Halo Mass and Velocity functions

Functional form for mass function:

$$\frac{dn}{dM} = f(\sigma)\,\frac{\bar{\rho}_m}{M}\,\frac{d\ln\sigma^{-1}}{dM}$$

Average matter density:

$$\bar{\rho}_m(z) \equiv \Omega_m(z)\rho_{\rm crit}(z) = \bar{\rho}_m(0)(1+z)^3$$

Numerical factor:

$$f_{\rm P-S}(\sigma) = \sqrt{\frac{2}{\pi}}\frac{\delta_{\rm c}}{\sigma}\exp\left(-\frac{\delta_{\rm c}^2}{2\sigma^2}\right)$$

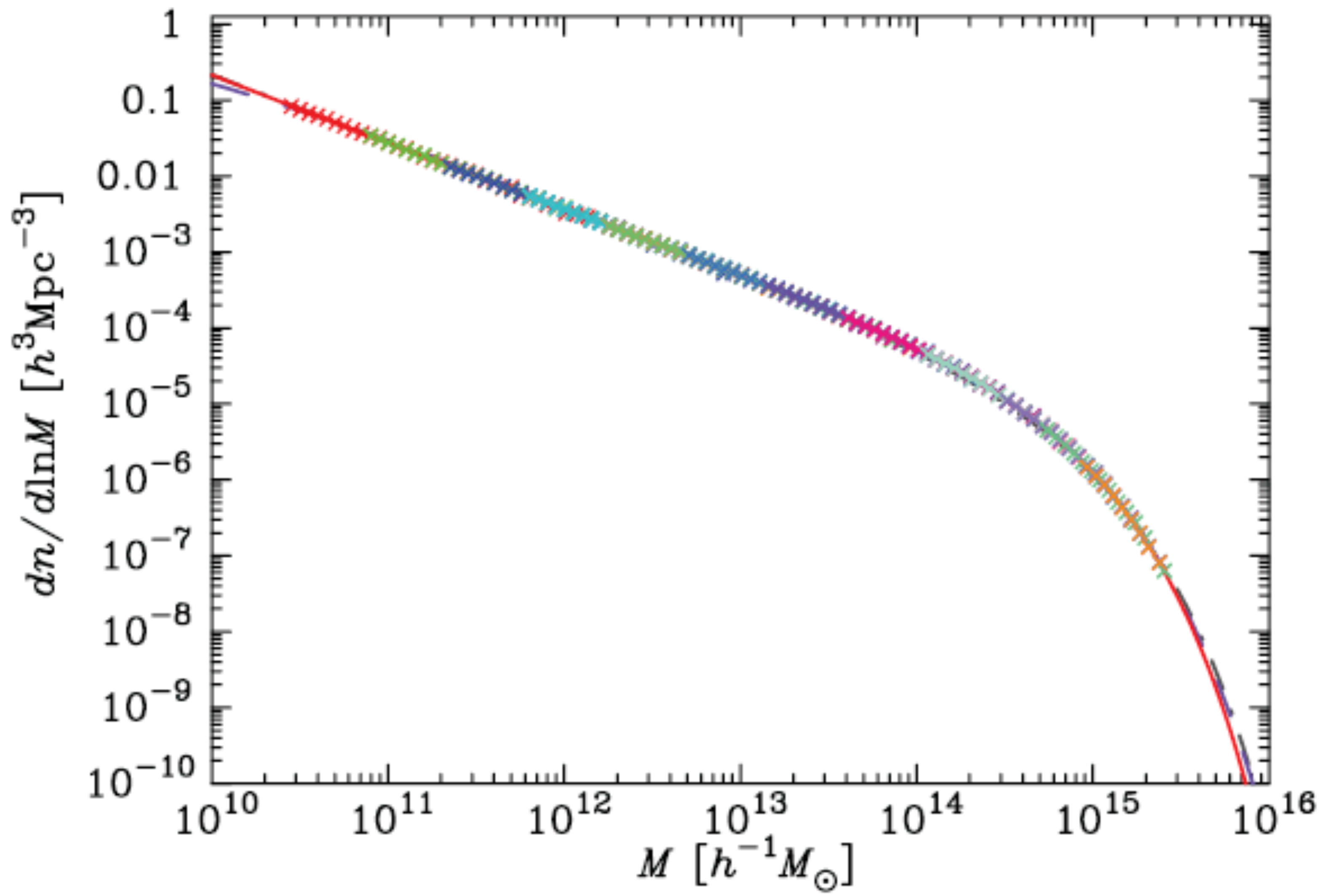
$$\delta_{\rm c} = 1.686$$

$$f_{\rm S-T}(\sigma) = A\sqrt{\frac{2a}{\pi}}\left[1+\left(\frac{\sigma^2}{a\delta_{\rm c}^2}\right)^p\right]\frac{\delta_{\rm c}}{\sigma}\exp\left(-\frac{a\delta_{\rm c}^2}{2\sigma^2}\right)$$

$$A = 0.3222,\ a = 0.707 \text{ and } p = 0.3$$

rms density fluctuation:

$$\sigma^2(M) = \frac{b^2(z)}{2\pi^2}\int_0^\infty k^2 P(k) W^2(k; M)\,\mathrm{d}k,$$



