Dynamical Friction

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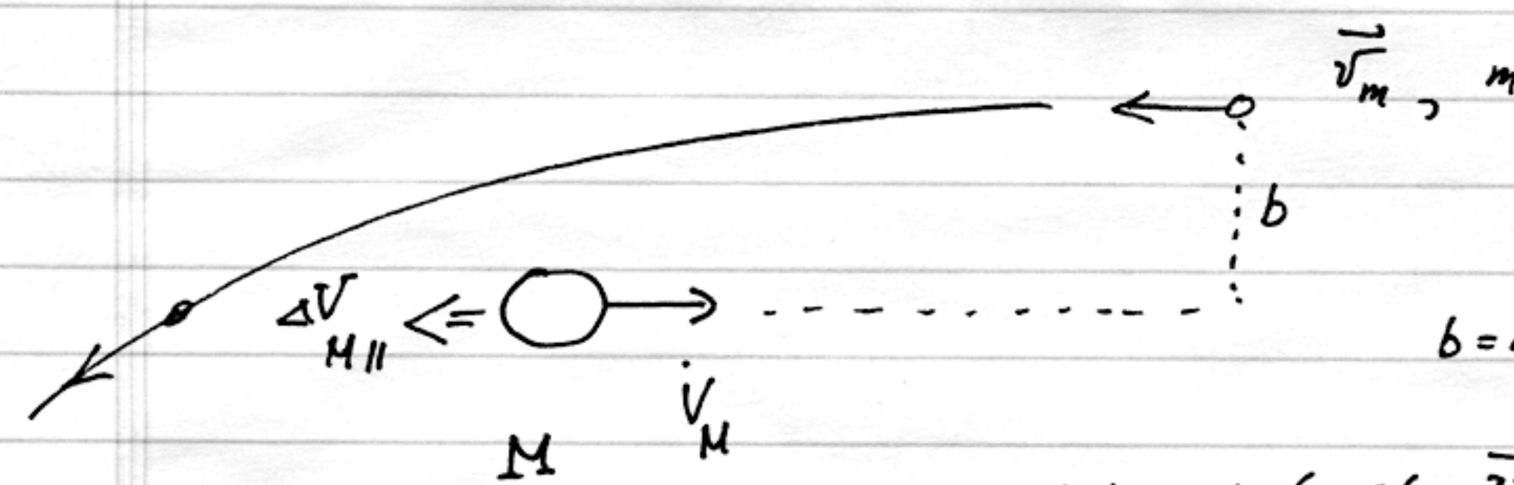
Object with mass M moves through a field of small objects. Mass of each small object is m



b = impact parameter

Relative velocity $\vec{V} = \vec{v}_m - \vec{v}_M$

$$b_{min} = \frac{G(m+M)}{V^2}$$

Result of a single collision gives

$$|\Delta V_{M\parallel}| = \frac{2m}{m+M} \cdot V \cdot \frac{1}{1+\left(\frac{b}{b_{min}}\right)^2}$$

Cumulative effect of many collisions is a sum of individual contributions:

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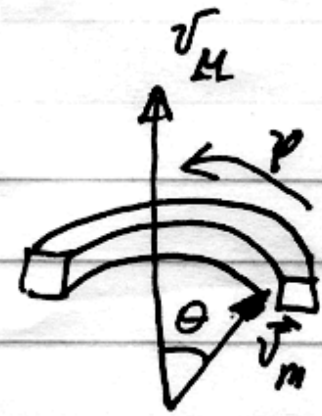
the number of collisions in dt is:

$$2\pi b\, db\, f(\vec{v}_m)\, d\vec{v}_m\, V dt$$

$$\frac{d\vec{v}_M}{dt} = \int d\vec{v}_m \int 2\pi b\, db\, f(\vec{v}_m)\, \frac{2m}{m+M} \cdot \frac{(\vec{v}_m - \vec{v}_M) V}{1 + \left(\frac{b}{b_{min}}\right)^2}$$

The integral over $\int db (\ldots)$ gives: $\ln\Lambda$

$$\frac{d\vec{v}_M}{dt} = 4\pi G^2 m(m+M) \int d\vec{v}_m\, f(\vec{v}_m)\, \ln\Lambda\, \frac{(\vec{v}_m - \vec{v}_M)}{|\vec{v}_m - \vec{v}_M|^3}$$



