**Homework 3. AST506**

- (1) System consists of two equal-mass spheres. Each sphere has a constant density. Mass and radius of each sphere is $M/2$ ans $R$. Distance between centers of spheres is $5R$. Find gravitational potential and acceleration in points:
  - A - center of the left sphere
  - B - at the surface of the left sphere, closest point to the second sphere
  - C - point between the centers of spheres ($2.5R$ from each center)
- (2) Prove that the total gravitational energy $\mathcal{W}$ of a spherical system can be written in the form:
$$\mathcal{W} = -\frac{G}{2}\int_0^\infty \frac{M^2(r)}{r^2}dr, \tag{1}$$
where $M(r)$ is the mass inside radius $r$.
- (3) Density profiles of dark matter halos are well approximated by the following function with two parameters:
$$\rho(r) = \frac{\rho_0}{x(1+x)^2}, \quad x \equiv \frac{r}{r_0} \tag{2}$$
  - Find gravitational potential $U(r)$
  - Find circular velocity $V_{\rm circ} = \sqrt{GM(r)/r}$ for the profile. Make a plot of it where on vertical axis you plot $V_{\rm circ}/V_{\rm max}$ and on horizontal $r/r_0$. Here $V_{\rm max}$ is the maximum circular velocity.
- (4) Density profiles of elliptical galaxies and bulges of spiral galaxies can be approximated with profiles:
$$\rho(r) = \frac{\rho_0}{x(1+x)^3}, \quad x \equiv \frac{r}{r_0} \tag{3}$$
Find the half-mass radius, the total mass, and the total gravitational energy $\mathcal{W}$ of the system.