Violent Relaxation

Violent relaxation

This process happens at early stages of evolution of a system. The key feature is the explicit dependence of gravitational potential $\varphi(x,t)$ on time t:

$$\varphi = \varphi(x,t)$$

Let's try to integrate energy of a particle along its trajectory: $E = \frac{v^2}{2} + \varphi(x,t)$

$$\left.\frac{dE}{dt}\right|_{\substack{\text{along}\\ \text{trajectory}}} = \frac{1}{2}\frac{dv^2}{dt} + \frac{d\varphi}{dt}$$

but

$$\frac{d\varphi}{dt} = \frac{\partial\varphi}{\partial t} + \frac{\partial\varphi}{\partial x}\frac{dx}{dt} + \frac{\partial\varphi}{\partial y}\frac{dy}{dt} + \frac{\partial\varphi}{\partial z}\frac{dz}{dt} \equiv \frac{\partial\varphi}{\partial t} + (\vec{v}\cdot\vec{\nabla})\varphi$$

and

$$\frac{1}{2}\frac{dv^2}{dt} = \vec{v}\cdot\frac{d\vec{v}}{dt} = -(\vec{v}\cdot\nabla)\vec{\varphi}$$

Here we used eq. of motion: $\frac{d\vec{v}}{dt} = -\nabla\varphi$

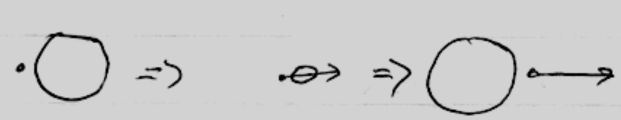
Thus, $\frac{dE}{dt} = \frac{\partial\varphi}{\partial t}$

$$\boxed{\left.\Delta E\right|_{\substack{\text{along}\\ \text{trajectory}}} = \int_{t_0}^{t} dt\cdot\frac{\partial\varphi}{\partial t}}$$

We cannot take this integral, because we do not know how φ changes with time. At early stages of evolution we expect that grav. potential changes significantly over short period of time. The shortest time is the dynamical time. Thus $\Delta\varphi \sim \varphi$ for $\Delta t \simeq t_{dynamical}$

Thus, our expectations: $\Delta E \simeq \int_0^{t_{dyn}} dt\,\frac{\Delta\varphi}{\Delta t} \sim \varphi \sim E$

Toy model for violent relaxation:



Particles, which come to the center after collapse, gain energy.

Particles, which cross the central region earlier, lose energy

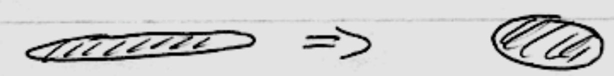
Properties of violent relaxation

$\Rightarrow$ $t_{\text{viol. rel.}} \simeq t_{dyn}$

$\Rightarrow$ no dependency on particle mass:

$$\langle v_1^2 \rangle = \langle v_2^2 \rangle \text{ for } m_1 \neq m_2$$

$\Rightarrow$ anisotropy is significantly reduced:



$\Rightarrow$ a fraction of particles may escape

$\Rightarrow$ Once the system reaches a quasi-static state, $\varphi = \varphi(x)$, violent relaxation shuts off. As such, it has a tendency to be incomplete. For example, initially anisotropic system will reduce anisotropy, but it will not become round.

$\Rightarrow$ redistribution of energy between particles is typical for violent relaxation: