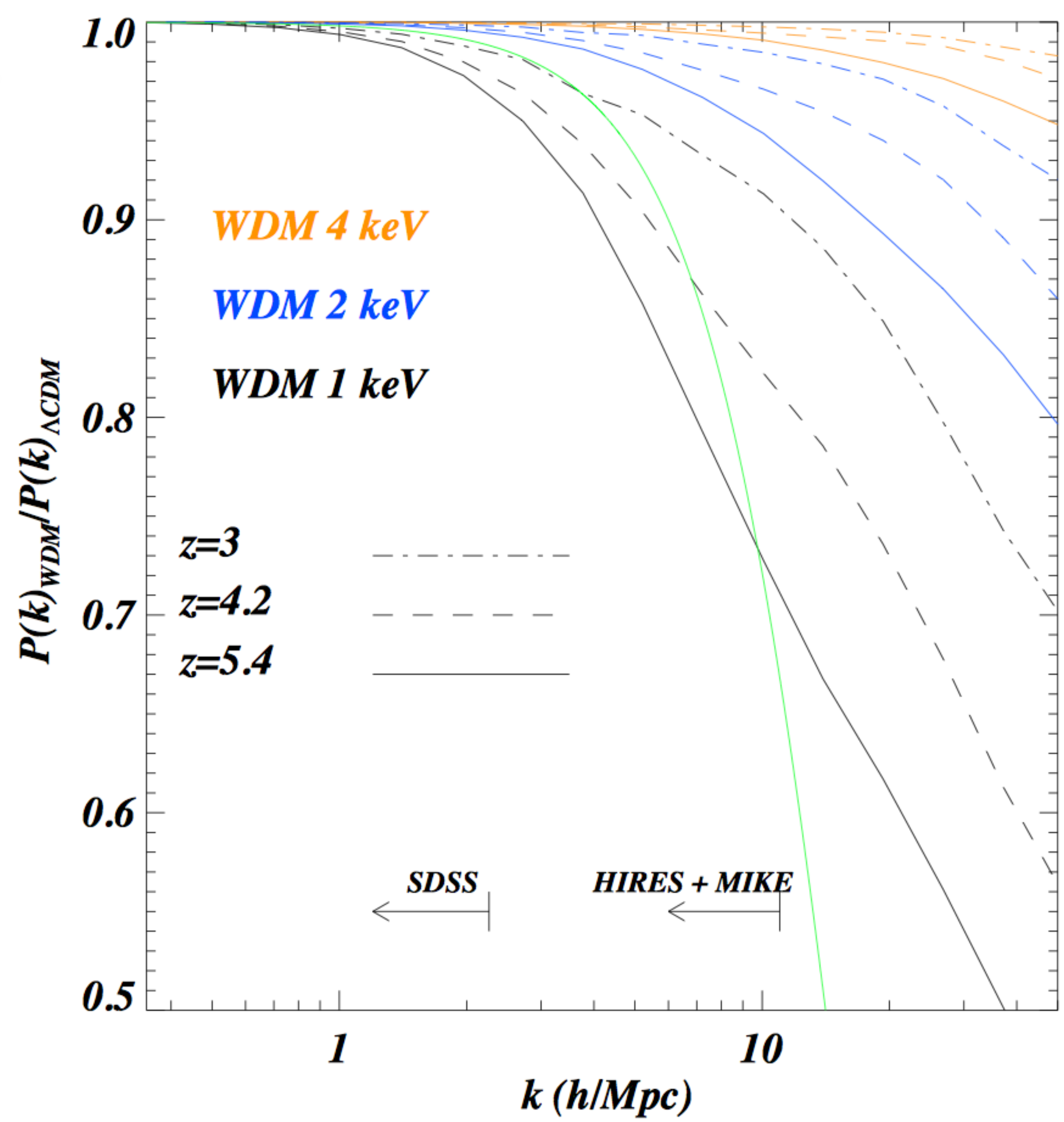
