Convection

Fluid Instabilities

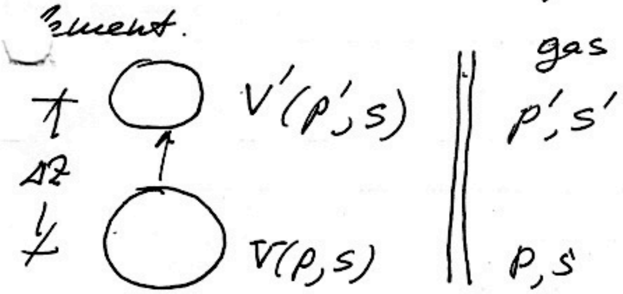
1) Convection
2) Rayleigh-Taylor
3) Kelvin-Helmholtz
4) Thermal

hydrostatics: $\boxed{\text{grad } p = \rho \vec{g}}$, $\vec{g}$ = grav. acceleration

Is it stable?
Let's assume, that all thermodynamical parameters depend only on one coordinate z (say, distance to the center of a star)

$p = p(z)$, $s = s(z)$, $V = V(z)$ s = entropy, V = specific volume

Consider an element of fluid, which is displaced from z to $z+dz$. The process is adiabatic – no energy exchange. The element is always in hydrodynamical equilibrium with surrounding fluid: its pressure is the same as the pressure of gas outside the element.



Stability condition: after the displacement the density of the volume element is larger than the density of gas.

$$V(p', s) < V(p', s')$$

Taylor expansion:

$$s' - s \simeq \frac{ds}{dz}\Delta z$$

$$V(p', s') = V(p', s) + \left(\frac{\partial V}{\partial s}\right)_p \frac{ds}{dz}\Delta z \Rightarrow \left(\frac{\partial V}{\partial s}\right)_p \frac{ds}{dz} > 0$$

Thermodynamics: $\left(\frac{\partial V}{\partial s}\right)_p = \frac{T}{c_p}\left(\frac{\partial V}{\partial T}\right)_p > 0 \Rightarrow \boxed{\frac{ds}{dz} > 0}$ Condition for absence of convection

For ideal gas this can be reduced to a condition for temperature gradient:

$$-\frac{dT}{dz} < g/c_p$$

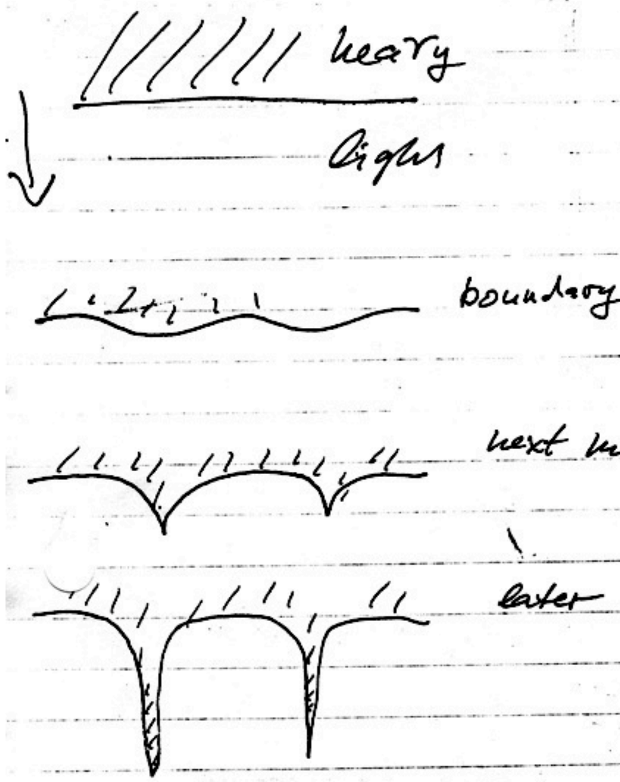
for stars: "radiative" temperature gradient

$$\left.\frac{dT}{dr}\right|_{rad} = \frac{L(r)}{r^2 T^3} \frac{3 \varkappa \rho}{4^3 \pi \sigma}$$

$\varkappa$ = opacity
σ = Stefan Boltz. const

if real temp. gradient $\left.\frac{dT}{dr}\right|_{ad}$ is larger than radiative gradient, the star is unstable: $\left.\frac{dT}{dr}\right|_{ad} > \left.\frac{dT}{dr}\right|_{rad}$ (instability)

<u>Rayleigh-Taylor instability</u>: heavy fluid atop of light one in presence of gravity g



Example: shock waves sweeps gas. Once the shock is exhausted, it leaves a low density gas around the star with dense shell outside

physically, R-T instability is the same as convective instability

<u>Convection</u> will tend to reduce the entropy gradient and temperature gradient. If convection takes place,

$$\frac{ds}{dz} = 0$$

<u>Convection</u> can be <u>laminar</u> or <u>turbulent</u> (The transition is defined by Rayleigh number, a combination, which includes typical temperature gradient, typical hight scale, viscosity and thermal conductivity coefficients)

$$R = \frac{\alpha g l^3 \Delta T}{\varkappa \nu}$$

$\frac{\delta\rho}{\rho} = -\alpha \frac{\delta T}{T}$

ΔT over l

$\varkappa$: thermal conduction

$\nu = \eta/\rho$: kinematic viscosity