Warren et al 2006

FIG. 1.—Central values of the binned mass functions from sixteen 1024^3 simulations of the ΛCDM universe as crosses, with simulations in different colors. The best-fit form for the mass function we calculate is shown as a solid line (*red*), the Jenkins fit as a dashed line (*purple*), and the Sheth-Tormen fit as a dot-dashed line (*dark gray*). Goodness of fit is poorly judged on this extreme log scale; it is more clearly resolved in the linear residuals of Fig. 2.

Halo mass function at z=0

Note the vertical axis scaling: the main trend of the mass function - $dn/dM \sim M^{-2}$ - has been taken away. We are looking at the deviations from the power-law.

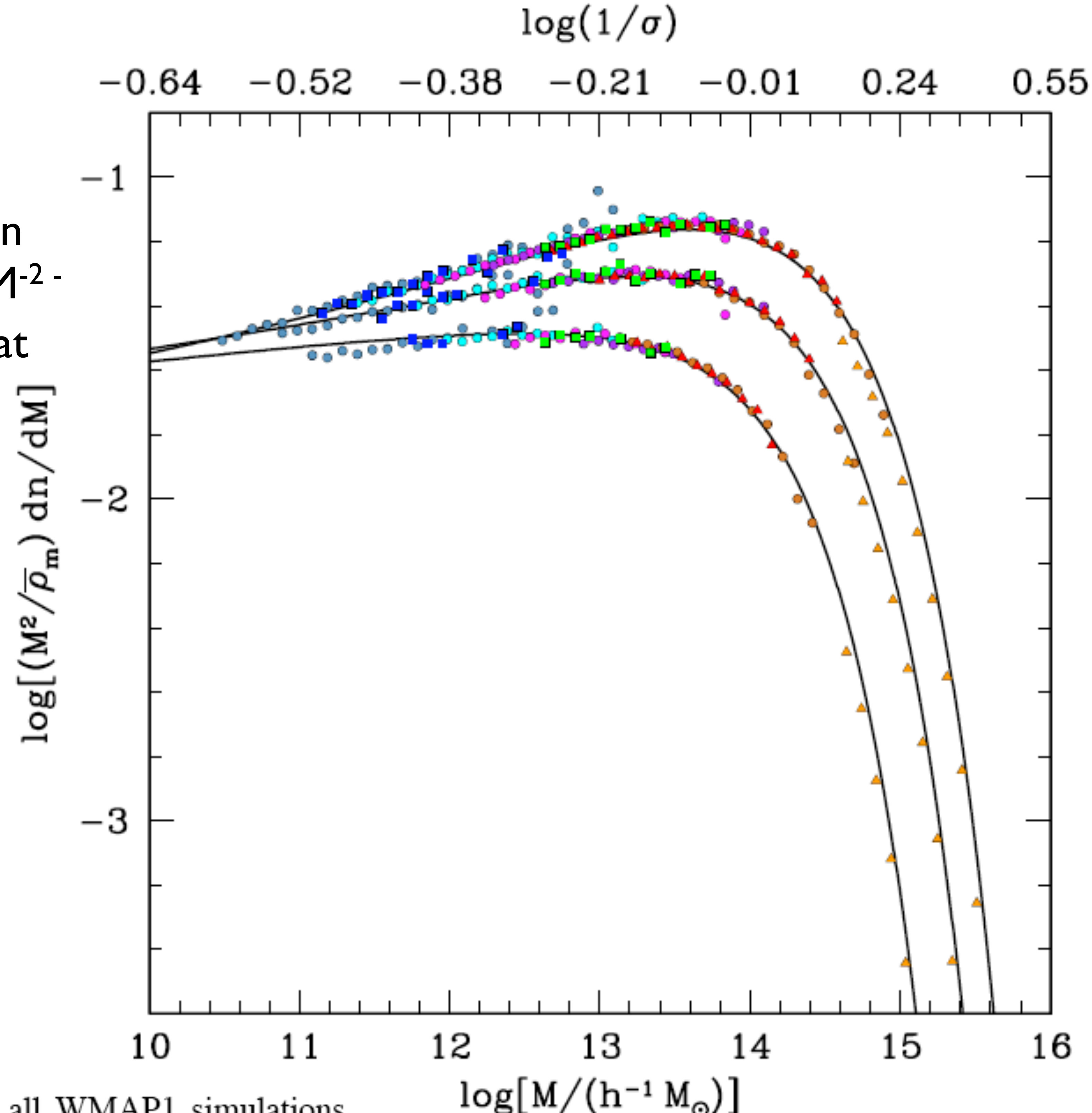
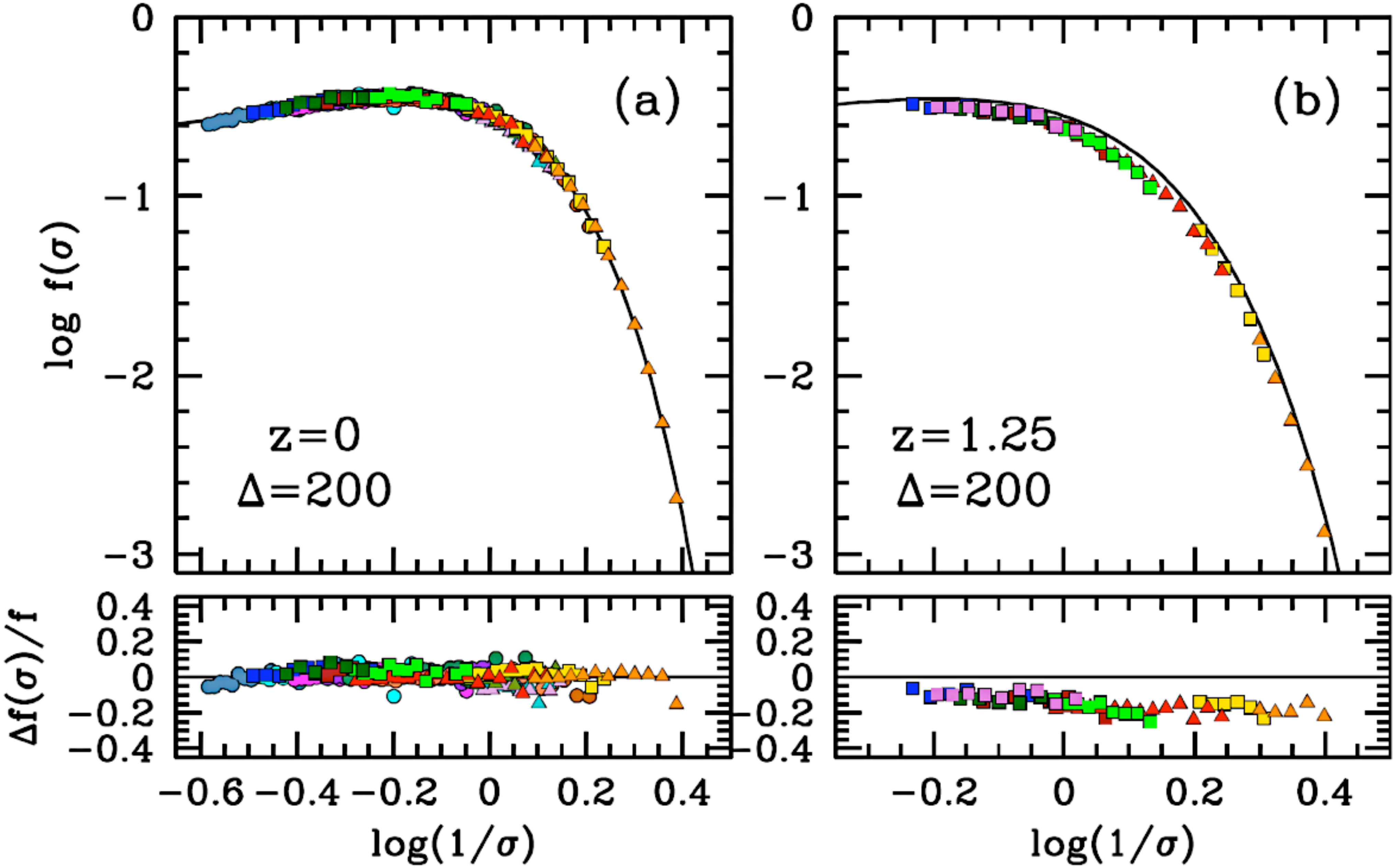


FIG. 5.— The measured mass functions for all WMAP1 simulations, plotted as $(M^2/\bar{\rho}_m)\,dn/dM$ against $\log M$. The solid curves are the best-fit functions from Table 2. The three sets of points show results for $\Delta = 200$, 800, and 3200 (from top to bottom). To provide a rough scaling between M and σ^{-1}, the top axis of the plot shows σ^{-1} for this mass range for the WMAP1 cosmology. The slight offset between the L1280 results and the solid curves is due to the slightly lower value of $\Omega_m = 0.27$.

