## Home work 1.

due Jan 24

1. Find general solution of equation:

$$
\begin{equation*}
\ddot{x}=\omega^{2} x+C, \tag{1}
\end{equation*}
$$

where $C$ is a constant.
2. Find general solution of equation:

$$
\begin{equation*}
\frac{d y}{d x}=-\frac{y}{x^{3}} . \tag{2}
\end{equation*}
$$

3. Find solution of equation:

$$
\begin{equation*}
\frac{d y}{d x}=-\frac{\sigma(x)}{x} \tag{3}
\end{equation*}
$$

where

$$
\sigma(x)=\left\{\begin{array}{lll}
\sigma_{0}\left(\frac{x}{x_{0}}\right)^{2}, & \text { if } & x<x_{0} \\
\sigma_{0}, & \text { if } & x>x_{0}
\end{array}\right.
$$

and $x=0-\infty$ and $y=0$ at $x=0$.
4. Find

$$
\begin{equation*}
\frac{\partial}{\partial x} \int_{0}^{\sin (x)} \frac{\ln (x)}{x} d x \tag{4}
\end{equation*}
$$

5. Analyze function

$$
\begin{equation*}
f(x)=\int_{0}^{x} \frac{x^{30}}{\left(1+x^{2}\right)^{15}} d x \tag{5}
\end{equation*}
$$

6. Integral

$$
\begin{equation*}
\int_{1}^{x} \frac{x^{3}+\sin (x)}{\left(1+x^{2}\right)^{2}} d x \tag{6}
\end{equation*}
$$

diverges at $x \gg 1$. Find an approximation for this integral at large $x$.
7. In the following problems $\vec{r}$ is the radius-vector in a 3D space.

$$
\begin{equation*}
\nabla\left(\frac{1}{|r|}\right)=? \tag{7}
\end{equation*}
$$

8. 

$$
\begin{equation*}
\nabla\left(r^{2}\right)=? \tag{8}
\end{equation*}
$$

9. 

$$
\begin{equation*}
\nabla\left(\vec{r}-\vec{r}_{0}\right)=? \tag{9}
\end{equation*}
$$

where $\vec{r}_{0}$ is a constant vector.
10.

$$
\begin{equation*}
\nabla^{2}\left(\frac{1}{r^{2}}\right)=? \tag{10}
\end{equation*}
$$

11. 

$$
\begin{equation*}
\nabla^{2}\left(\frac{1}{\left|\vec{r}-\vec{r}_{0}\right|}\right)=? \tag{11}
\end{equation*}
$$

where $\vec{r}_{0}$ is a constant vector.