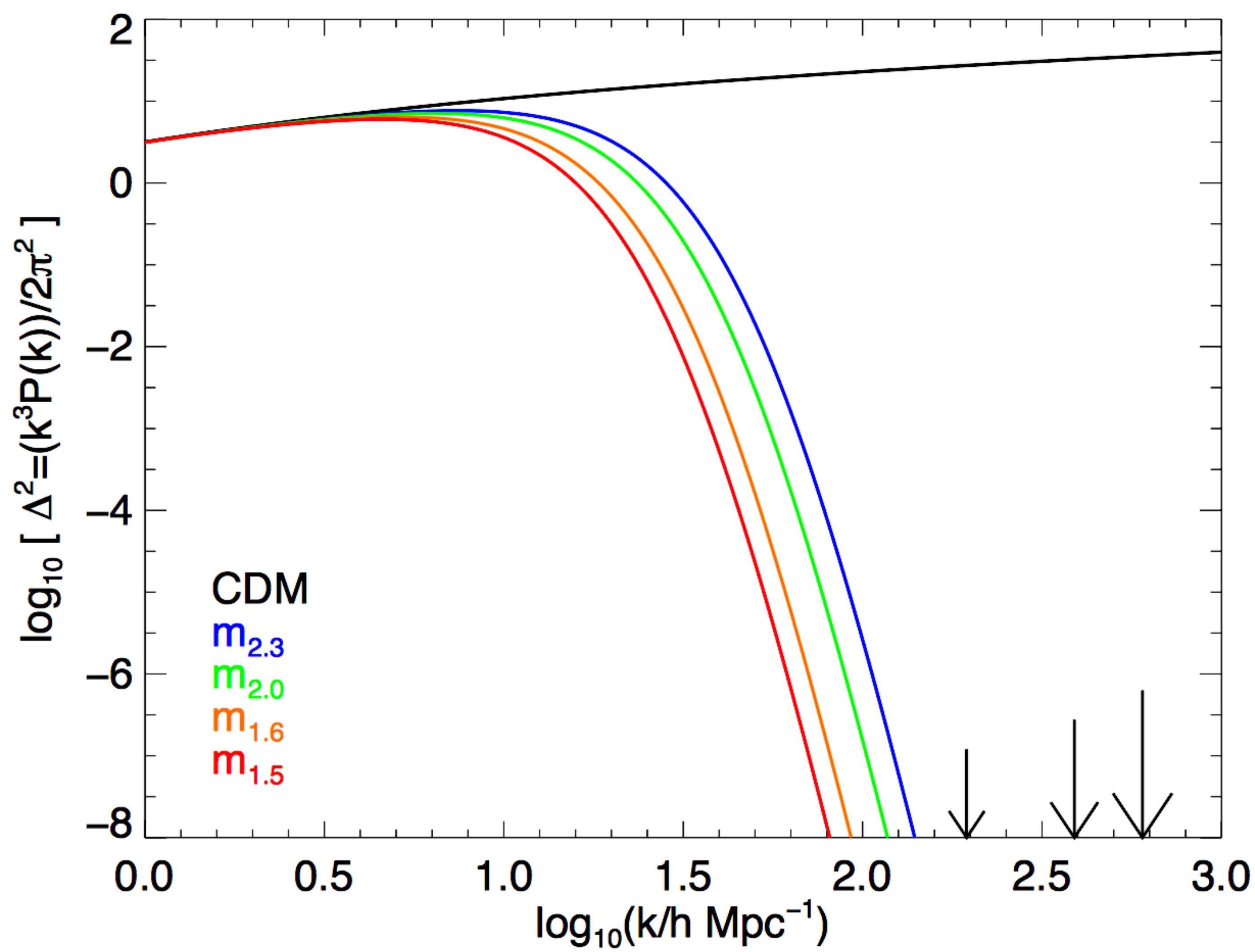


