Circular velocity, dynamical time, free-fall time, crossing time.

Circular velocity is defined by: $\frac{v_c^2}{r} = g$,

where $g(r) = \frac{GM(r)}{r^2}$, $M(r) = \int_0^r 4\pi\rho(r)\, r^2 dr$

Dynamical time: First consider a homogeneous sphere with constant density: $\rho = \text{const}$. Take a particle at rest at distance r and drop it:

$$\frac{d^2 r}{dt^2} = -\frac{GM(r)}{r^2} = -\frac{4\pi G}{3}\rho r$$

This is the equation for a harmonic oscillator with frequency ω: $\omega^2 = \frac{4\pi}{3} G\rho$. The period of oscillation is equal to $T = \frac{2\pi}{\omega} = \sqrt{\frac{3\pi}{G\rho}}$.

Dynamical time is defined as $\boxed{T_{dyn} \equiv \frac{T}{4} = \sqrt{\frac{3\pi}{16 G\rho}}.}$

Note that for a homogeneous sphere T_{dyn} does not depend on initial radius. This is not the case for general distribution of mass. We treat T_{dyn} as a scale for motion of particles.

Free-fall time: Let the sphere collapse.

$$\| \, t_{free\text{-}fall} = \frac{t_{dyn}}{\sqrt{2}} \, \|$$

Crossing time: If R is a typical size of a system and v is typical velocity, then $\left| t_{cross} = \frac{R}{v} \right|$