Home work 1. **due Jan 24**

1. Find general solution of equation:
$$\ddot{x} = \omega^2 x + C, \tag{1}$$
where C is a constant.

2. Find general solution of equation:
$$\frac{dy}{dx} = -\frac{y}{x^3}. \tag{2}$$

3. Find solution of equation:
$$\frac{dy}{dx} = -\frac{\sigma(x)}{x}, \tag{3}$$
where
$$\sigma(x) = \begin{cases} \sigma_0 \left(\frac{x}{x_0}\right)^2, & \text{if} \quad x < x_0 \\ \sigma_0, & \text{if} \quad x > x_0 \end{cases}$$
and $x = 0 - \infty$ and $y = 0$ at $x = 0$.

4. Find
$$\frac{\partial}{\partial x} \int_0^{\sin(x)} \frac{\ln(x)}{x} dx \tag{4}$$

5. Analyze function
$$f(x) = \int_0^x \frac{x^{30}}{(1+x^2)^{15}} dx \tag{5}$$

6. Integral
$$\int_1^x \frac{x^3 + \sin(x)}{(1+x^2)^2} dx, \tag{6}$$
diverges at $x >> 1$. Find an approximation for this integral at large x.

7. In the following problems $\vec{r}$ is the radius-vector in a 3D space.
$$\nabla \left(\frac{1}{|r|}\right) = ? \tag{7}$$

8.
$$\nabla \left(r^2\right) = ? \tag{8}$$

9.
$$\nabla \left(\vec{r} - \vec{r}_0\right) = ? \tag{9}$$
where $\vec{r}_0$ is a constant vector.

10.

$$\nabla^2 \left(\frac{1}{r^2} \right) = ? \tag{10}$$

11.

$$\nabla^2 \left(\frac{1}{|\vec{r} - \vec{r}_0|} \right) = ? \tag{11}$$

where $\vec{r}_0$ is a constant vector.