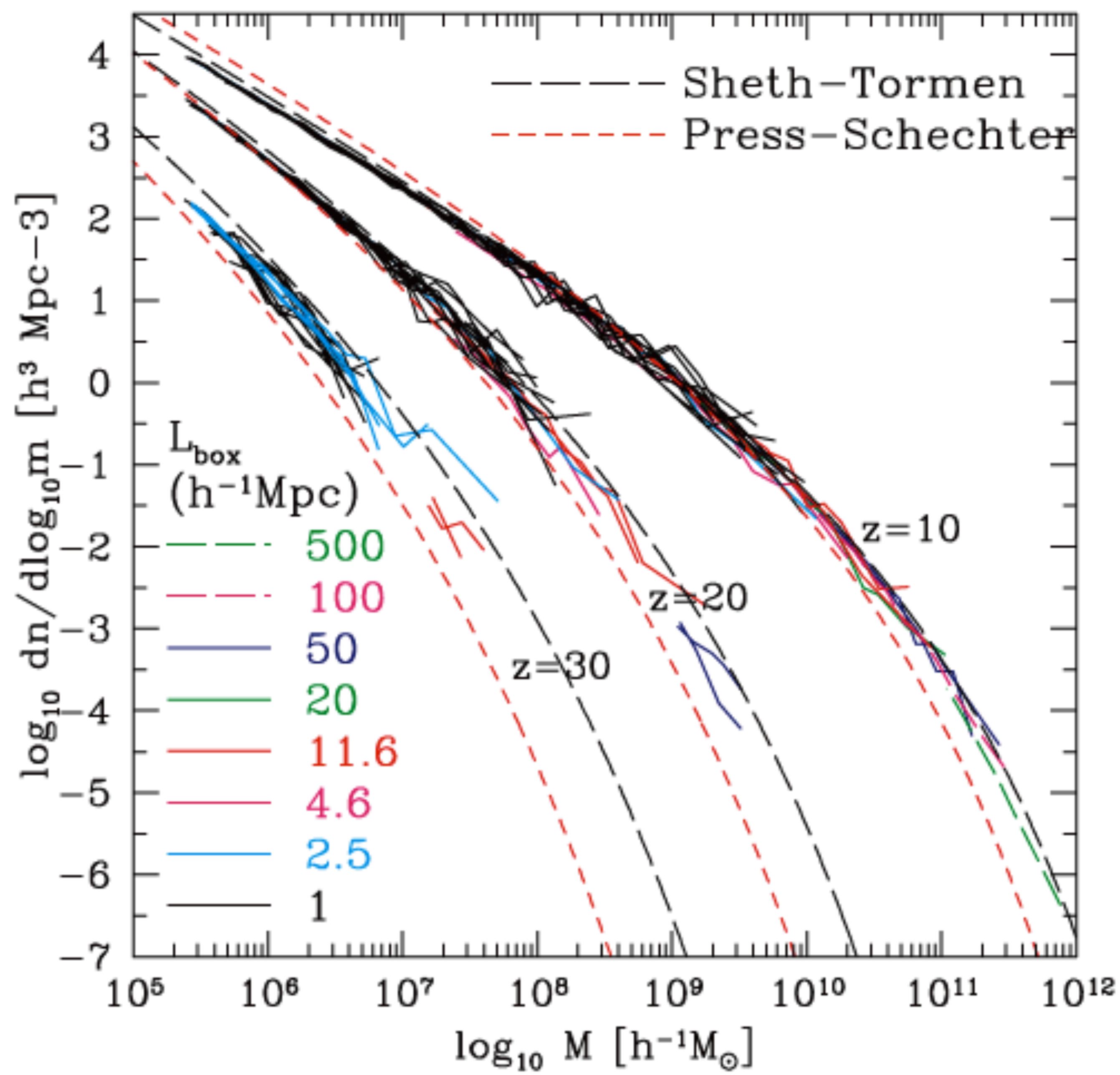
Tinker etal 2008



Nearly universal shape of mass function when we plot $f(\sigma)$

$$\frac{dn}{dM} = f(\sigma)\,\frac{\bar{\rho}_m}{M}\,\frac{d\ln\sigma^{-1}}{dM}$$