Integral over angles θ and φ between vector $\vec{v}_M$ and $\vec{v}_m$ gives dependence on $|\vec{v}_m|$

$$\frac{d\vec{v}_M}{dt} = -16\pi^2 G^2 m(m+M)\ln\Lambda \cdot \vec{v}_M \cdot \Psi(v_M, v_m)$$

where

$$\Psi(v_M, v_m) = \frac{\int_0^{v_M} f(v_m) v_m^2 dv_m}{v_M^3}$$

For large v_M (as compared with v_m) and for Maxwellian distribution of velocities

$$f(v_m) = \frac{n_0}{[2\pi\sigma_v^2]^{3/2}} \exp\left(-\frac{v_m^2}{2\sigma_v^2}\right)$$

we get

$$\frac{d\vec{v}_M}{dt} = -\frac{4\pi G^2 m(m+M) n_0}{v_M^3} \cdot \vec{v}_M \left[\mathrm{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2}\right]$$

Here $x \equiv \frac{v_M}{\sqrt{2}\sigma_v}$ and $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$

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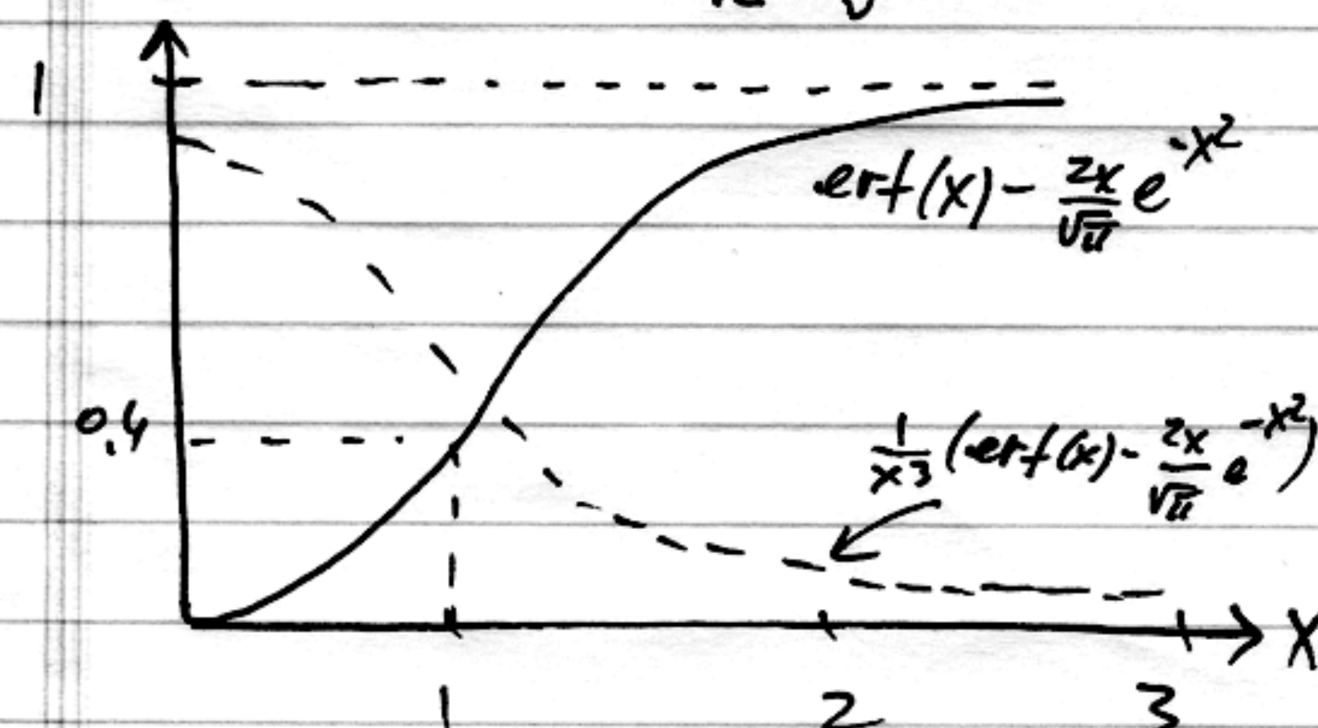


$\operatorname{erf}(0) = 0$

$\operatorname{erf}(\infty) = 1$

$\operatorname{erf}(x) \simeq \frac{2x}{\sqrt{\pi}}$ for $x \ll 1$

$\operatorname{erf}(x) \simeq 1 - \frac{e^{-x^2}}{x\sqrt{\pi}}$, $x \gg 1$

⇒ For large v_M and $M \gg m$ the effect scales as

$$\frac{dv_M}{dt} \propto \frac{\rho M}{v_M^2} \quad , \text{ where } \rho = n_0 m = \text{density of small objects}$$