FIG. 1: Ratio between the 3D non-linear matter power spectrum of 3 different WDM models (1, 2 and 4 keV, black, blue and orange curves) at 3 different redshifts (z = 3, 4.2, 5.4, represented by the dot-dashed, dashed and continuous curves) and the corresponding ΛCDM model. The green curve represents the linear redshift independent suppression in terms of matter power for a $m_{\mathrm{WDM}} = 2$ keV model obtained using

Viel et al 2013

Warm dark matter

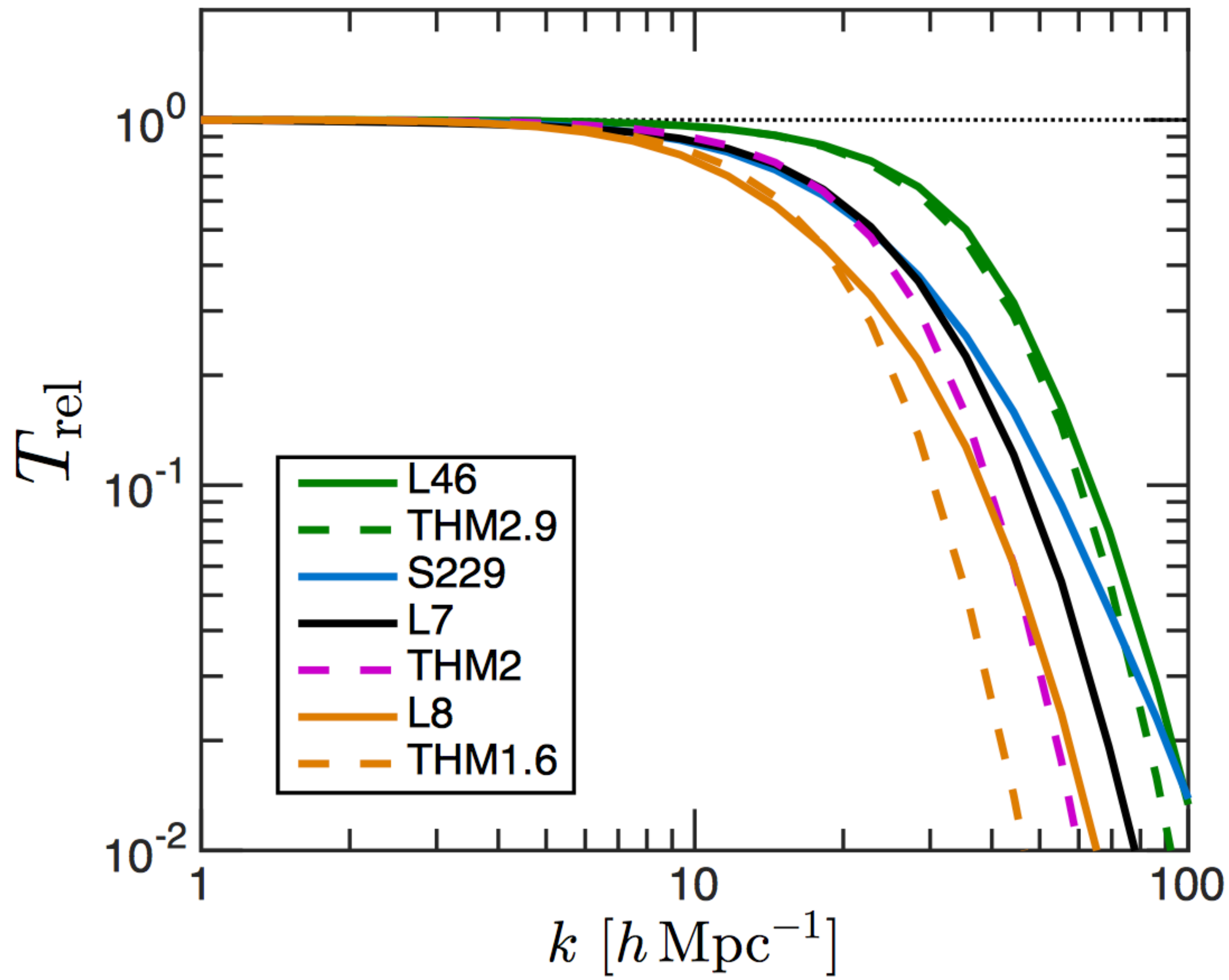
Simulation	m_{WDM}[keV]
CDM-W7	–
$m_{2.3}$	2.322
$m_{2.0}$	2.001
$m_{1.6}$	1.637
$m_{1.5}$	1.456



Lovell et al 2014

Warm dark matter

Figure 1. Suppression of the linear matter power spectrum of resonantly-produced sterile neutrino models (solid lines) and their best-fit thermal equivalent model (dashed lines) relative to CDM, where $T_{\rm rel} = \sqrt{P_{\rm WDM}/P_{\rm CDM}}$. The L46 (solid; green), L7 (solid; black), and L8 (solid; orange) models are based on the sterile neutrino DM production calculations of Abazajian (2014), while S229 (solid; blue) model is based on the more accurate treatment of Venumadhav et al. (2015). The L7 and S229 models both share an equivalent thermal WDM model of $m = 2$ keV (THM2/dashed; magenta). The shape and large-k behavior of the WDM transfer functions vary among the sterile neutrino models and compared with their thermal equivalent models.