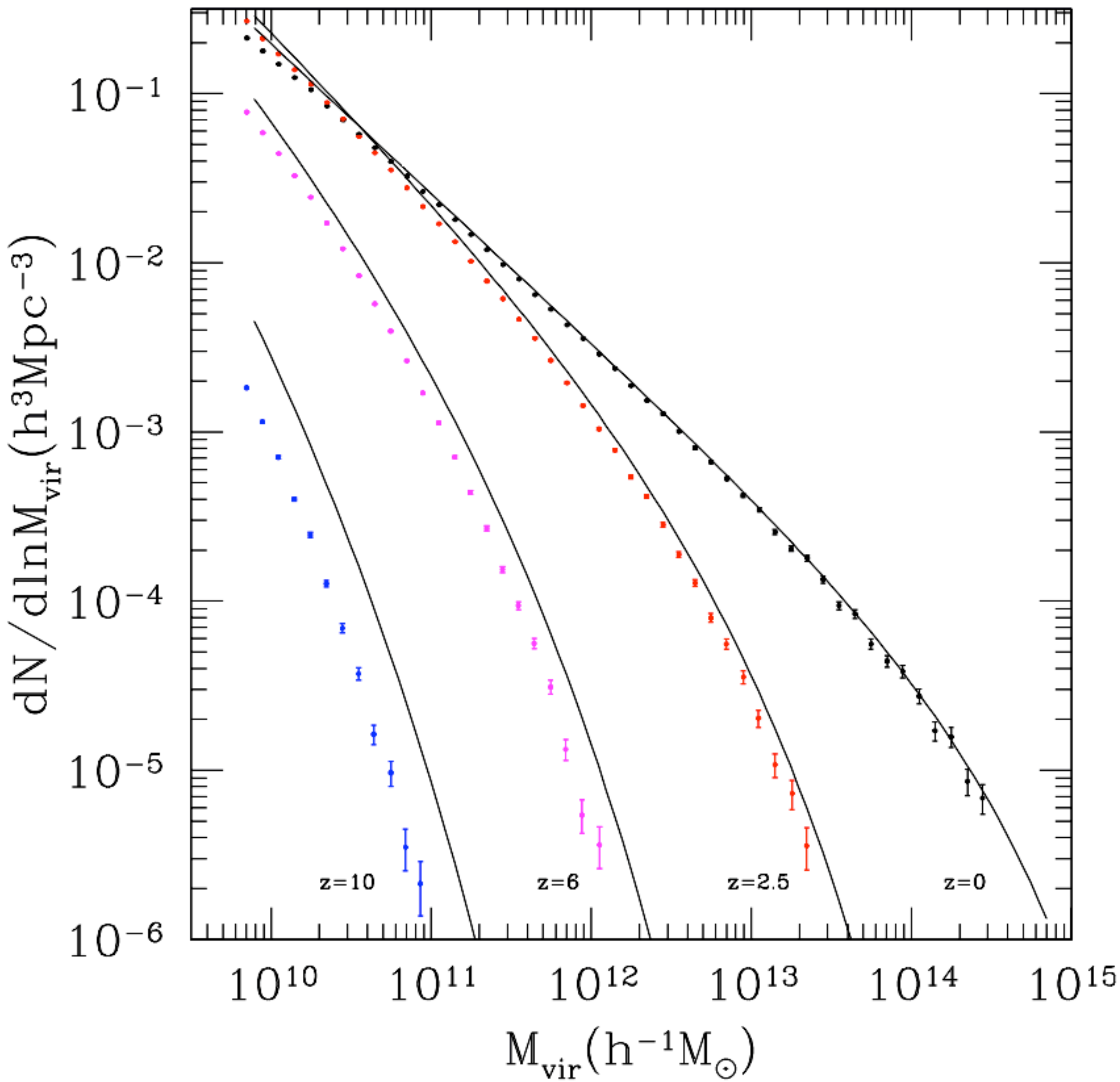
Evolution of the mass function with redshift