⇒ For quasi circular orbit in system with $\rho \propto r^{-2}$

$$t_{fric} \simeq \frac{1.17\, r_{init}^2\, v_{circ}}{G M \ln\Lambda} \simeq$$

$$\simeq \frac{2.64^{11}\,\text{yrs}}{\ln\Lambda} \left(\frac{r_{init}}{2\,\text{kpc}}\right)^2 \left(\frac{v_{circ}}{250\,\text{km/s}}\right) \left(\frac{10^6 M_\odot}{M}\right)$$

⇒ For non-circular orbits the eccentricity of the orbit does not decrease with time

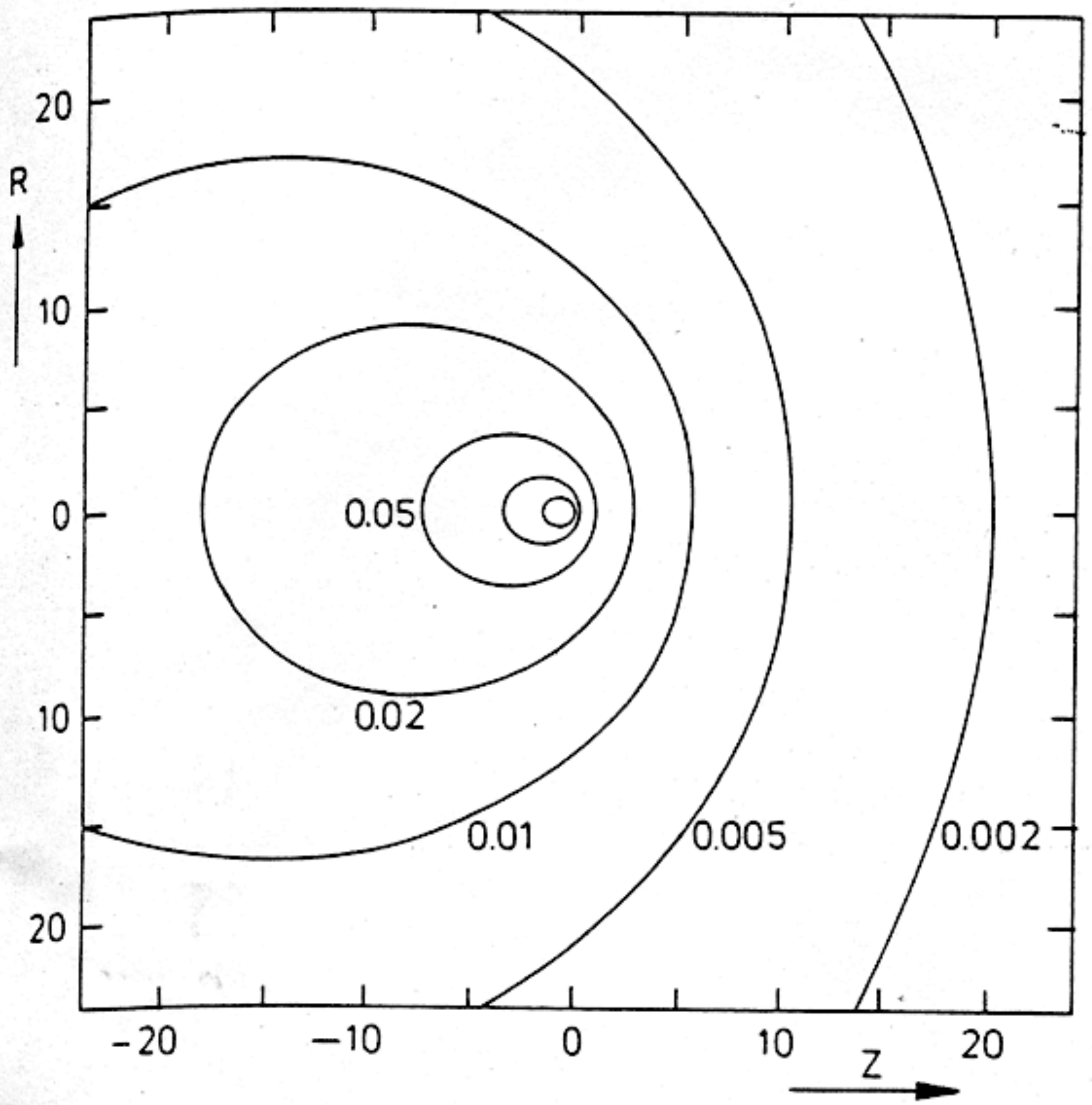


Figure 7-3. A mass tra
from left to right at spee
v through a homogeneou
Maxwellian distribution o
with one-dimensional disp
$\sigma = v$. Deflection of the
by the mass enhances the
density downstream more
upstream. Contours of ec
stellar density are labeled
the corresponding fractior
density enhancement. (Fr
Mulder 1983.)

$$\frac{d\mathbf{v}_M}{dt} = -\frac{4\pi \ln \Lambda G^2 \rho M}{v_M^3} \left[\mathrm{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2} \right] \mathbf{v}_M$$

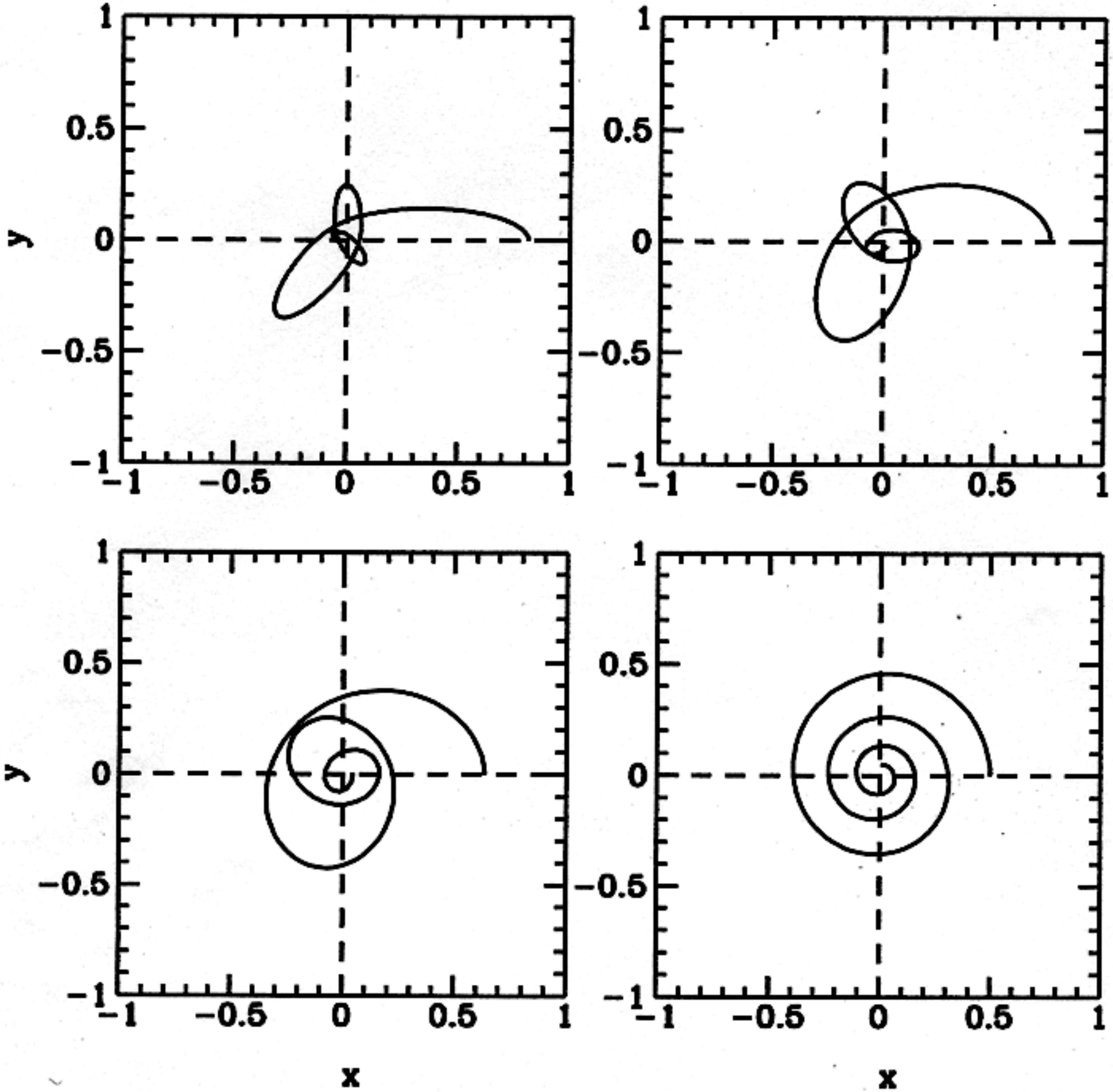
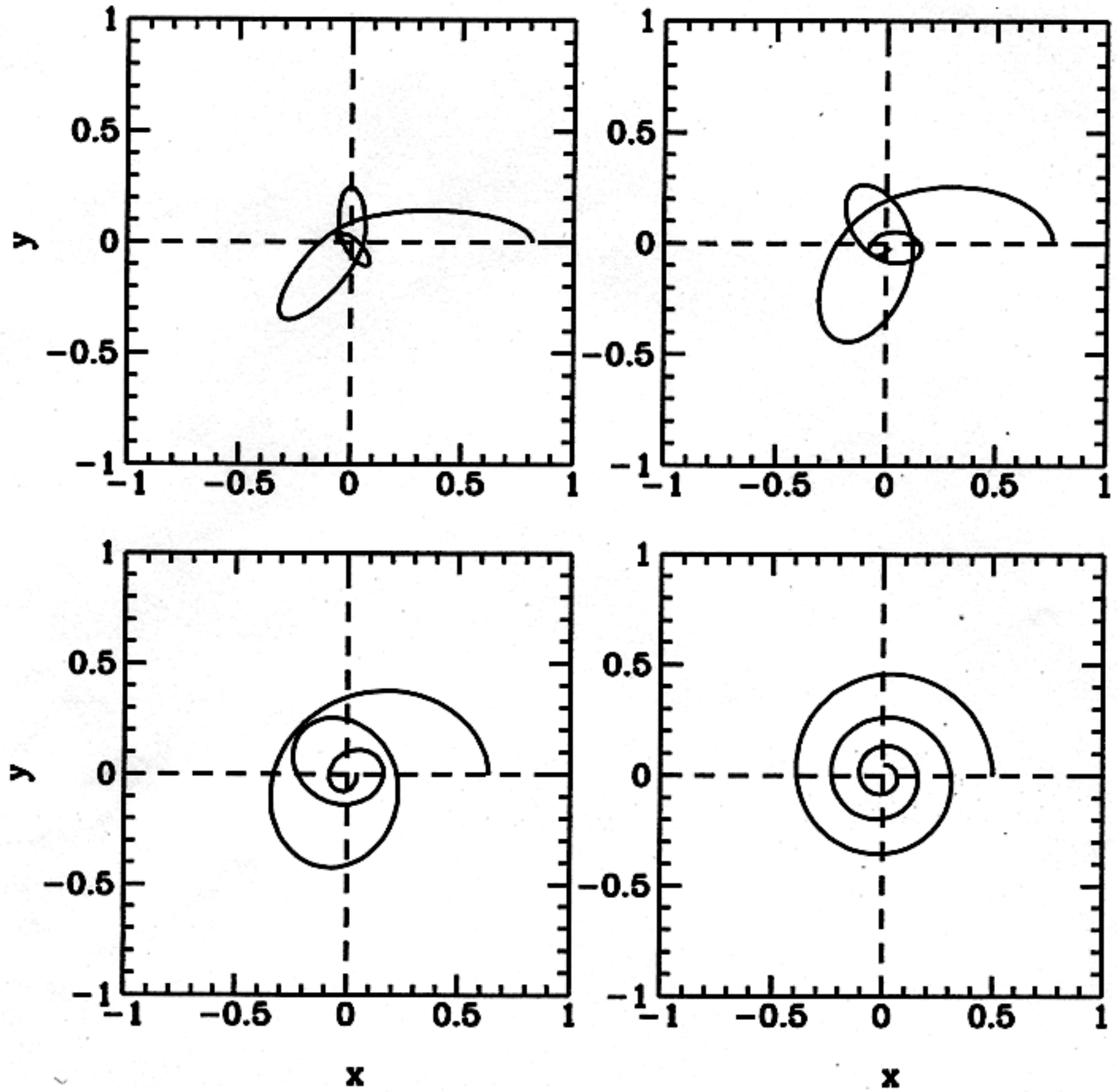


FIG. 1.—Collection of orbits in the plane (*x*, *y*) computed within TLR, for $r_{cir}/r_t = 0.5$ and initial eccentricities $e_{orb} = 0.8$ (*top left*), 0.6 (*top right*), 0.3 (*bottom left*), and 0 (*bottom right*). Length is in units of r_t.



—Collection of orbits in the plane (x, y) computed within TLR, for $r_{cir}/r_t = 0.5$ and initial eccentricities $e_{orb} = 0.8$ (*top left*), 0.6 (*top right*), 0.3 t), and 0 (*bottom right*). Length is in units of r_t.

$$\tau_{DF} = T_c\, \varepsilon^{0.78} \equiv 1.17\, \frac{r_{cir}^2 V_{cir}}{GM \ln(Nm/M)}\, \varepsilon^{0.78}$$

ε is orbital eccentricity