Resonant Sterile Neutrino Dark Matter in the Local and High-z Universe

Bozek et al 2015

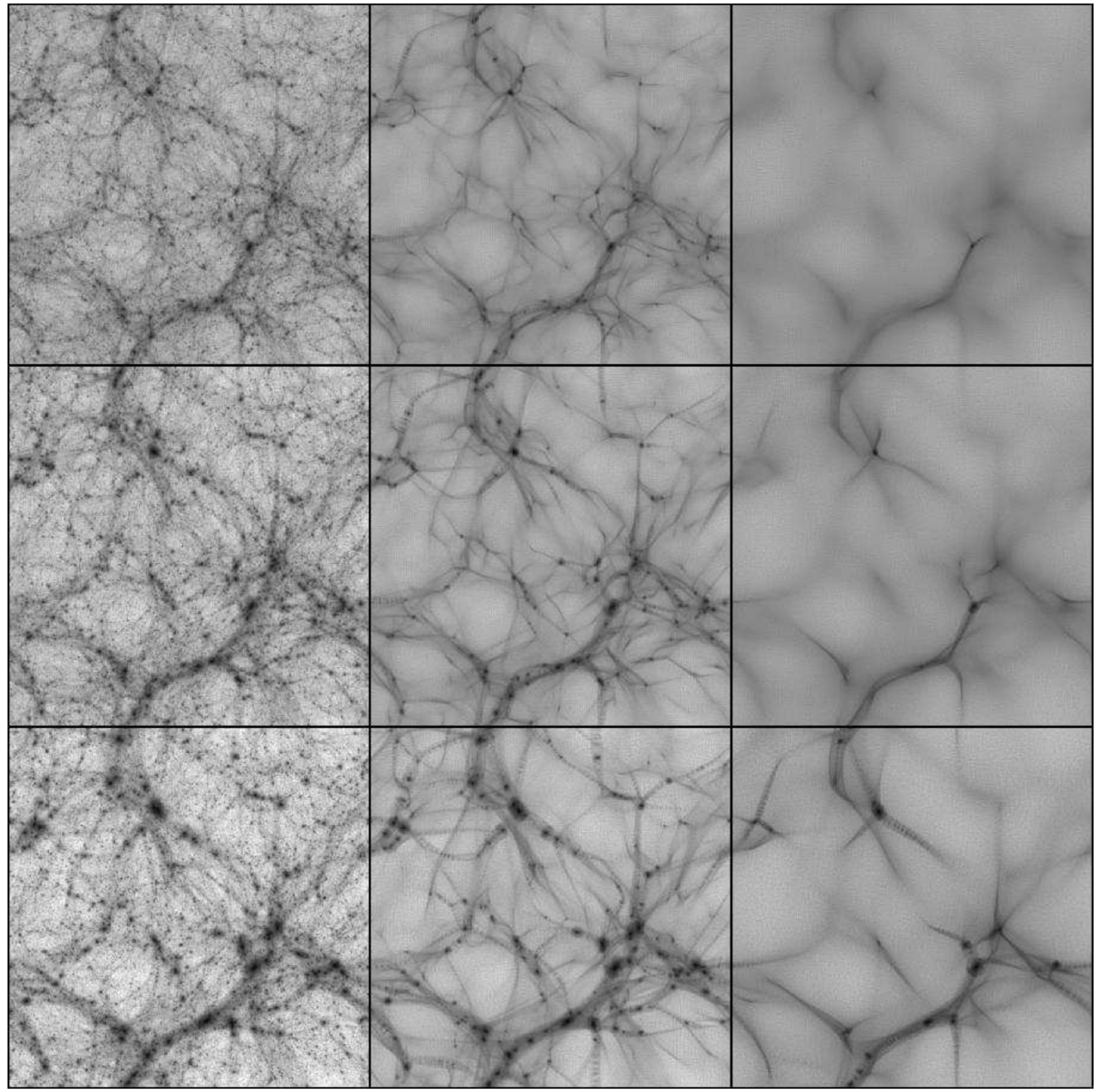
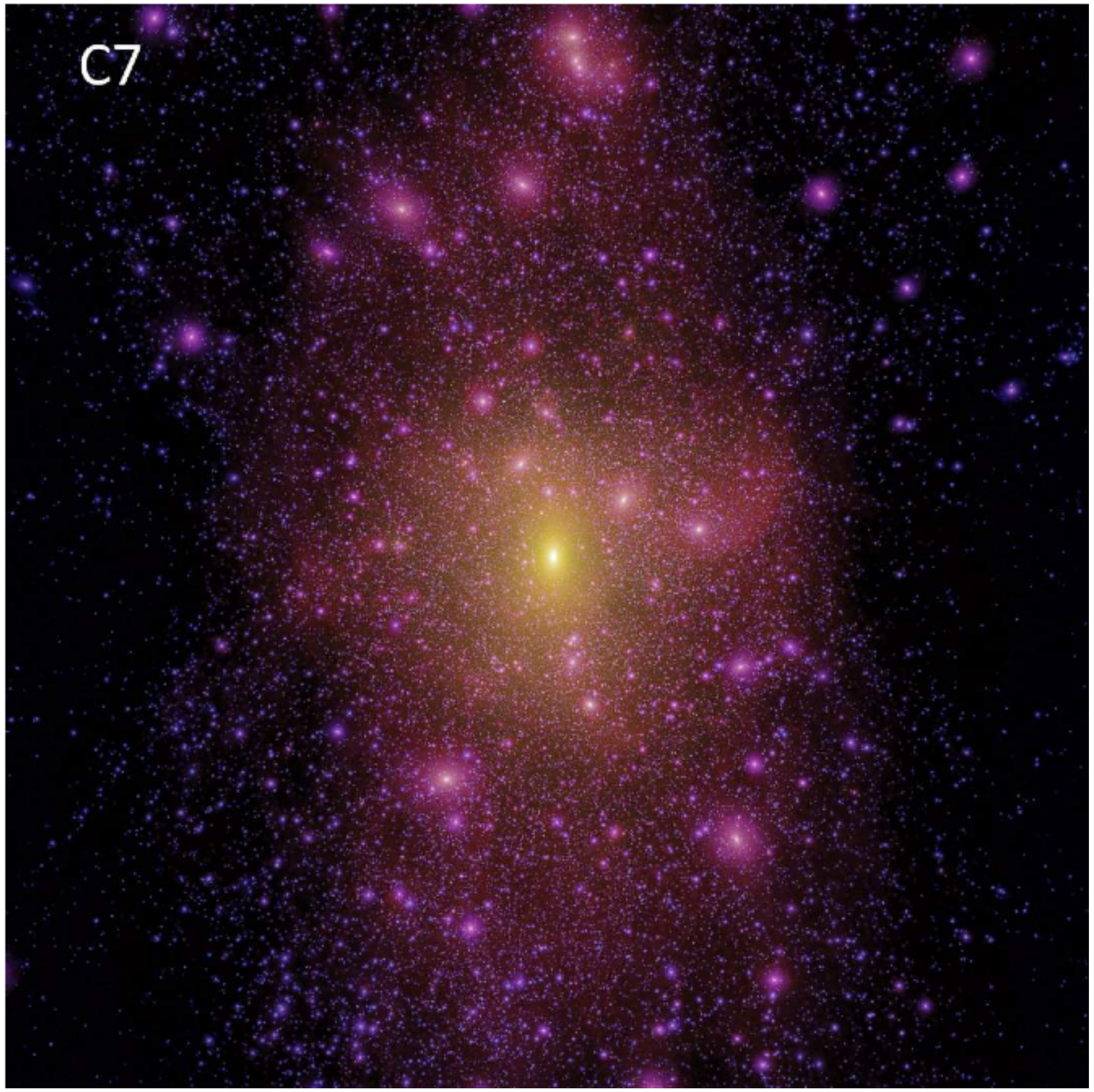