Read et al 2007

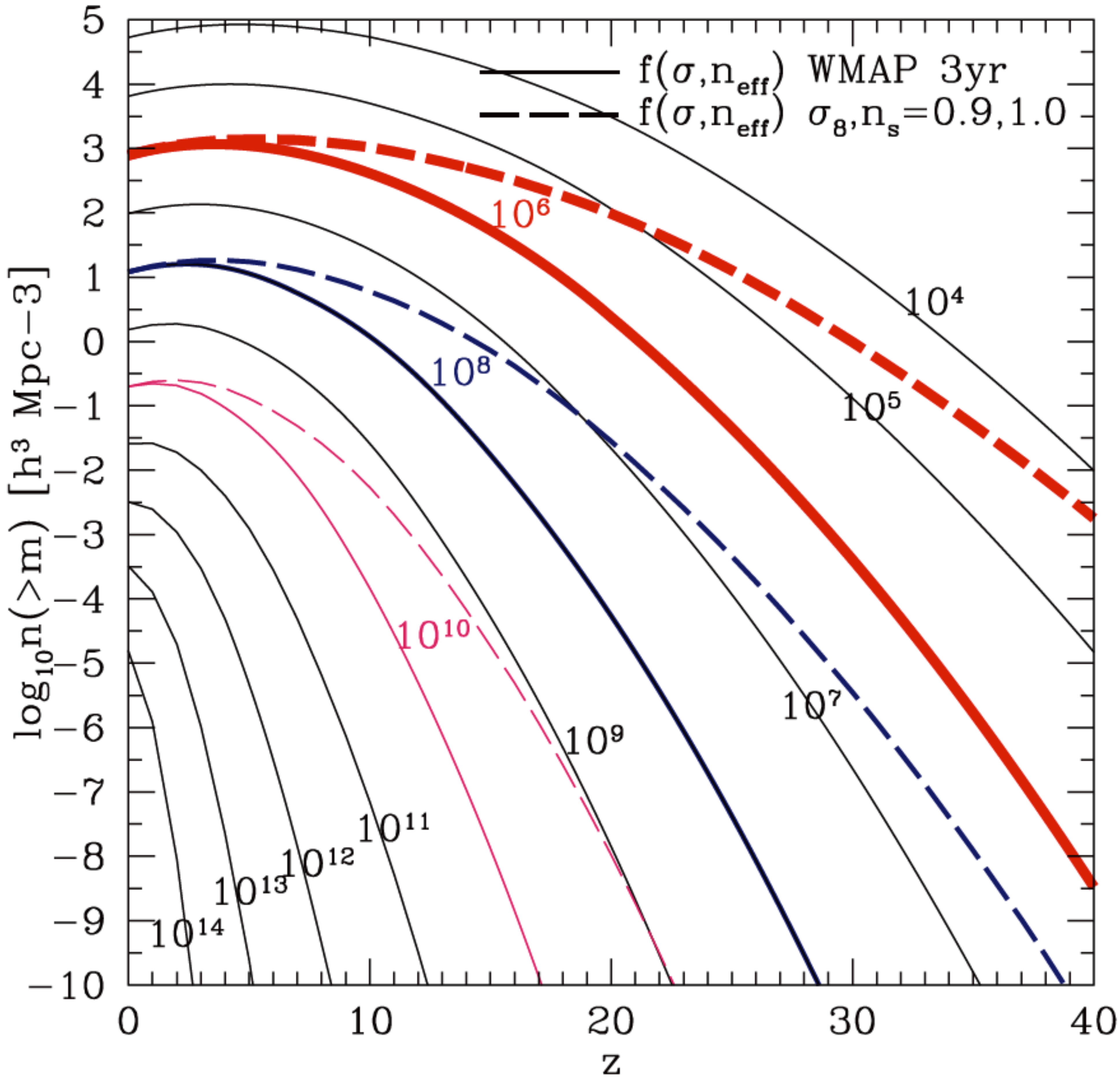
Evolution of the mass function with redshift

Full curves are for the Sheth-Torman approximation



Symbols are from N-body simulations for distinct halos defined using Spherical Overdensity algorithm

Figure 8. Cumulative abundance versus redshift for the *WMAP* three-year cosmological parameters (Spergel et al. 2006) implied by our new analytic mass function (equation 11). Also shown is the abundance of haloes for the standard cosmological parameters assumed in the simulations of this work for three masses. The curves are labelled by mass in units of $h^{-1}\,\mathrm{M}_{\odot}$. The curves marked '*WMAP* 3yr' assume the complete preferred *WMAP* three-year parameters ($\sigma_8,\ n_\mathrm{s} = 0.74,\ 0.951$), including the effect of the preferred parameters (e.g. $\Omega_\mathrm{m},\ \Omega_\mathrm{baryon}$, etc.) on the transfer function. The abundances at high redshift are most sensitive to cosmological parameters.

