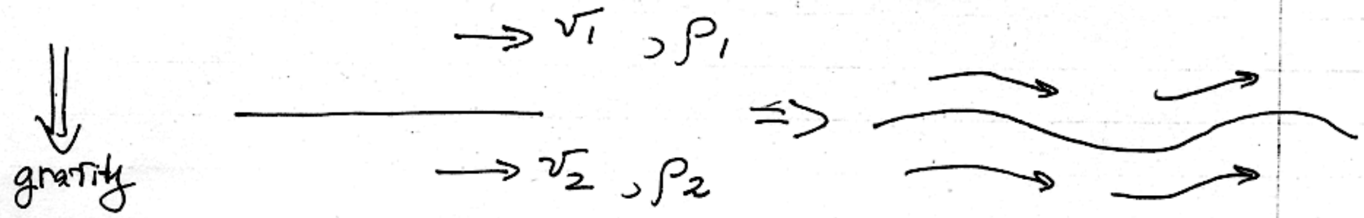
Kelvin-Helmholtz instability

Kelvin-Helmholtz instability: stability of a boundary between two fluids moving one relative to another in a gravitational field

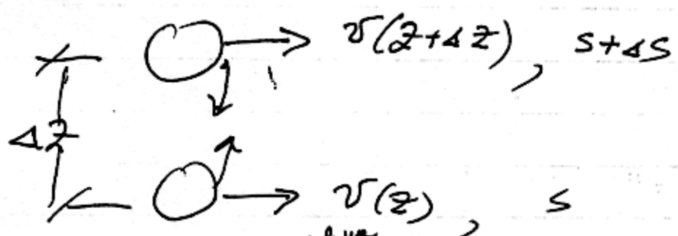


if $\rho_1 < \rho_2$, there is no Taylor instability, but if $v_1 - v_2$ is too large, the flow is unstable

The system is stable in the sense of R-T instability:

$$\frac{ds}{dz} > 0$$

This means, that one needs to do a work dW to bring two blobs from different hights



the force per unit volume to move a blob is $g\left(\rho\big|_{blob} - \rho\big|_{gas}\right) = -g\left(\frac{\partial\rho}{\partial s}\right)_p \Delta s$

Thus, per unit mass the work is

$$dW = -\frac{g}{\rho}\frac{\partial\rho}{\partial s}\Delta s\,\Delta z$$

At the same time, by mixing those two blobs, one releases shear energy of the flow:

$$dE_{kin} = \frac{1}{2}\left(v^2 + (v+\Delta v)^2\right) - \frac{1}{2}\left[2\left(v+\frac{\Delta v}{2}\right)^2\right] = \frac{1}{4}\Delta v^2$$

kin. energy per unit mass

Thus the condition for stability: difference in shear energy is smaller than the energy needed to displace the blobs;

$$\frac{-g\left(\partial\rho/\partial s\right)_p \frac{ds}{dz}}{\rho\left(\frac{dv}{dz}\right)^2} > 1/4$$

$$\left(\frac{\partial\rho}{\partial s} < 0\right)$$

$$\left.\frac{\partial\rho}{\partial s}\right|_p = \frac{\left(\partial\rho/\partial T\right)_p}{\left(\partial s/\partial T\right)_p} = \frac{T}{c_p}\left(\frac{\partial\rho}{\partial T}\right)_p < 0$$

Conclusions: - entropy gradient stabilizes the instability
- gravity helps to prevent the instability

For a sharp discontinuity the condition for instability:

$$(v_2 - v_1)^2 > \frac{2(\rho_2 + \rho_1)}{\rho_2 \rho_1}\left[T g (\rho_1 - \rho_2)\right]^{1/2}$$

T = surface tension

For air and water the difference in velocity is

$$650 \ \frac{cm}{sec}$$