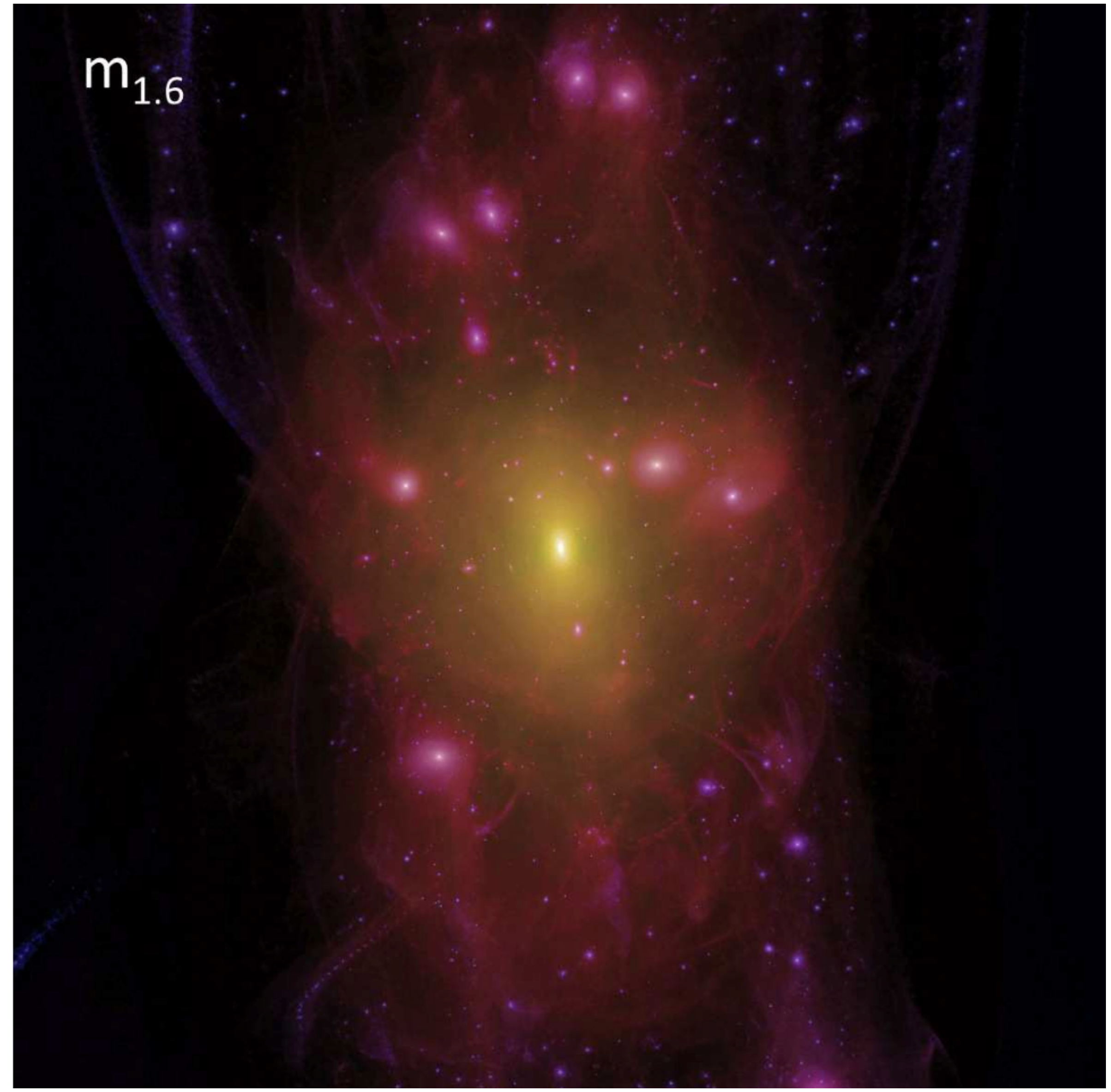
Fig. 4.— Projected density of 20 h^{-1}Mpc boxes, on a logarithmic scale of surface density. Left to right: ΛCDM, m_X=350 eV and m_X=175 eV ΛWDM. Top to bottom: redshift Z =3, 2, and 1. A simulation with $m_X \sim 1$ keV would have an appearance intermediate between the left and central columns. (A higher resolution version of this Figure is available at the web site referred to in the introduction.)

Bode et al 2001

LCDM



WDM



Lovell et al 2014