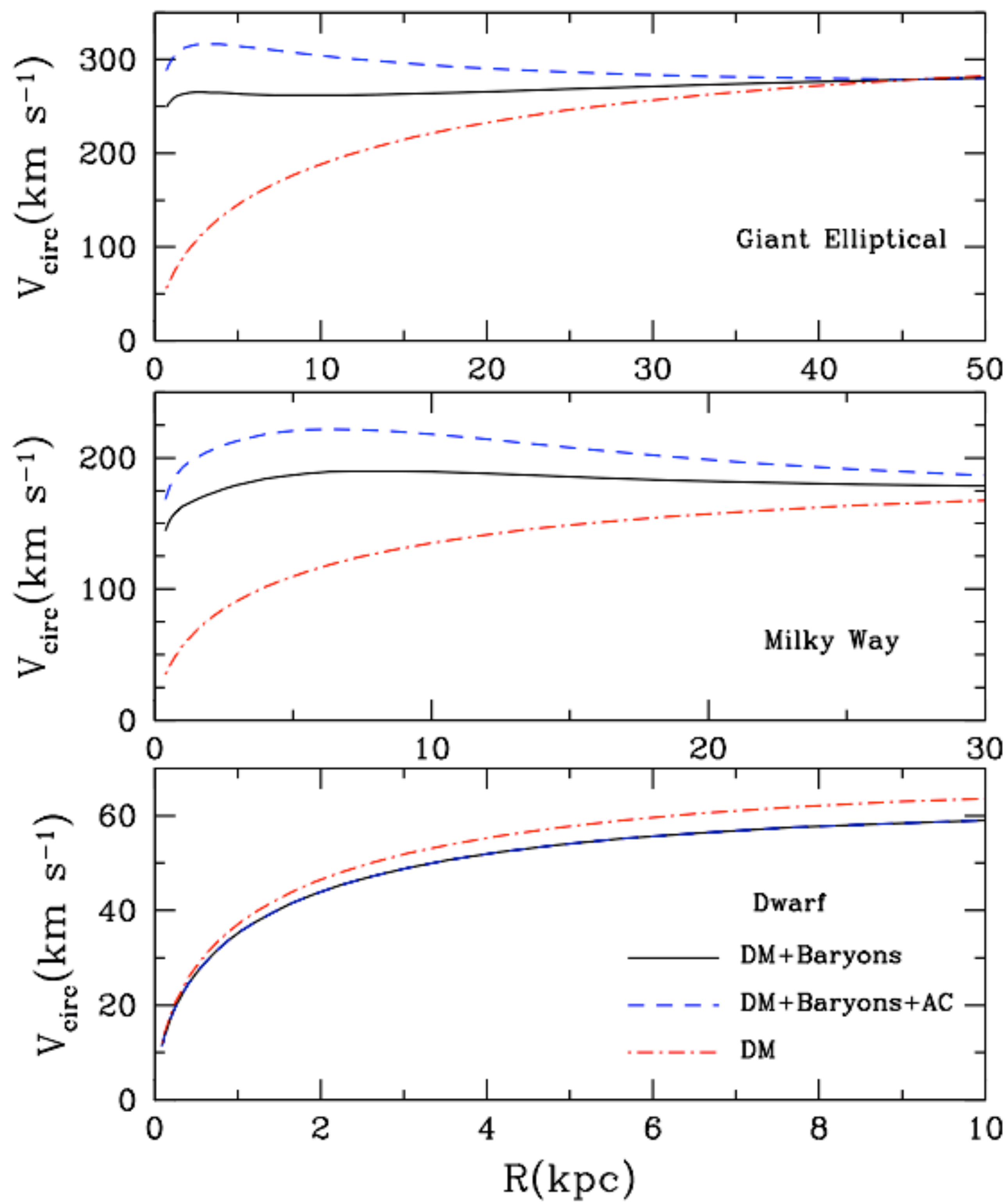


f(σ,n_eff) WMAP 3yr
f(σ,n_eff) σ_8,n_s=0.9,1.0
10^4
10^5
10^6
10^7
10^8
10^9
10^10
10^11
10^12
10^13
10^14
log_10 n(>m) [h^3 Mpc−3]
z

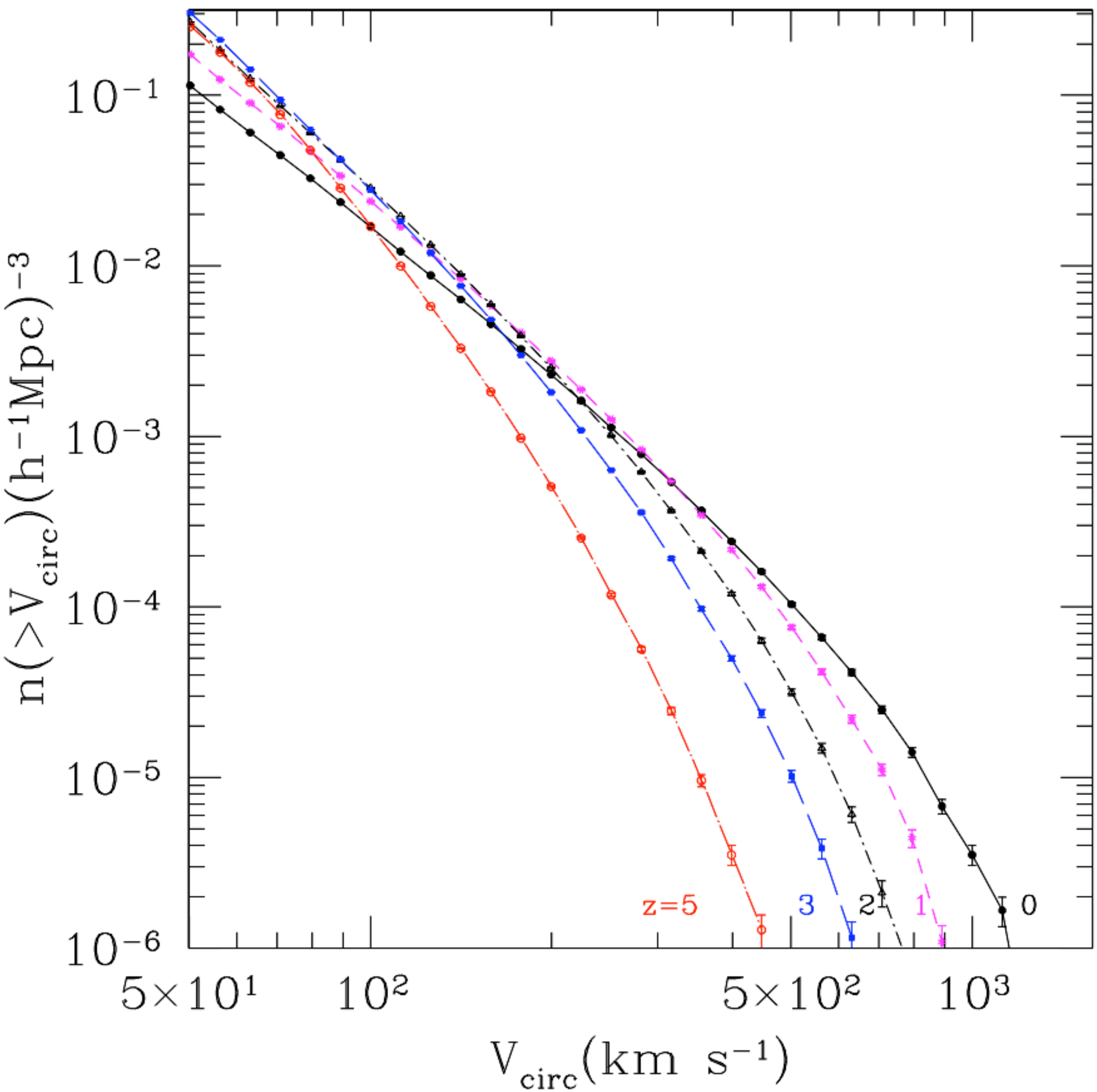
For each halo we find mass within given radius M(<r) and then find maximum of circular velocity

$$V_{\rm circ} = \sqrt{\frac{GM(<r)}{r}}\Bigg|_{\rm max}$$

Fig. 5.— Effect of cold baryons on circular velocity profiles for three characteristic models of galaxies with virial masses 10^{13} $M_\odot$ (top), 1.7×10^{12} $M_\odot$ (middle), and 7×10^{10} $M_\odot$ (bottom). The "DM" curves include a cosmological fraction of baryons that trace the dark matter distribution. The cold baryon mass is added to the true dark matter mass in calculating the circular velocity ("DM+Baryons"). The effect of adiabatic compression of the dark matter is included in the models named "DM+Baryons+AC". After adding the cold baryons the circular velocities are rather flat in the inner 5 – 10 kpc regions.



Halo Velocity function



$$n(>V) = AV^{-3}\exp\left(-\left[\frac{V}{V_0}\right]^{\alpha}\right)$$

where the parameters A, V_0, and α are functions of redshift. For $z = 0$ we find

$$\begin{aligned} A &= 1.82 \times 10^4 (h^{-1}\ \mathrm{Mpc/km\ s^{-1}})^{-3}, \\ \alpha &= 2.5, \qquad (16) \\ V_0 &= 800\ \mathrm{km\ s^{-1}}. \end{aligned}$$